

NETWORK AND ROUTE OPTIMIZATION IN WIRELESS SENSOR NETWORKS

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by

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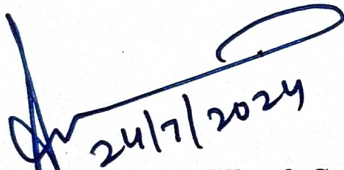
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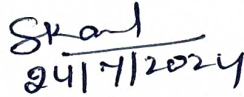
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CERTIFICATE

Certified that **Ms. Radhika Kavra** (2K19/PHD/AM/03), has carried out her research work presented in this thesis entitled “**Network and route optimization in wireless sensor networks**” for the award of **Doctor of Philosophy in Mathematics** from Delhi Technological University, New Delhi, under our supervision. The thesis embodies results of original work and studies are carried out by the student herself and the contents of the thesis do not form the basis for the award of any other degree to the candidate or to anybody else from this or any other University/Institution.



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A handwritten signature in black ink, appearing to read 'Radhika', with a horizontal line underneath.

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Abstract

Wireless sensor network (WSN) is a new class of wireless networks that are made up of tiny sensor nodes ranging from tens to thousands in number which are meant to monitor physical or environmental conditions by establishing connection with each other to exchange information and data. These sensor nodes obtain information on the environment such as temperature, pressure, humidity or pollutant, and send this information to a base station. The latter sends the information to a wired network or activates an alarm or an action, depending on the type and magnitude of data monitored. This characteristic of wireless sensors makes them very popular and trendy with a huge number of civilian and military applications in the present world.

Another characteristic of wireless sensor nodes is that they are power constraint as the only source for their energy is non-rechargeable battery. Once the power of nodes gets drained they would not be able to perform any task in the network and so the network functionality gets broken. Therefore issues related to power conservation of nodes, redundant power consumption, network lifetime etc have been tried to resolve since past many years under different factors and still in progress.

The work in the thesis has been greatly motivated by the existing models and optimization techniques, introduced in the field of WSNs to deal with the network issues. We have addressed more issues/challenges and introduced new graph theoretic models along with new algorithmic approaches of lesser time complexity to provide the optimal solution to many problems. This thesis provides good amount of analysis on nodes' energy consumption and network disconnectivity conducted via simulation work on different scenarios possible in WSNs.

We have compiled the work of the thesis in 6 chapters where chapter 1 is introductory in nature. Chapter 2-6 provide the study on different problems and requirements of WSNs and put forth satisfactory solutions to the problems along with analysis. Furthermore chapters 1, 2, 5 and 6 are based on the research work published in the

form of research articles in reputed refereed journals and rest of the chapters are communicated research papers.

After chapter 6, we have presented the summary of the research work carried out in the thesis. In addition, the future scope of the thesis has been discussed briefly. Finally the thesis is ended with the bibliography and list of publications.

Dedicated
to
My Family

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Chapter 1

Introduction

The facilitation of low-cost multi-functional battery-powered wireless sensors that are small in size and communicate untethered using radio signals brought us to wireless sensor networks (WSNs). WSNs have gained worldwide attention in recent years due to emergence of its application in various field of real life. Despite of its growth since several years, issues related to power consumption of power constraint wireless sensors, connectivity of the network, network lifetime and many more have always been challenging. These pertaining issues and challenges give rise to many optimization problems in WSNs at topology as well as routing level and so many optimization techniques have been developed along with many models. The present introductory chapter¹ initially provides the overview of WSNs, wireless sensors and trending applications of WSNs. Then the chapter provides brief understanding on various issues as well as challenges of WSNs and optimization problems in WSNs. The chapter explicitly discusses existing optimization techniques at both the topology and routing level in WSNs along with research gaps and limitations. Henceforth, the fundamentals of this chapter are proficient in providing a platform to motivate the research work carried out in the thesis.

¹The content of this chapter is based on a review paper “Systematic study of topology control methods and routing techniques in wireless sensor networks”, *Peer-to-Peer Networking and Applications*, **15(4)**, 1862-1922 (2022). <https://doi.org/10.1007/s12083-022-01325-4> (SCIE, **Impact Factor: 4.2**)

1. Wireless sensor network (WSN)

Wireless sensor network (WSN) can be defined as an infrastructure-less network, made up of few tens to thousands of battery-powered omnidirectional wireless sensors. These wireless sensors work together to monitor a region to obtain data about the environment and cooperatively pass their data through the network to a main location (sink or base station) where the data can be observed and analysed. A sink or base station acts like an interface between users and the network. One can retrieve required information from the network by injecting queries and gathering results from the sink [5]. An illustration of a WSN is shown in Fig. 1.1.

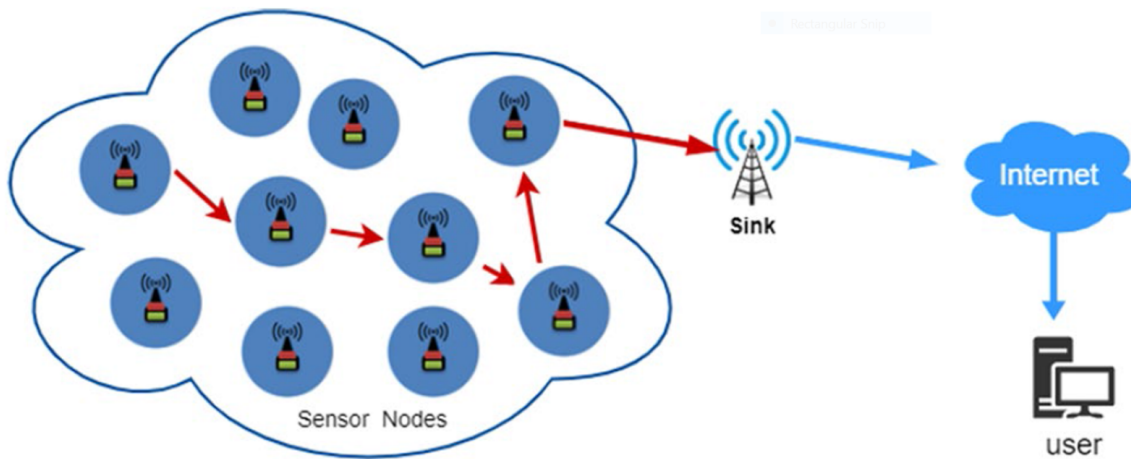


Figure 1.1: An arbitrary structure of a WSN

1.1 Wireless sensors

Wireless sensors are tiny sensor nodes, equipped with sensing and computing devices, radio transceivers and power components which leverage the idea of WSNs based on collaborative effort of large number of sensor nodes. Each sensor node consists of three subsystems: sensing, processing and transmission (see Fig. 1.2 redrawn from [6]). A sensing subsystem has transducers in it to collect information about the environment. A processing subsystem has an embedded processor to process local sensed information and transmission subsystem has a transceiver embedded in it to exchange sensed/processed information with external systems (base station). Some sensor nodes have two additional units for location finding and mobility so that

they can carry out the tasks assigned to them [7]. This way, sensors can be static or mobile depending on the application goals.

When sensors are deployed in a region, they are responsible for self-organizing an

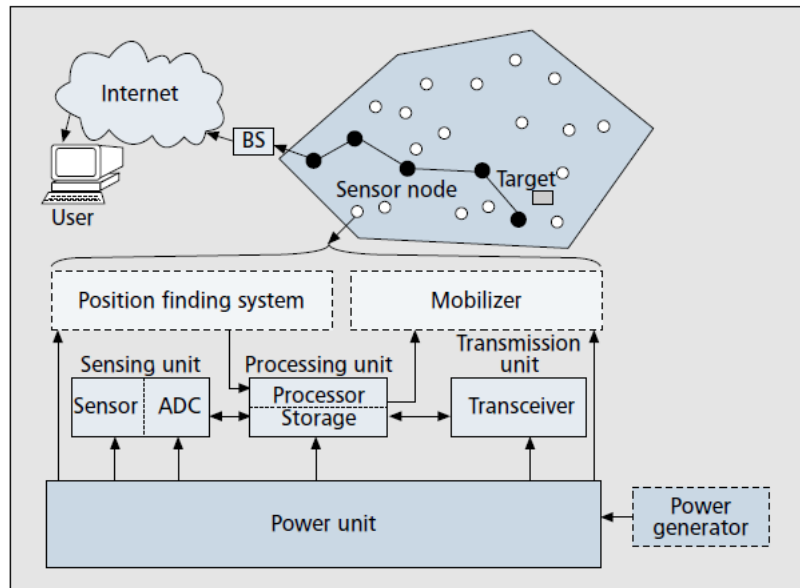


Figure 1.2: Components of wireless sensors

appropriate network infrastructure, called topology and often start multi-hop communication among themselves, called routing.

Each individual node in WSN is inherently resource constrained i.e. it has limited processing speed, limited energy, restricted communication bandwidth and storage capacity. Therefore more is the power expenditure of a node earlier it will die in the network. It is because the only source of power for wireless sensors is battery which is usually not rechargeable or irreplaceable especially when sensor nodes are expected to operate with no human intervention for a longer period of time.

Despite of the limitation of sensor nodes, these have been widely engaged in our everyday life activities and bringing out many applications of WSNs. It is because wireless sensors can be densely deployed in any region to communicate and transmit to longer distance using radio signals in lesser time as compared to traditional wired networks.

1.2 Application of WSNs

Wireless sensors are greatly engaged in sensing, tracking and monitoring tasks like indoor and outdoor environment sensing, health and well-ness monitoring, power monitoring, inventory location monitoring, military surveillance, tracking objects, ani-

mals, humans and vehicles etc [8]. They have been very useful in our day-to-day life as they offer cctv surveillance kind luxury to us, event and weather prediction, military security and like so on as mentioned in Fig. 1.3. Though all applications of WSNs are connected to real-life activities yet some of them need to very realistic and fast responded to the events like environment sensing, bomb detection, country border security etc so they are called real-time application of WSNs.

A brief on few applications of WSNs is following:

Military application: WSN is an integral part of military command, control, commu-

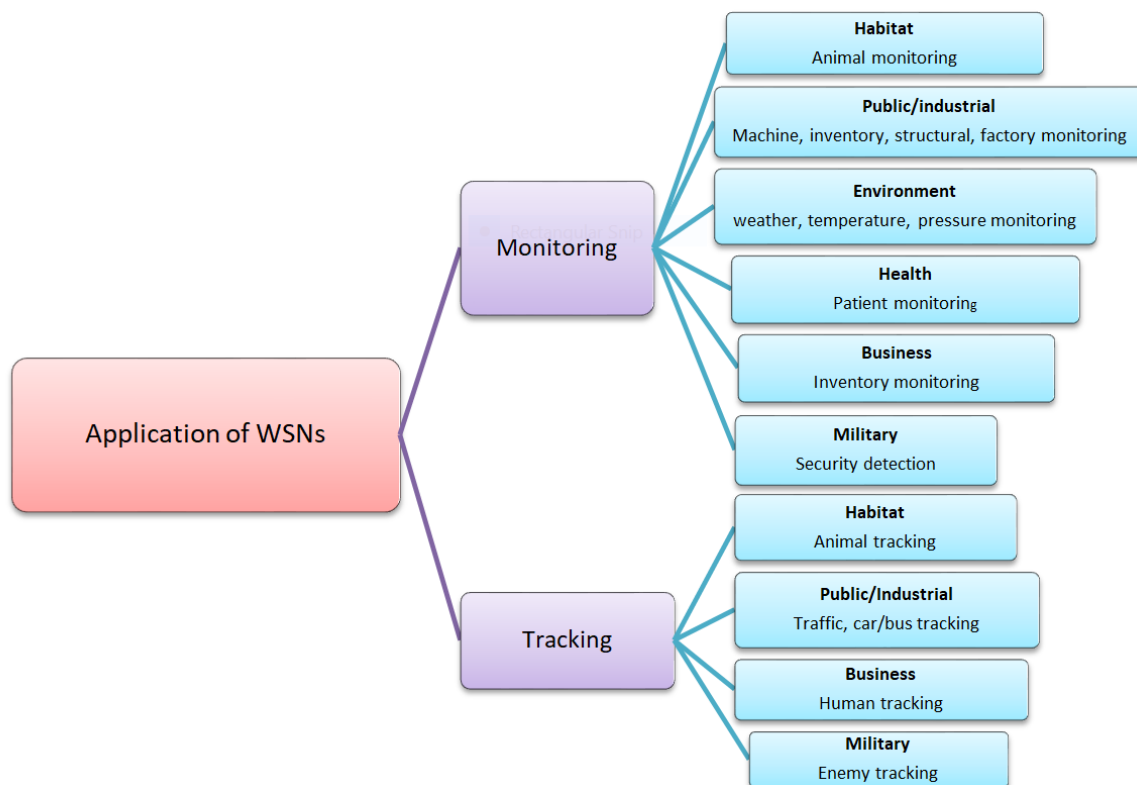


Figure 1.3: Applications of WSNs

nications, computing, intelligence, battlefield surveillance, reconnaissance and targeting.

Area monitoring: In area monitoring, sensor nodes are deployed over a region where some phenomenon are to be monitored. When sensors detect an event being monitored (heat, pressure etc), then event is reported to one of the base stations, which then takes appropriate action.

Transportation monitoring: Real-time traffic information is being collected by WSNs to later feed transportation models and alert drivers of congestion and traffic problems.

Environment sensing: This includes sensing volcanoes, oceans, glaciers, forests

etc.

Health application: Some of the health applications for sensor networks are supporting interfaces for the disabled, integrated patient monitoring, diagnostics, and drug administration in hospitals, telemonitoring of human physiological data, tracking and monitoring doctors or patients inside a hospital.

Though sensor nodes have been used for continuous sensing, event detection, event ID, location sensing and local control of actuators for past many years yet deployment of these nodes to target area put-forth many issues/challenges which have been discussed in next section.

1.3 Challenges in WSNs

Engagement of wireless sensors in various application fields retain lots of limitations and challenges related to topology as well as routing in WSNs like connectivity, transmission loss, network lifetime, energy consumption, fault-tolerance, transmission delay and many more. This is because of limited power storage of sensor nodes and once their power get depleted, they would no more able to perform their activity in the network which would cause several issues as mentioned in Fig. 1.4 [9, 10]. Some of the issues like connectivity, coverage, node failure, energy consumption, scalability, fault-tolerance etc can be dealt at topology construction phase or underlying network acknowledgment phase of WSNs, these challenges are known as ‘topology control (TC) issues’ and the issues which are occurred and tackled only at routing phase like transmission delay, transmission cost, reliability, throughput, load balancing etc are called ‘routing issues’.

Providing brief detail on some of the challenges of WSNs:

- **Topology control (TC):** TC in WSNs is the problem of determining an appropriate topology or network structure with different topological goals like minimum interference and energy consumption with increment in network capacity like network lifetime, energy efficiency, scalability while maintaining the network connectivity and/or coverage. TC is the initial stage for any topology aware routing.

- **Energy consumption:**

Energy consumption comprises mainly of 3 tasks in the network:

1. Each node consumes energy to process/decode the received coded data.
2. To transfer data successfully to another node, each node needs sufficient

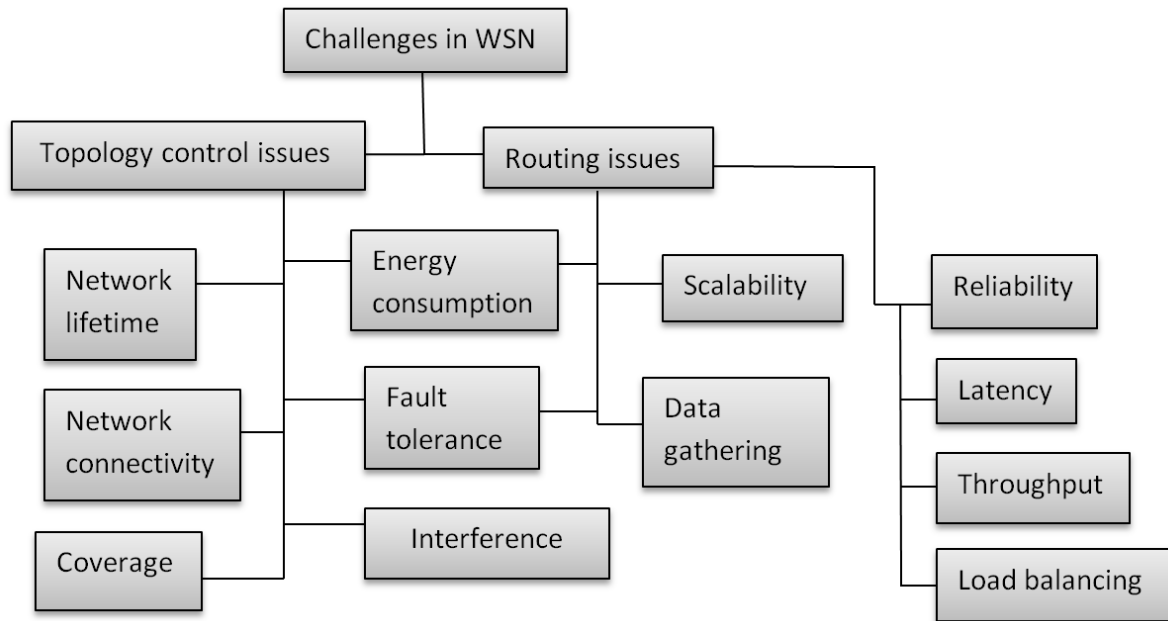


Figure 1.4: Topology control and routing challenges in WSNs

power to compensate the path loss of signal along its way.

3. Then energy is expended by receiver node to process the signal and acknowledge back to its source node.

If any topology or a route from source node to its receiver node consumes more energy, it would never be an energy efficient and/or energy optimal. So energy conservation via minimization of node's energy consumption has been a prime requirement to have an energy efficient and/or energy optimal topology or routing in WSNs.

- **Network connectivity:** Connectivity refers to wireless approach of each sensor to atleast another sensor using its power to maintain the network connectivity. Loss of connectivity causes loss of sensors, data and misroute of forwarding data.
- **Network lifetime:** It is measured by the lifetime of sensor who dies first in the network due to early power drainage. Death of a single sensor may disturb the whole network functioning as it may lead to transmission loss and disconnectivity.
- **Interference:** It is a kind of hindrance or obstacle existing in the topology between the nodes due to collision of data packets transmitting over the same route by multiple nodes and hence retransmission of data packets is required.

This leads to more energy consumption. Also more dense the network, more is interference.

- **Load balancing:** Some of the nodes are loaded with more data to process and transmit than others leading to fast energy depletion hence nodes get failed, lost network connectivity and network lifetime. So load balancing refers to uniform distribution of sensing and transmission loads between the nodes for a uniform power consumption to take place.
- **Coverage:** Coverage refers to an area covered by a sensor to sense and process the information. A network needs to have an optimal coverage i.e. minimum number of sensors are deployed to cover the whole target region. Lack of coverage in any part of target region causes energy hole in the topology i.e. no information is being collected and forwarded from that part of region due to loss of coverage.
- **Data gathering:** Multiple nodes gather data from a same source or node. This causes unnecessarily data collection and transmission therefore it must be done in such a way that reduces data redundancy and data retransmission so that nodes energy can be conserved.
- **Energy hole or Hot-spot problem:** Nodes near sink, have to spend extra power in data aggregation and transmission. So they die very early and their space leave energy hole or hot-spot issue in the network which degrades routing process. Henceforth nodes with more power consumption bring energy holes on their places if they get died in the network.
- **Latency/delay:** Latency refers to an end-to-end delay in data delivery, also called routing delay. Delay at topology level where transmission delay between each pair of sensor nodes have been considered, is called topology delay. Real-time applications of WSNs like military surveillance, environment sensing, require least delay at both the routing and topology level.
- **Scalability:** For applications where tens to thousands of sensor nodes are deployed in a target area, routing protocols should be designed in a way that they will be able to handle and respond to such greater number of events or addition or deletion of nodes.

- **Reliability:** Reliability of a WSN is related with a probability of successful data transmission between sender to receiver node. Reliability factor of WSNs can be enhanced by mitigating interference factor among the sensors.
- **Throughput:** Throughput refers to how much data can be transferred from source to destination node within a given time frame. Interference between two nodes adversely effects the throughput between them.
- **Packet delivery ratio:** It is a percent of data packets delivered to base station to those generated by source nodes. Network interference degrades packet delivery ratio.
- **Quality of service (QoS):** QoS is related to reliability, packet delivery ratio and throughput factor with minimum error in transmitted data.
- **Deadline miss ratio (DMR):** It refers to a fraction of data packets which don't reach destination node within a given time-frame. Both the QoS and DMR have been effected by interference and energy hole problem.
- **Fault tolerance:** It is an ability of routing protocols to respond quickly and discover new routes for transmission of data in case of nodes failure in the network. Fault tolerance is the most required factor for trendy WSNs.

It can be said that a root cause of all the challenges mentioned here is limited power storage of sensor nodes. Still WSNs which aim to minimize latency, maximize throughput or QoS, bring more power consumption of sensor nodes. This is because nodes have to exert maximum power to communicate to the destination node in least time possible. Hence this brings out a tradeoff between optimization of node's power consumption and delay in real-time applications based routing in WSNs.

As every task from topology setup to data transmission requires nodes energy that is why topology control and routing issues become challenging to handle in WSNs.

1.4 Optimization in WSNs

Limited available energy to sensor nodes has attracted the attention of many researchers to appropriately deal with energy consumption of sensor nodes, network lifetime, network connectivity, link/network interference, transmission cost, data reliability, latency, load balancing, fault tolerance etc in designing the network topolo-

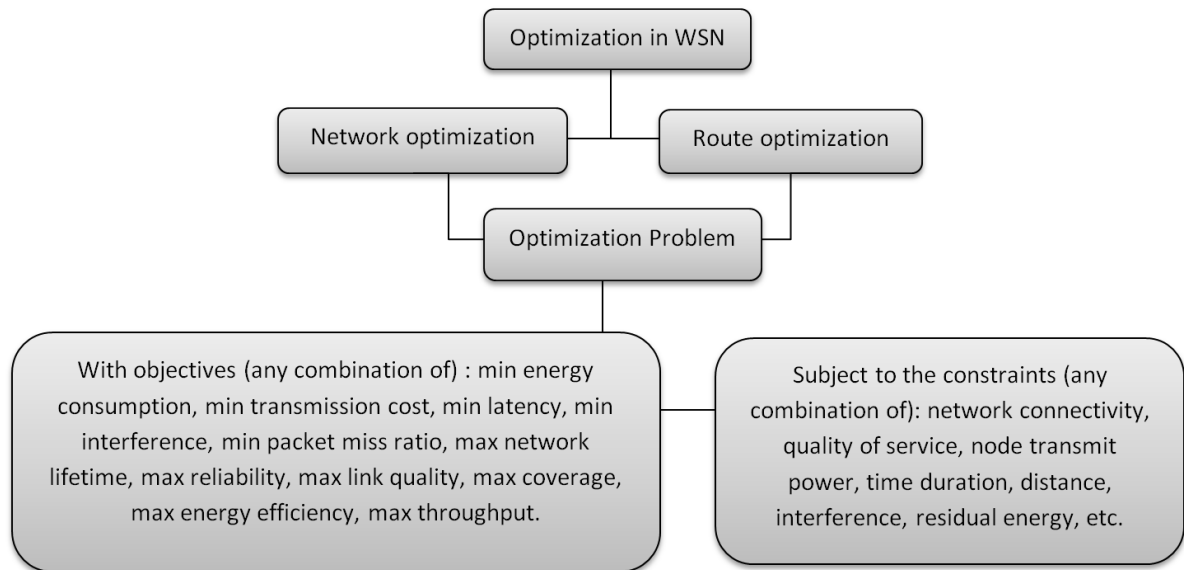


Figure 1.5: Optimization in WSNs

gy and routing protocols. These pertaining issues in WSNs give rise to many optimization problems whose objectives can be (any combination of) maximize energy efficiency, minimize nodes' transmit power or energy consumption, maximize link quality, maximize network lifetime, maximize reliability, maximize coverage, maximize throughput, minimize delay, minimize transmission cost, minimize interference, minimize transmission loss, minimize unbalanced energy consumption etc subject to the constraints (any combination of) network connectivity, transmission delay, quality of service (QoS), transmit power, distance, coverage, (static/mobile) topology etc. These objectives in WSNs bring optimization in WSNs are of 'Network' and 'Route' optimization kinds.

Network optimization is related to achieve a desired topology goals of randomly distributed sensors in a target area that minimizes energy consumption, node's transmit power, transmission cost, maximizes network lifetime subject to the constraint of network connectivity and/or coverage where as **Route optimization** aims to find an optimal routing between source sensor node to destination node in such a way that it results in lesser energy consumption, interference along with application based WSNs requirement like minimum transmission delay, maximum throughput etc.

In this respect, various optimization techniques have been developed in history. The techniques which work on network optimization are called 'topology control (TC) methods/techniques' [3, 4, 11, 12, 13, 14] whereas the techniques which work on route optimization is called 'routing techniques' [9, 10, 15, 16, 17]. Both the topology control methods and routing techniques are used to enhance WSNs performance by saving

energy of battery constraint sensor nodes in their own way, former works on underlying network/topology of WSN and later is engaged to find end-to-end routing path between source to destination node.

1.5 Preliminaries

Before going further in this chapter, we would like to introduce general terminologies, graph related models, required definitions and abbreviations.

Definition 1.1. Centralized and distributed method: A methodology which requires a master node to run and control other nodes in the network and every other node needs to take permission or informs to master node first to work further is called centralized method whereas in distributed method, each node has same authority to work and methodology can be initiated by any node independently as per the requirement.

Definition 1.2. Homogenous and Heterogenous WSN: A WSN is said to be homogeneous if all the sensors have same battery power, sensing, processing and communication capabilities. A WSN which is not homogenous is said to be heterogenous.

Definition 1.3. Static and Dynamic WSN: A WSN where each sensor node, even sink node is not allowed to move or kept fixed after deployment in the target area is called static/immobile WSN while in a dynamic or mobile WSN, nodes can move, enter or exit and if nodes are static then atleast a sink node is mobile in nature.

Definition 1.4. Graph model of a WSN $G(V, E)$: It is a graphical layout $G(V, E)$ of any WSN which can be directional/bidirectional connected graph depending on the transmission between the nodes. Each sensor node is taken as a vertex, forming a vertex set $V(G)$ and each communicating directional/bidirectional link is taken as an directed/undirected edge, forming an edge set $E(G)$. An illustration of graphical layout of a WSN $G(V, E)$ is shown in Fig. 1.6 where $|V(G)| = 16$ and $|E(G)| = 32$.

Definition 1.5. Weighted Graph model of a WSN: When each edge in a graphical layout of a WSN has been weighted with some positive quantity like interference, Euclidean distance and/or delay, depending upon the objective of the problem which is to be solved is called weighted graph model of WSN.

Definition 1.6. Transmission cost function c : A transmission cost function c is defined as $c : V(G) \times V(G) \rightarrow R^+ \cup \{\infty\}$ such that $c(u, v)$ is the power emission necessary for node u

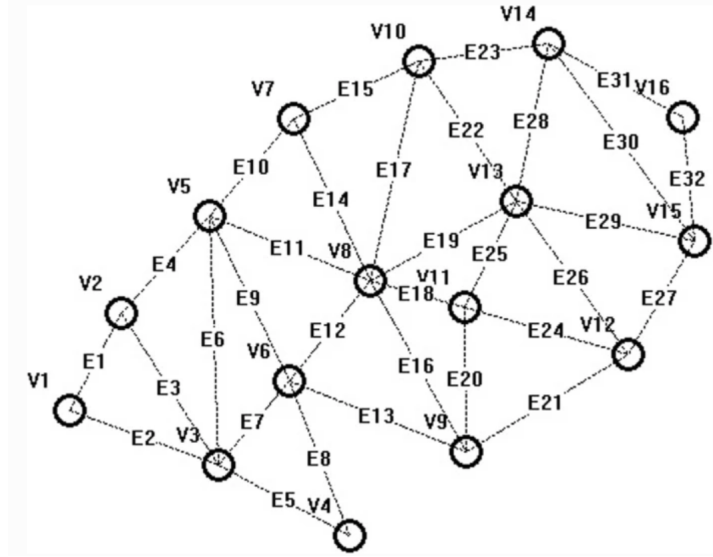


Figure 1.6: A bidirectional graph model of WSN having 16 sensors and 32 bidirectional links

to send packet to node v in a directional graph where $u, v \in V(G)$. For a bidirectional graph model of WSN, $c(u, v) = c(v, u) = \max\{c(u, v), c(v, u)\}$ can be taken symmetrically [3] and written as $c(uv)$ where $uv \in E(G)$ and uv denotes a bidirectional edge. Moreover, $c(u, v) = \infty$ implies node u cannot communicate to node v directly.

Definition 1.7. Power dissipation over a link (uv): Power required for a direct transmission from u to v is a polynomial function of the distance d between u and v i.e. $c_1 + c_2 d^\alpha$, where $d = \|uv\|$, α ($2 \leq \alpha \leq 6$) [11] is the path loss gradient depending on transmission environment, c_1 and c_2 are some constant depending on electronic characteristics of wireless devices [18]. This $c_1 + c_2 d^\alpha$ corresponds to a transmission cost on the real line of the link uv , according to definition 1.6. The exact value of c_1 , c_2 and α are not known, only the Euclidean distance between the nodes can be computed therefore the transmission cost $c(uv)$ is approximated by the value $\|uv\|^2$ for a bidirectional link uv [3].

Definition 1.8. Power function r : A power function $r : V(G) \rightarrow R^+$ is used to assign sufficiently high transmission power to a node $u \in V(G)$ i.e. $r(u) = \max_{uv \in E(G)} c(uv)$. The assigned transmission power to each node is called transmit power or a power level of a node [11].

Definition 1.9. Transmission range: When a sensor node say u has been assigned with a power level $r(u)$ then u will be able to sense and communicate directly till $r(u)$ distance so $r(u)$ is said to be a transmission range of node u .

Definition 1.10. Transmission disc $D(u, r_u)$ of node u : A circular disc $D(u, r_u)$ is assumed to exist, taking node u as a centre and $r_u = r(u)$ as a radius. Every other node which comes inside this disc is known as direct-neighbour of node u .

Definition 1.11. Coverage of a node u via link uv : Coverage of a node u via link uv is the number of nodes covered by disc $D(u, ||uv||)$ i.e. $cov_{uv}(u) = \{w | w \in D(u, ||uv||)\}$. Coverage illustration is given Fig. 1.7, redrawn from [3].

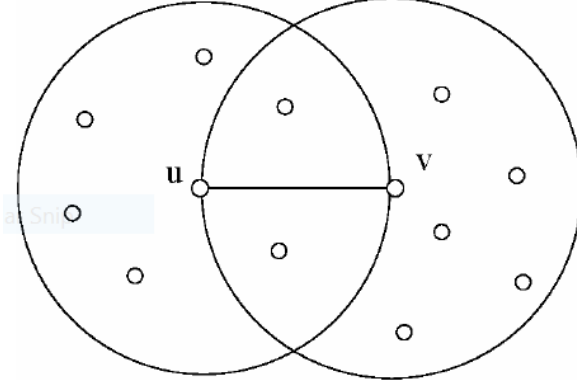


Figure 1.7: Coverage of a link uv

Definition 1.12. Coverage based interference of a link uv : Panda and Shetty in [3] have introduced two link interference models based on coverage: 1) MAX interference model (I_{MAX}) has been given by Eq. (1.1) and 2) SUM interference model (I_{SUM}) has been given by Eq. (1.2). Link interference computation by I_{MAX} and I_{SUM} models on bidirectional link uv given in Fig. 1.7 are 7 and 13 respectively.

$$I_{MAX}(uv) = \max\{cov_{uv}(u), cov_{vu}(v)\} \quad (1.1)$$

$$I_{SUM}(uv) = \{cov_{uv}(u) + cov_{vu}(v)\} \quad (1.2)$$

Definition 1.13. Node interference via link coverage: The interference of a node u via link is the maximum of coverage based interference of all links incident on u i.e. $IC(u) = \max_{uv \in E(G)} IC(uv)$.

Definition 1.14. Sender interference (I_S) model: This model gives the measurement of interference exists on each sender node u by counting nodes other than u which belongs to the transmission disc of u . Mathematically it is denoted as $I_S(u) = \{v \in V \setminus \{u\} | v \in D(u, r_u)\}$. In Fig. 1.7, $I_S(u) = 7$ when $r_u = r_{uv}$.

Definition 1.15. Receiver interference (I_R) model: This model gives the measurement of interference exists on each receiver node v by counting the transmission discs of other nodes to which v belongs. Mathematically it is given by Eq. (1.3).

$$I_R(v) = \{u \in V \setminus \{v\} | v \in D(u, r_u)\} \quad (1.3)$$

where $D(u, r_u)$ is a transmission disc of another node u with transmission range r_u [19]. A topological illustration of receiver interference is given in Fig. 1.8, redrawn from [4]. Also measure of receiver interference on each node of the illustrated topology is given in Table 1.1.

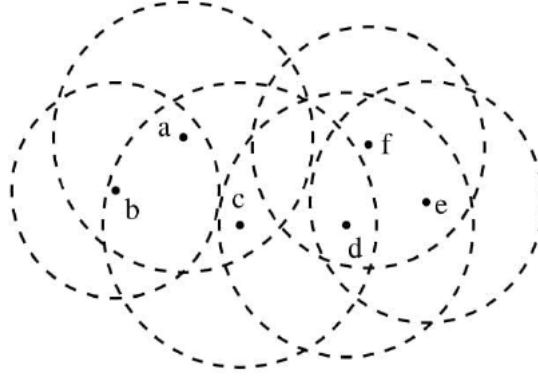


Figure 1.8: A topological illustration of receiver interference

Node	a	b	c	d	e	f
$I_R(v)$	2	2	2	3	2	2

Table 1.1: Measure of receiver interference of topology given in Fig. 1.8

max	maximum	min	minimum
avg	average	BS	Base Station
r_{min}	minimum transmission range	CH	Cluster head
r_{max}	maximum transmission range	MS	Mobile sink
SN	Sensor node	MN	Member nodes
MST	Minimum spanning tree	ngh	neighbourhood
H_p	High power	L_p	Low power

Table 1.2: Abbreviations used in this chapter

1.6 Topology control (TC) methods

Coming to topology control methods in WSNs which have been developed to optimize WSN by establishing energy efficient or energy optimal topology of sensor nodes with the maintenance of connectivity among the sensors, conservation of nodes' power, enhancement of network lifetime, minimization of nodes' transmit power and network interference for past many years.

In this section, we have provided a detail discussion on 'Graph based approaches'

in TC of WSNs which include those methods/algorithms which are either depending on some graph-based models or utilizing existing concepts and algorithms of graph theory like the concept of minimum spanning tree (MST), hamiltonian graph/cycle, shortest path methods etc. For example, Yuanyuan et al. in [20] utilize the concept of dominating set of graph theory to obtain a connected topology, Abu-Affash et al. in [21] utilize the existence of second hamiltonian cycle to obtain low-powered connected network, Abderrahim et al. in [22] introduce Dijkstra algorithm based energy efficient technique, Yang and Cai in [23] introduce minimum spanning tree (MST) based QoS required topology, likewise Cheng et al. in [24] develop MST based algorithm to achieve Strong Minimum Energy Topology (SMET).

Moreover discussion on explicitly designed interference models and interference minimizing algorithms in TC of WSNs have also been provided which have been greatly engaged in conserving nodes power, extending network lifetime, eliminating redundant transmission and transmission cost in directional/bidirectional connected graph models of WSNs.

1.6.1 Graph based approaches in TC

TC regarding energy consumption has been related to minimize nodes power consumption as nodes are power constraint and depletion of their power make them die soon hence it effects network lifetime adversely. Attempts, related to minimize and balance nodes power consumption by determining an appropriate network topology which is energy efficient or cost efficient, results an optimal power assignment to each node belonging to the topology or allow nodes to adjust their power level to design their own topology, come under '**Transmit power management in TC**'.

Transmit power management TC methods establish and maintain the topology of underlying network of WSNs by minimizing nodes power consumption either by reducing redundant transmission links or managing cost efficient transmissions depending on the required aim. This consequently results reduced and appropriate transmit power to each node in the resultant topology. Cheng et al. in 2003 [24] formulated a 'Strong Minimum Energy Topology (SMET)' problem which has been related to transmit power/power assignment to each node in the network. **SMET** problem states that for a given set of sensors, compute transmit power of each sensor such that there exists atleast one bidirectional link between every pair of sensors and total transmit power gets minimized. Total transmit power is the linear sum of transmit power of each sen-

sensor node in the network. Cheng et al. proved the NP-completeness of SMET in bidirectional graph model of WSNs and proposed two algorithms to find the approximate solution of the problem. Santi in her survey paper [11] has provided the discussion about '**Range assignment problem**', which is same as SMET with the only difference is that the graph model can be directional as well as undirectional/bidirectional. Rong et al. in 2004 [35] formulated **DPMP** (Dual power management problem) and proved the NP-hardness of the problem. It can be said that DPMP is the extended problem of SMET, dealing with optimal assignment of transmit power to each node from the discrete set of two power levels i.e. $\{H_p$ (high power), L_p (low power) $\}$. **DPMP** states that for a given set of sensor nodes and two power levels, H_p and L_p , find the optimal assignment of these two power levels to the nodes of induced graph model of WSNs such that the total power consumption gets minimized. Once the transmit power either high or low has been assigned to the nodes, it is kept on fixed. Generalising the DPMP to k -distinct power levels assignment problem, named as K -**Distinct SMET** which put forth problem of optimal assignment of a transmit power to each node in the network from the discrete set of k distinct power levels such that the resultant network is strongly connected and total power consumption gets minimized. Then some authors [26] come-up with fault tolerance in power management problem by introducing k -edge connectivity concept of graph theory.

Moreover a detail discussion of each technique related to the above mentioned problems has been documented in the Table 1.3 including objective/contribution of the developed work in the field, detail of methodology opted and limitations with its future scope.

Going through each technique of Table 1.3, following are the key highlights

- Although the developed work has provided the energy efficient topology according to the desired objective and has lesser complexity, yet is not suitable in dynamic WSNs. Topology gets disconnected with the nodes movement hence they are not adaptable to nodes mobility as once the power is assigned to the nodes, it remains fixed in the network during data transmission.
- Transmit power assignment and optimization are limited to discrete power levels [21, 25, 28, 29].
- Existing TC in WSNs have been greatly influenced by tree topologies having fixed power level where nodes cannot vary their transmission range.
- Issues related to fault-tolerance of topology have been resolved by undertaking edge-connectivity [26]. It can be further taken to vertex-connectivity with the consid-

eration of node failure.

- TC methods lag behind in term of network heterogeneity, scalability, fault-tolerance, mobility and delay.

1.6.2 Interference minimization approaches in TC

As interference in WSNs has been a major issue to energy consumption, delay and reliability in data transmission, various attempts have been made to develop techniques for reducing interference in history of WSNs. Authors have designed many interference models and graphical algorithms to minimize max interference, total interference and avg interference at both the node or link level.

The I_S model (refer definition 1.14) and I_R model (refer definition 1.15) are the most opted models to minimize nodes/network interference [12]. In [36], authors introduced energy efficient algorithms to minimize total and max network interference under I_S model subject to a connectivity constraint of directional graph models. Authors in [37] developed a new technique to minimize max sender interference of a connected network. This technique adds some new nodes to the topology in such a way that the interference can be reduced while ensuring the network connectivity. Panda and Shetty in [3] come-up with new interference models namely SUM and MAX (refer definition 1.12), to compute coverage based link interference under I_{SUM} and I_{MAX} models. After this, Sun et al. in [38] introduced another link interference model which computes number of interfering links instead of nodes which come inside the transmission disc of any node. This new link interference models outperform the previous models as being more practical [38].

The mentioned interference minimizing models result min interference topology with the issue of network sparsity and fault-tolerance. Then Bao and Deng in [39] introduced fault-tolerant and reduced interference topology by introducing k -disjoint paths between every pair of nodes of min interference. However, the topology formed using these techniques keeps high latency factor. Therefore Wu et al. in [40] propose polynomial time algorithms to result delay constraint and min interference topology. But the assumption that all links are having same transmission power, limits the use of the work. Lakshmi et al. in [4] minimize sender and receiver interference using two distinct power levels; nodes are either assigned with L_p or H_p in such a way that the formed-MST minimizes max interference. An interference cancellation based data aggregation technique has been introduced in [41] to reduce aggregation delay and

energy consumption simultaneously under a physical interference model. The work has been defined for directional graph model of WSNs.

However, existing techniques and algorithms lag their functionality in dynamic WSNs as they are designed for static network only. Also, work on interference minimization can be extended to nodes with multiple power levels or continuous range of power level like an interval and the concept of examination of link quality and nodes residual energy can be taken into consideration for fault-tolerance and reliability.

The thorough discussion on research work concerned with minimization of max interference, total interference and avg interference has been documented in Table 1.4 revealing the contributions made by researcher in this direction along with description of used/designed methodology, limitations and future scope.

Following are some defined terms which have been used in Table 1.4:

Highway model: It is obtained when nodes are distributed linearly in one-dimension only. It is established to obtain a lower bound on network interference.

Unit disk graph (UDG): A WSN has been modelled as UDG when two nodes communicate if and only if the distance between them is at most $r \leq 1$, where r is the transmission range which is equal for all nodes.

Gabriel graph (GG): A WSN has been modelled as GG when two nodes u and v communicate if and only if the disk obtained using the uv as a diameter doesn't contain any point.

Yao graph (YG): A WSN has been modelled as YG with an integer parameter $k \geq 6$ which defines k cones. In each cone, node v chooses closest neighbour node u with distance atmost one.

1.7 Routing techniques

On the basis of topology/network structure formation, routing in WSNs has been classified into flat-based, location-based and hierarchical-based. Flat-based routing protocols have been developed for homogenous WSNs where each sensor node has same functionality and operational capacity. It is also known as data-centric routing because BS sends queries to certain regions and waits for data from the sensors located in the selected regions. Some popular data-centric or flat-based routing protocols are Flooding, Gossiping [47], SPIN [48], Directed Diffusion [49], COUGAR [50], ACQUIRE [51]. However, flat-based routings show the limitations regarding reliable

data delivery and fault-tolerance, result routing overhead and latency with the increase in node density.

In location-based routing, also called geographic routing, location information of sensor nodes are required for route discovery, data forwarding and maintenance of sensor nodes in the network. Energy consumption can be estimated by calculating distance between two particular nodes that helps in reducing energy consumption and optimizing whole network. MECN, SMECN [52], EAGRP [53], LBRP [54] are some energy aware location based routing protocols. Existing strategies tend to increase extra routing overhead in order to obtain location information of destination nodes through the location service algorithm especially in dynamic WSN. In addition, energy hole problem and location inaccuracy problem also occur in geographic routing [10]. To tackle these issues, a novel fuzzy logic-based geographic routing protocol (FLGR) [55] has been proposed. Compared with existing protocols, FLGR protocol has reduced routing overhead and increased packet delivery ratio in a sparse network. Location-based routings are having drawbacks in large dynamic WSNs since requirement of accurate address of each sensor node makes the routing topology costly. Also performance degrades in term of latency in large network.

Hierarchical routing protocols also known as cluster-based routing, have overcome many limitations of flat based and location based routing protocols in term of scalability, energy consumption, latency, dynamic topology, algorithmic complexity, load balancing and network heterogeneity. The basic methodology and aim involve in this type of routing is the formation of optimal number of clusters (group of nodes), cluster size, appropriate selection of cluster head (CH) node in each cluster, optimal assignment of member nodes (MNs) to each CH, known as cluster member and optimal intra-cluster and inter-cluster routing. CH gathers information/data from its MNs via intra-cluster routing protocol and then CHs transmit gathered data to the BS/final destination node via inter-cluster routing protocols [56, 57]. A basic network structure of cluster based/hierarchical routing is shown in Fig. 1.9. In hierarchical routing, cluster of nodes can take any shape like chain, tree or grid since partition of target area has been done in many shapes like square, circles, grids and concentric circles [58, 59, 60].

Some authors introduced the concept of rendezvous points (RPs) and sojourn points (SPs) selection in cluster-based routing where mobile sinks (MSs) are engaged to aggregate data from MNs and/or CH nodes by halting at each of RP or SP [61, 62]. Basically RPs and SPs are stoppage point for MS node and authors have been dealing

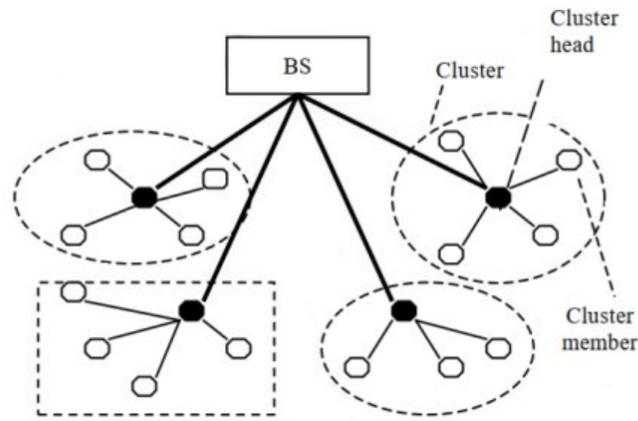


Figure 1.9: Network structure of cluster based/hierarchical routing

to find optimal RPs/SPs to improve load balancing and reduce energy hole problem specifically. Both RPs/SPs and CHs are chosen on the basis of distance factor, MNs density and energy factor. Irrespective of this, intra-cluster and inter-cluster routing are need to optimize. Some authors [63] involve time division multiple access (TDMA) mechanism to intra cluster routing where MNs transmit data in a specified time slot to CHs and keep themselves inactive otherwise and code division multiple access (CDMA) [64] to inter-cluster routing where a CH sends its data only by acknowledging code associated to it. Both TDMA and CDMA avoid unnecessary data collision, transmission, congestion at CH node or BS and consequently save nodes energy.

Following are the key points about classical hierarchical routing:

- These techniques form routing topology for both homogenous and heterogenous WSNs [65, 66].
- Some routing protocols are suitable for both static and mobile WSNs by offering mechanism of re-clustering and re-routing and some limit themselves to static network only [67, 68].
- Cluster formation makes the network more scalable and energy efficient yet they are showing limitation when timely data delivery is required. For this to achieve, some author brings the concept of engaging mobile sinks (MSs) so that they themselves come to collect data from the shortest route possible but not on the urgency basis. Therefore timely data delivery is the persisting issue [69, 70].
- These routing causes overhead and energy-hole problem when the hierarchy formation is chain and tree kind [71, 72].
- The most evolutionary part of these techniques is to have fault-recovery mechanism using beacon message exchange. This makes the network more reliable and fault-

tolerant.

However more techniques to deal with message overhead issue, fault-tolerance and energy-hole problem in dense, heterogenous and dynamic WSNs need to develop. Moving to more stable, scalable, robust, evolutionary and multi-objective 'ant colony optimization (ACO)' based routing techniques.

1.7.1 ACO based routing techniques

Routing topology formed using classical hierarchical routing protocols is not robust to nodes movement and failure. Also classical hierarchical routing techniques lag their performance while solving more complex multi-objective problems. So to overcome these issues and to find optimal solution of multi-objective problems, ACO based routing techniques have been in trend for past many years. ACO techniques mimic the intelligence and social behavior of natured species like swarm and ants as they develop knowledge on their own to communicate and feed themselves. These routing techniques resolve complex clustering and routing issues very efficiently in large dynamic WSNs [73]. As these techniques are robust, adaptable and flexible with respect to topology changes so they are called topology aware optimization techniques.

Definition 1.16. ACO techniques: ACO is the abbreviation of 'ant colony optimization'. This technique is introduced imitating the foraging behavior of ants seeking a path between their colony and source food. When ant moves in search of food, it leaves pheromone (a kind of liquid marker) along the path which evaporates over time. So other ants follow the same path by smelling the pheromone concentration. Moreover shortest path carries more pheromone quantity as more ants opt that path. With the movement of ants, quantity of pheromone updates everytime, this way a reliable and shortest path can be achieved using ACO technique. ACO has been used for a long time to solve route optimization problems in WSNs. Pheromone update function depending on distance factor, nodes' energy, nodes residual energy and/or node degree, has been formulated to discover optimal route/path in both static as well as mobile WSNs. A general mechanism of ACO technique is demonstrated in Fig. 1.10, taken from [74].

Detail discussion on well established ACO based routing techniques has been provided in Table 1.5 by mentioning objectives/contributions of the developed technique, detail of methodology used with persisting limitations.

Following are some key points, we have drawn from Table 1.5:

- **Optimal routing by ACO technique:** ACO technique is having best perfor-

speed and distances to other nodes. Undoubtedly this makes the topology more robust and stable with lots of expense.

Though ACO technique has been involved with many factors to optimize routing in WSNs yet some prominent factors like node interference, link interference, routing delay, have not been considered explicitly with ACO technique.

1.7.2 Real-time routing protocols

Real-time routing protocols are particularly concerned with delay-constraint or delay-sensitive data transmission. These protocols are designed to minimize end-to-end data delivery with the maximization of throughput and reliability factor to support real-life wireless communication in a more efficient way [15, 88, 89].

A thorough review of well established real-time routing protocols has been conducted and analysis of each technique has been documented in Table 1.6.

The definition of estimating end-to-end delay has been distinct according to the problem definition like if the methodology is based on to determine packet travel time then distance becomes the prime concern; some authors consider hop counts i.e. number of intermediate nodes between source to destination node; when methodology uses duty-cycle then total time spend during switching from sleep to awake has been considered for delay estimation. In some research work, delay has been taken equivalent to propagation delay + transmission delay + queuing delay. In Table 1.6, we have considered all aspects to estimate end-to-end delay.

Real-time routing protocols are well adaptable to dynamic WSNs. Most of them don't prefer to establish topology and do not have topology awareness like cluster-based routing. RAP [90], SPEED [91], MMSPEED [92] which are the very initial real-time routing protocols, determine velocity of data packets based on delivery deadline, urgency factor and distance from the destination node at the time of data transmission without establishing initial topology. However, their network lifetime don't last long as nodes die soon. After this, some authors [93, 94, 95] have introduced delay constraint techniques which consider nodes residual energy as well while prioritising data delivery or to determine shortest route for data transmission. This reduces earlier node failure. Then author in [96] have introduced a technique by involving location awareness and link quality to minimize routing error and packet loss at destination node. This maximizes throughput with reduction in end-to-end delay. Some authors [97, 98, 99, 100, 101] have proposed delay minimization techniques by opting cluster

formation but these techniques are only suitable for static network. Technique in [102] engages MSs to collect data to minimize redundant waiting time, makes itself suitable for dynamic and delay-harsh network by ignoring energy saving factor. Moreover authors in [103] have introduced a non-topology aware protocol that minimizes end-to-end delay by considering RSSI (received signal strength indicator), delay per hop and residual energy simultaneously to make next move. Then technique in [104] considers more factors like packet delivery, nodes energy, nodes degree, distance, retransmission etc to determine next routing hop to balance energy consumption with delay minimization. Authors in [105, 106, 107] consider hop counts to determine min delay path while authors in [108, 109] estimate end-to-end delay based on sleep and wakeup cycle of nodes.

After going through each technique of Table 1.6, we have drawn some conclusion:

- Prioritizing end-to-end delay minimization effects the nodes energy adversely in such a way that nodes die soon in the network so saving nodes energy should be taken equally important. This improves overall network performance.
- Topology aware real-time routing techniques balance end-to-end delay and energy consumption in an optimal way but they are lacking their performance in dynamic network. Also some are designed for homogenous network only.
- Indeed proposed techniques in this field enhance packet delivery ratio, improve throughput, reliability, reduce end-to-end delay at the cost of more energy consumption than usual or by making network costly by opting MSs or backup nodes.

1.8 Motivation

We have analysed various topology control methods and routing techniques in optimizing underlying network and routes in WSNs. Though the literature of optimization techniques in WSNs is abundant, we have focused to cover and study well established and evolutionary work which minimizes max nodes' power consumption, assigns optimal transit power to nodes, minimizes total transmission cost of the network, total link interference, max node interference under different models, minimizes end-to-end delay, maximizes throughput, reliability and network lifetime. Moreover, the work we have carried out in this thesis, majorly inclined towards the graph based and algorithmic

approaches which optimize power consumption, nodes' transmit power, transmission cost, delay and interference at both the topology and routing level.

Among the graph based approaches which have been focussed to minimize power consumption via range assignment problem and transmit power control, some of the work have adopted single transmit power assignment with fixed transmission range whereas some bring out the strategy of discrete power assignment to have energy efficient routing topology. So when it comes to transmit power assignment to nodes, work has been limited to discrete power management in topology control with fixed transmission range. Although some TC techniques [114] involve variation of transmission range/power level of nodes in an implicit way to reduce power consumption and extend network lifetime. Therefore in this thesis, we have explored new graph models of WSNs and TC methods with continuous power levels of nodes and shown the optimization of node's transmit power explicitly throughout.

TC in WSNs has been greatly influenced by energy efficient tree topologies engaged with immobile sensor nodes, lack dynamic applicability of WSNs due to nodes immobility and less stable with the removal of old nodes or addition of new nodes. As each vertex and edge of tree plays a crucial role in maintaining connectivity among the sensors therefore removal of any intermediate node due to its death from these structures brings disconnectivity and so loss of data during transmission among the sensors, energy hole and unreliability issue. Therefore reconnectivity/reorganization of disconnected topology in lesser time complexity and sensitivity analysis with respect to change in optimal value of power consumption has been a prime objective of the work in the thesis. Also addition of new nodes are required to enter to an existing optimal tree-topology on urgent basis due to some application of WSNs then extension of the optimal topologies with respect to new nodes has been demanded in the field of WSNs. Research work on the same has been an important part of the thesis as well.

Existing TC methods which achieve minimum cost routing topology by minimizing nodes' power consumption or engaging smaller transmission range result longer transmission delay between the nodes in large network whereas delay-sensitive routing protocols which work on minimizing routing delay from source to receiver node, neglect rest of the nodes' topology and power consumption. This therefore motivates us to bring both factors in a frame in this thesis.

Also existing interference minimization solutions based on graph approaches focus

on optimization on topology level only. So in present thesis interference minimization has been promoted to routing level with new optimization technique and model.

Henceforth **Chapter 2** introduces a new ‘Interval graph model of WSNs’ where each sensor node has been assigned with a continuous power level in the form of interval and satisfies the properties of being an interval graph. New graph models and algorithms have been proposed to result power optimized topology of sensor nodes such that the maximum power consumption/transmit power of the node gets minimized subject to the connectivity among the sensors with minimum number of bidirectional links. Also optimization of max and total transmission cost have been shown under worst case scenario of power consumption.

Then **Chapter 3** presents sensitivity analysis on ‘energy optimal spanning tree topology’ of immobile nodes which has been obtained in chapter 2, with respect to each intermediate node removal. This is because node gets died in the network due to drainage of its power and stops working so either it assumes to be removed or it is actually removed from the topology. Therefore chapter 3 reveals all disconnectivity aspects and methods to deal with these disconnectivities with minimum possible effect on optimal value of max power consumption. This work supports the fault-tolerance of the energy optimal tree topology on interval graph model.

Meanwhile **Chapter 4** introduces new algorithmic approaches to extend ‘energy optimal spanning tree topology’ of immobile nodes which has been obtained in chapter 2 when some new nodes are required to enter to the topology on any urgent basis in WSNs in an energy efficient way. Sensitivity analysis has been done to show possible effects on optimal value of max power consumption under different cases of addition of new nodes. This work shows the scalability factor of the energy optimal tree topology on interval graph model by the proposed algorithms.

Chapter 5 introduces a new methodology to result an energy and delay optimized bidirectionally connected routing topology of WSNs where nodes are having continuous power levels, heterogeneity and immobility in nature. We have introduced ‘sorted interval data based graph model’ to explicitly deal with nodes power consumption and considered distance with interference model in estimating delay during transmission. This therefore reduces both the power consumption and delay between the nodes on topology level.

Last but not the least, **chapter 6** brings a new interference minimization routing technique by engaging two distinct interference models with ACO algorithm. This

minimizes link as well as receiver node interference along with transmission cost on the route from source to destination node on bidirectional connected graph model of WSNs. Also a new interference model based on disc interruption has been introduced to result the same in lesser time complexity than the ACO based routing technique.

A summary followed by the future scope of the research work is evinced to conclude the thesis after chapter 6. In the end, the thesis provides a bibliography and a list of author's publications.

Table 1.3: Brief on Graph-based approaches in ‘Transmit power management in TC’

Reference	Contribution	Description	Limitation/Future scope
2012 [25]	Authors proposed a fully distributed power adjustment algorithm named as BRASP to form a connected and reliable topology of minimum cost by undertaking the factor of lossy links by computing link qualities locally. Worst case time complexity is (E) and cost is $(E + V)$. Outperform other Deterministic TC algorithms in term of network energy efficiency and connectivity with decreased node degree.	Among the K — distinct power levels, each node starts with minimum power level and gradually enhance its power to any of the available power level to satisfy the connectivity constraint. Connectivity constraint is the value between 0 to 1 such that the value more than 0 between the the pair of nodes depicts their connection and closer the value to 1, stronger is the connection. Authors used Dijkstra’s algorithm to find reliable path of minimum cost between the pair of nodes by following the restriction of hop counts means if 2-hop restriction is imposed, nodes can adjust their power to reach 2-hop away nodes only.	Defined for static network only. BRASP will consume more energy when larger threshold of hop count is used. Work on improving end-end delivery by accounting time-dependent lossy links might be undertaken in future.
2015 [26]	Introduction of fault-tolerance in DPMP through k —edge connectivity, named as k —EDP. Authors proved the NP-hardness of k —EDP in geometric case and provided an approximate solution to the problem in polynomial time. Moreover, proposal of ‘Minimum-Cost-H Path’ algorithm to obtain 2-edge connected spanning subgraph.	Author formulated the problem as LPP and proposed an algorithm using the iterated rounding method, given in [27]. Algorithm keeps on applying ‘Iterated rounding technique’ on LPP to convert its fractional solution into integer until a k -connected graph is obtained. Minimum-Cost-H Path algorithm successively adds H-paths with the idea of picking low power edges first to obtain 2-edge connected spanning subgraph H .	Suitable for static WSNs only. The problem can be concerned with k -vertex-connectivity undertaking the node failure in future.
2015 [28]	Proposal of an approximate solution to DPMP by introducing SCDPA (strongly connected dual power assignment) algorithm and an algorithm concerned with the assignment of 3-power levels which are low, middle and high power level following the similar steps of the former algorithm.	MST of the induced graph is found by using Prim’s algorithm and transmission range is assigned to each node using distance factor in MST. Now nodes whose transmission range is less than or equal to low power level is assigned with L_p and remaining with H_p .	Designed for static network only. No concern of topology fault-tolerance has been shown. A work to have fault-tolerant and minimum delay topology can be explored in future.
2015 [29]	Provides the study of NP-completeness of K -Distinct SMET problem. A hierarchical structure of K -Distinct SMET is introduced, named as ‘ k -Distinct- r SMEHT’ problem and an approximate solution is provided which reduces total power consumption. Moreover, a polynomial time solution has been shown for the 2-Distinct-SMET problem.	Methodology of algorithm is using extension tree concept and idea of joining r -distinct spanning tree to form a single spanning tree of a complete weighted bi-directional graph model of WSNs.	Proposed solution is not optimal and suitable for static network only.
2016 [30]	Authors proposed topology adaptation algorithm (TAA) and preprocessing algorithm (PA) to obtain minimum power consumption and connected bidirectional routing topology in heterogeneous WSNs with time complexity $(n^3 \log^2 n)$. This work is extendable to different types of nodes having different power levels. More fault tolerant than conventional sparse topology.	Each super nodes are supposed to keep k -disjoint paths with sink nodes and ordinary node keeps a single path to sink node using graph transformation and max-flow theory. TAA algorithm follows MST initially to establish connectedness among all the nodes and sink, then more links have been added to MST to achieve k —connectedness of super nodes. Graph transformation converts node-disjoint paths to edge-disjoint paths using max flow theory and best k -paths have been kept. PA algorithm is used after TAA to remove unnecessary long links without disturbing connectivity if required.	Super nodes makes the network quite expensive. Proposed work is suitable in static network only.
2017 [31]	Authors proposed two algorithms for SMET problem, first one is the improvisation of Prim-incremental heuristic in term of time complexity and second is the hybrid of Boruvka and Prim algorithms.	Methodology of first algorithm joins the nodes in greedy fashion of increasing total power consumption to form a spanning tree. There exists a set for every node to choose its closest neighbour directly based on increased power level. Second algorithm forms MST using hybrid methodology of Prim and Boruvka algorithms. Each vertex is taken as a single component and edge of least power consumption is selected from each component to form a new contracted graph. Selection of edges from contracted graph is done to form MST using Prim’s.	No approximation bound on optimal solution has been provided. Algorithms are restricted to planar graphs only. Also nodes are assumed to be immobile in behaviour.

Table 1.3: Continued

Reference	Contribution	Description	Limitation/Future scope
2017 [32]	Proposal of an integrated power adjustment and clustering based TC method known as ‘Collaborative Topology Control Protocol (CTCP)’. This approach reduces power consumption and improves stability factor in large heterogeneous WSNs.	The methodology starts with minimum connected domination set (MCDS) theory to select dominated nodes as CHs and establish a base topology of CHs. Then power adjustment algorithm has been initiated to assign distinct transmission range to each MN within a group by its CH. Transmission range of each node can be varied to achieve desired topology as smaller range of MN causes many CHs of long transmission range which causes more energy cost and long range of MN reduces energy cost caused by CHs with the increment of hop distance.	This method reduces hop distance at the cost of keeping dense network which causes network interference. Also the work is not suitable when nodes are mobile since it complicates the working of algorithms in topology establishment.
2018 [21]	Authors improved approximation bound of achieving optimal solution of DPMP and developed a polynomial time algorithm which uses the concept of second hamiltonian cycle.	The methodology is using the contracted set theory with second hamiltonian cycle with minimum number of nodes with high transmit power on an induced directional graph model of WSNs to obtain connected subgraph.	Designed for static and directed network. The work can be extended to mobile topology with bidirectional links.
2019 [33]	Proposal of an virtual topology based routing protocol where nodes can vary their transmission range. The proposed algorithm is distributed in nature and improves network life, routing cost and having polynomial time complexity.	In this, authors modelled the WSN as a disk graph and engaged minimum connected dominating set (MCDS) to have the connected topology among the nodes which results quick search of routes with shorter distance by keeping minimum disc radius and variable transmission range.	This approach is not much suitable for mobile WSNs. Even though nodes can have variable transmission range but the effect of topology construction on nodes power level is not shown explicitly.
2019 [34]	Authors proposed two polynomial time algorithms to obtain the solution of DPMP under two cases, 1. Power levels are chosen from the given discrete set and 2. Power levels are taken as input. Lower and upper bounds on optimal number of sensor nodes assigned with H_p for DPMP have been obtained. The second algorithm performs better than first to minimize total power consumption with connectivity constraint.	First algorithm starts with finding of MST then assignment of transmit power to each node in MST. H_p is taken maximum of assigned transmit power and rest transmit powers work to find L_p . Each node whose transmission range is less than H_p would be assigned with L_p , so this way minimum number of sensors is assigned with H_p and total power gets minimized in the connected network. The methodology of second algorithm is to assign L_p to all the nodes inducing a spanning forest which consists of one or more components. Assignment of H_p is done to only those nodes which effect maximum number of connected components.	The obtained solution is approximate, lacking optimality. Both the algorithms are suitable for static network only.
2019 [4]	Formulate a k -power level assignment problem and provide a near-optimal solution by proposing two polynomial time algorithms. First is MST-based and second is incremental heuristic. Incremental heuristic performs better in minimizing total power consumption than MST-based.	The methodology of first algorithm starts with the set of k -power levels in ascending order and computation of MST. According to the weight on each edge of MST, transmit power from the set of k -power levels are assigned using divide and conquer rule i.e. if weight of node u is lying between p_i and p_{i+1} , then u is assigned with p_{i+1} . The methodology of second algorithm forms a spanning tree with one by one addition of edges in increasing order of their weights. The difference from the former algorithm is that this algorithm maintains two sets for vertices, one shows visited vertices and other unvisited.	In MST-based algorithm if weight on any edge is more than the available power level then no feasible solution exists and algorithm gets terminated. Also it is designed for static network only.

Table 1.4: Brief on ‘Interference minimization approaches’ in TC

Reference	Contribution	Description	Limitation/Future scope
2009 [42]	Proposal of two polynomial time algorithms MI-S and LS-IM to minimize max interference. Both the algorithms show good performance in some instances. Analysis of the proposed algorithms in best and worst case scenario has been shown.	MI-S algorithm works to join edges in increasing order of their weight to form a spanning tree which minimizes max interference of the network. LS-IM involves local search technique to improve the solution obtained by MI-S if there is any change in network topology because of addition or deletion of edge.	Algorithms don’t minimize interference optimally in every instances and is limited to 2-D topology.
2011 [43]	Proposal of first exact algorithm to minimize avg interference and provide proof to the bound on max interference while minimizing avg interference in 2-D network. Authors presented study of minimization of avg interference in connected topology using no-cross property and proposed another algorithm to minimize the avg interference for 1-D model in polynomial time.	Using the concept of mutual exclusion of long edges, computational geometry based on distance and transmission range, first algorithm obtains low interference spanning tree. Second algorithm considers mutual interference and right side topology for the nodes lying from left to right to construct a low interference topology.	Suitable for static nodes deployment and low dimension topology. Assumption of interference range is same as transmission range is limiting this work.
2011 [44]	Proposal of exact algorithm to minimize receiver interference for Highway model. Multiple planar connected topologies with reduced avg and max interference can be constructed using the algorithm.	Utilizing no-cross property to obtain connected planar topology and dynamic programming, algorithm yields optimal spanning tree for a highway model.	Algorithm is designed for static network using 1-D interference models only. Minimizing max interference for the highway model is still an open problem.
2011 [45]	SBYaoGG (Smart Boundary Yao Gabriel Graph) for low interference connected network has been proposed. This optimizes transmission cost of symmetric links and resultant topology has energy efficient and low-obstacle routing.	SSYaoGG algorithm is the hybrid of GG and YG algorithms adopting smart region boundaries. This algorithm involves neighbour discovery utilizing max transmission range, computation of GG from the UDG yields planar topology with linear number of edges then computation of avg direction vector and YG on reduced topology yields power spanner connected planar topology.	Reduced topology is sparse and every node sets to min transmission range reduces link reliability and QoS.
2012 [46]	Proposal of an interference minimization technique by reducing the communication radius of nodes which therefore reduces network interference while maintaining connectivity constraint. This defined technique is applicable to asymmetric case also.	Technique works in localized way i.e. let r_{min} is the boundation set on communication radius of each node which is actually the longest edge distance and the sensor field is partitioned into cubes of side length r_{min} which therefore results communication radius of each node inside any cube less than r_{min} .	Assumption of uniform range assignment to each node limits the applicability of the proposed technique. Also working analysis using mobile nodes has not been given.

Table 1.4: Continued

Reference	Contribution	Description	Limitation/Future scope
2013 [3]	Proposal of ‘Minimum Interference Strong Bidirectional Topology (MISBT)’ algorithm with two new interference models, SUM and MAX, to estimate coverage-based and transmission-based nodes interference. Polynomial time MISBT algorithm under MAX model optimally minimizes max nodes interference and yields 2-approximate solution to min-avg nodes interference.	MISBT algorithm involves the computation of complete weighted graph of the WSN where the weight on each edge is found by adopting either the MAX or SUM model, then computation of MST yields minimum interference connected topology and reduced power assignment to each nodes.	Designed for static WSNs. The work can be extended by allowing interference and transmit power to result in a real number considering real-life situations.
2015 [38]	Introduced a new link interference model which counts the actual number of interfering links with the given link by discarded the previously designed link interference model based on coverage. Author proposed a centralized greedy algorithm to construct an interference-optimal topology.	Working of the algorithm is to sort links in increasing order of their weights which is depending on link interference and distance, then insertion of links one by one to obtain an energy spanner connected topology. This reduces the chances of loop formation and redundant addition of links as one by one insertion of links is adopted.	Algorithm is defined for static WSN. Individual focus on interference reduction makes network sparse and energy hole problem.
2019 [37]	Adopted the strategy of additional nodes deployment in the sensor field at those position which helps in minimization of sender interference without disturbing the nodes position. Authors proposed a polynomial time algorithm which minimizes max sender interference while preserving connectivity.	Working of the algorithm named as MSD, is to connect nodes to their nearest nodes which results each edge is Gabriel and induced a spanning forest. A MST is obtained from the resultant GG such that some edges belong to MST, not to spanning forest. Then a new node is positioned at the bisection point of that edge which further reduces the range assignment of the end nodes ensuring connectivity requirement.	Designed for static network. Deployment of more nodes causes more hops to cross which may lead transmission delay.
2016 [39]	Proposed a fault-tolerance and interference-aware topology control (FICTC) algorithm which is having polynomial time complexity. FICTC minimizes max link interference in bidirectional network and meeting the fault tolerant requirement.	The algorithm works to achieve different k -connectivity between different pair of nodes by establishing k -disjoint paths between any pair of nodes. This is done according to obtain minimum vertex separation set with use of nodes splitting technique. This splitting technique employs two nodes like v' and v'' in place of a node v in the original topology to fulfill the connectivity requirement. Then Prim’s algorithm has been used to find augmented paths between the nodes on the resultant topology, obtained after splitting operation and updation of links capacity on the obtained augmented path. All the augmented paths merged together to obtain final k -connective topology with reduced link interference.	Defined for static network only. Adoption of k -disjoint paths between every pair of nodes causes more data transmission so more energy consumption.

Table 1.4: Continued

Reference	Contribution	Description	Limitation/Future scope
2018 [40]	Proposed one centralized and two distributed TC algorithms which jointly work on reducing link interference and transmission delay in the connected network. Authors undertook three optimization problems: minimize max interference, minimize avg interference and minimize path interference.	The first algorithm MDTC is centralized in nature adopted greedy strategy in which links are sorted in ascending order of their weights. Weights are link interference and links join themselves if they satisfy delay bound in the increasing order of their weights. Second algorithm is DMST which builds a delay-constraint MST to minimize avg links interference with restricted hop counts to satisfy the delay bound. DMST adds links of min interference whose hop counts must be less than imposed delay. A local version of DMST is also proposed by author in which nodes make next move on the basis of neighbour information i.e node choose another node if the corresponding path has min interference satisfying delay constraint. A polynomial time LDBF algorithm is proposed to minimize path interference. LDBF utilizes the concept of Bellman-ford algorithm in a localized way with the inclusion of transmission delay restriction over the path.	MDTC has higher time complexity. All links are assumed to have same transmission power. Delay is measured on basis of hop counts only. Although delay is based on link quality and distance.
2019 [4]	Proposal of MST-based algorithms to minimize max sender and receiver interference with DPMP. Algorithms yield min interference connected topology in polynomial.	Working of proposed algorithms is to partition edges into two sets i.e. H_p set and L_p set according to their weights where weight on each edge is defined on the basis of Euclidean distance. Edges with L_p are added first to construct topology following the computation of sender (or receiver) interference of each node. Then H_p vertices are sorted in ascending order of their sender (or receiver) interference and added accordingly to the topology with H_p assignment until a loop free connected topology has been obtained.	This work is limited to interference minimization with dual power assignment to stationary nodes.

Table 1.5: Brief on ‘ACO based routing techniques’ in WSNs

Reference	Contribution	Description	Limitation/Future scope
2017 [75]	Introduced an improved ACO based routing method which reduces energy consumption and extends network lifetime.	For optimal route evaluation, authors formulate a fitness function which depends on nodes residual energy and route length simultaneously. Larger the fitness value, more pheromone quantity gets update and more optimal the route is. In this improved version, authors not only consider transmission distance of every next node but also distance between node and sink to save more energy. Also to reduce algorithmic complexity and energy consumption, authors limit the search space of ants by introducing the search direction angle which should be smaller than 90° between the line connecting a node and sink and the boundary line.	Assumption of a square field where nodes and sink become static after deployment limits the applicability of algorithm. Author ignores the effects of other energy levels (e.g., avg energy, min energy). Extension to mobility and real-time constraints can be done in future work.
2017 [76]	Proposed an ‘ACO Energy Aware (ACOEAE) routing algorithm’ which offers multiple transmission path from source to sink. ACOEA, optimizes ACO in terms of angle and distance between nodes to obtain better multiple paths. This achieves balanced energy consumption and extended network lifetime.	In ACOEA, three types of ants are engaged, 1. Search path ants: search multiple energy efficient paths from source to sink by computing transition probability (having pheromone and distance parameters) at every next move, 2. Back-off ants: sent by sink node to source along the same path for updation of pheromone value and take record of nodes residual energy, 4. Repair ants: do the routing maintenance task and in case of node failure, path is repaired by replacing dead node with one of its suitable neighbour node. Authors introduce ‘angle factor’ so that any node which forms the largest angle between sink and source node taken as the next node. This helps in reduction of energy consumption and optimize the path performance. To search more paths, remaining links that are unused, selected successively to receive ants using the transition probability formula.	Authors assume the static model to design the algorithm. Disconnectivity and hole problem can be the issue due to nodes replacement to repair path since these factors are ignored.
2018 [77]	Introduced a ‘new data gathering method (ACO-MSPD) based on ACO technique’ for non-uniform data producing sensors at different time period. ACO-MSPD finds the optimal no. of RPs and optimal trajectory (tour) for MS within an imposed constraint of max tour length. ACO-MSPD minimizes avg MS tour length, reduces the complexity to find suitable RPs. It adopts reselection RPs dynamically during change of weights of RPs.	The algorithm mainly consists of four phases, 1. Directed Spanning Tree (DST) formation: each SN chooses one forwarding node from n _{gh} of max load, 2. Optimal selection of RPs: RPs selection is done by improved ACO method where the defined probability function returns the highest value with the updation of pheromone value between the current node and the selected RP node, 3. MS path formation: an optimal tour by MS visiting each RPs to collect data using ACO technique and 4. Reselection of RPs: If MS tour exceeds defined constraint length and/or when nodes generate non-uniform data then reselection of few RPs takes place.	Complexity of Algorithm increases with increase in ant population and iterations. This method is designed for those network where sensors become static after deployment and a single MS has to engage.
2018 [78]	Proposal of an energy efficient clustering and tree-based routing protocol named as ACOPSO. This method prolongs network lifetime.	The algorithm first clusters the nodes based on remaining energy, then inter-cluster data aggregation using pheromone factor. Involved ACO technique discovered a minimum cost spanning path between CHs and sink with the updation of pheromone after every iteration.	Designed for static network where each sensor and sink remain static after deployment. Due to consideration of distance factor only, it may cause transmission delay when path of low energy is taken.

Table 1.5: Continued

Reference	Contribution	Description	Limitation/Future scope
2019 [79]	Proposed ACO based QoS aware energy balancing secure routing (QEB-SR) algorithm. This algorithm improves network lifetime, end-to-end delay, reliability and load balancing.	QEB-SR used improved models for calculating end-to-end delay based on lesser number of hop counts during transmission and trust worthy nodes has been chosen based on energy level and distance factors for data transmission to reduce packet dropping. While moving from source to sink, these ants store information about the quality of nodes traveled such as energy consumption, residual energy, packets generated, packets received, packets transmitted and end-end delay and so update the pheromone level on their way back from destination to source. After max iteration, pheromone level keeps on updating and the path with max pheromone quantity becomes an optimal path for data transmission.	Designed for static and homogenous WSNs.
2019 [80]	Proposal of a novel 'Energy-Efficient Load Balancing Ant-based Routing Algorithm (EBAR)' for WSNs. This balances energy consumption, prolongs network lifetime and speed up the convergence of route discovery under the constraints of limited energy supply.	This new algorithm involves three components to achieve its aim: 1. An improved pheromone trail update method, based on energy levels of SNs and path length information, 2. An effective heuristic strategy which reinforces low energy cost routes in the route discovery phase based on expected cost metric and 3. An opportunistic broadcast scheme is adopted to control the packet overloading and congestion.	Assumptions of links are symmetric and sensors are isomorphic in functionality and capacity, restrict the applicability of the algorithm.
2019 [81]	Proposed an improved ant colony algorithm to optimize mobile WSNs in term of life cycle, imbalance energy consumption of sensor nodes and data gathering. Proposed method is having good dynamic adaptability and load balancing. A low power low error geographic routing based on ACO to enhance data reliability is seen.	Authors adopted both, global pheromone and local pheromone update strategies. To avoid the concentration of pheromone in the path that can be too high or too low, the pheromone concentration is set to range from some min to some max which in turn accelerate the convergence of algorithm. Updation of pheromone happens after every transmission along the path. A probability function has been defined to choose next move based on pheromone quantity. When the data needs to be transmitted, the optimization mechanism runs to calculate the current communication quality and distance of neighbour node.	Opted multi-hop communication routing may cause latency issue while maintaining load balancing and stability factors.
2019 [82]	Proposal of an 'ACO Optimized Self-Organized Tree-Based Energy Balance (AOSTEB)' algorithm in which an energy optimized intra-cluster route is selected using ACO algorithm by MNs to communicate with nearest CH. AOSTEB algorithm involves rotation of CH when its energy becomes low, offering balanced load-distribution and helps in improving network lifetime.	The ant's movement creates multiple paths between MNs and their CH, optimal path is selected dynamically among the multiple paths on the basis of min energy consumption during transmission along the path. AOSTEB improves CH selection by introducing a parameter called as Probability of Election (PE) based on residual energy and neighbor nodes so a node which is having maximum PE value selected as a CH. Also selected CHs keep min distance between each other so as to have a balanced distribution of clusters inside the network. Data transmission are done when CHs form a tree-like structure in which only one CH on behalf of all CHs communicates with the BS whose residual energy is greater among all.	AOSTEB is designed for static network only. Initial random route selection causes redundant data transmission. Moreover, only one CH transmits data to BS directly, it may cause congestion at CH node as well as transmission delay.
2020 [83]	Proposed a 'distributed data gathering technique (GTAC-DG)' where the decision making skill of game theory are used to find optimal set of RPs and enhanced ACO technique is used to find optimized route among the RPs for MS visit. GTAC-DG helps to reduce redundant data transmission, energy consumption and delay in data delivery. A reliable and intelligent trajectory of MS has been resulted, extending the lifetime and energy conservation. As selected RPs directly transfer data to MS, this leads to better balance in energy consumption and reduces communication overhead.	The technique starts with game theory Based RP-selection algorithm where nodes involve in a local-game with its neighbors competing with each other to be a RP depending upon the relative energy and load of player nodes. Larger the relative energy and load, more is the probability of being a RP. After the suitable RPs selection, trajectory design phase is executed i.e. travel path for MS visiting all RPs, same as TSP problem. So a modified ACO method is proposed by authors, formulating a tradeoff function between considered parameters that are load factor of RPs, pheromone value and distance between RPs and MS to have an optimized path. Also RPs with higher load are prioritized first to visit by MS to reduce retaining data to buffer for a longer period of time.	Assumptions of sensors are immobile and have same communication range restrict the applicability of algorithm.

Table 1.5: Continued

Reference	Contribution	Description	Limitation/Future scope
2021 [84]	Proposal of ‘Multi-sink placement and any-cast routing (MPAR)’ and ‘Extended multi-sink placement and any-cast routing (EMPAR)’ involving two level architecture where lower level comprises of all cluster groups and upper level comprises of sink nodes and their related activities. With the increase of MNs and decrease in cluster size, algorithms can balance nodes energy consumption, reduce transmit power and prolong network lifetime.	Methodology of the proposed algorithms are as follows, 1. Sensors cluster themselves based on Euclidean distance where MPAR selects CHs whose energy level is max while EMPAR considers both residual energy and distance factors along with the history of sensors selected as CHs, 2. Formation of shortest path tree at lower level through which each sensor transmits its data to the corresponding CH, 3. This chooses optimal locations for multiple sinks to place where a fitness function considers distance between the CHs and sinks with data length of each cluster and 4. Construction of high level any cast routing tree connecting all the CHs with multiple sinks for low cost data transmission has been done by improved ACO algorithm using pheromone based probability function. The employed probability function jointly considers residual energy and Euclidean distance for selecting next hop CH.	With the increment of sensors, time complexity increases. Involvement of multiple sinks makes network costly. As the number of sinks increases, finding the best location for the sinks becomes more complicated.
2021 [85]	Proposed an energy-constrained and delay-sensitive data aggregation approach. This approach gives shortest possible MS traversal route in polynomial time. Latency is minimized by employing one hop communication between CHs and MS and shortest path traversal of MS.	This method finds SPs such that tradeoff between energy consumption and delay gets balanced. This formulates the optimization problem of minimizing no. of SPs with one-hop connectivity to CHs. Since more SPs causes delay and less SPs causes energy hole issue so this approach aims to find optimal SPs and shortest path for MS to visit each SPs. SPs nomination is based on density factor i.e. density of nodes around CHs and coverage overlapping. An ACO method has been employed to find shortest travelling path for MS, using transition probability function which is further based on pheromone concentration and distance. CHs use TDMA schedule for data transmission among themselves.	Applicable where nodes remain static. Obtained traversal route for MS can be less reliable since nodes residual energy has been ignored.
2021 [86]	Proposal of ‘Energy-efficient cluster-based routing protocol using unequal clustering and improved ACO techniques (ECRP-UCA)’. ECRP-UCA is able to achieve global optimal solution and prevent quick convergence to local solution. Moreover it treats connectivity, fault tolerant issues of large network, balances energy consumption and improves network lifetime.	This method uses a new parameter i.e. no. of backward relay nodes in previous round with distance from the sink, residual energy and node degree to determine cluster radius. This measure gives unequal cluster formation. MN selects its CH according to the multi-objective cost function which is based on residual energy and distance parameters. Also to manage connectivity in large-scale network, ECRP-UCA refers higher energy nodes as relay nodes for ensuring multi-hop connectivity between isolated nodes and distant CH nodes. Strategy like replacement of dead nodes by already backup nodes without reconfigure whole network is adopted to ensure fault-tolerant network. For energy efficient inter-cluster routing, author formulates improved ACO (I-ACO) method with an addition of important parameter called NPPRN (No. of preferred probable relay nodes) with the energy of next hop node and distance from the current node to the next node.	Designed for static homogeneous network only. Computational time increases with the increment in the value of parameters.
2021 [87]	Proposal of an extended ACO-based mobile sink path construction for event-driven WSNs, abbreviated as ACO-MSPD. This helps to improve network lifetime, hot-spot issue, load balancing and throughput. The present work outperform its previous version and EAPC.	The first task is to construct a connected directed spanning tree (DST) of the given network. Thereafter ACO is used to generate the RPs location by updating the ants pheromone quantity iteratively. A probability function has been constructed to trace delay-aware path for MS to collect data from each RPs and MS prioritized RPs based on available data size. Concept of virtual RPs (VRPs) is the new approach used in this work. VRPs are those SNs which are more closer to MS than the RPs so they directly transfer their data to MS when comes under its communication range. This reduces the buffer time, redundant transmission and delay as well.	Nodes except sink node are supposed to be static in nature after deployment. So the proposed work is not suitable in mobile WSNs. Also the preferred DST model of this work is less fault-tolerant.

Table 1.6: Brief on ‘Real-time routing techniques’ in WSNs

Reference	Contribution	Description	Limitation/Future scope
2004 [110]	Proposal of an ‘energy-aware delay constraint routing algorithm’ for both static and dynamic sink engagement. This approach provides least cost path meeting end-to-end delay constraint, reduces avg delay and packet miss ratio. Also this limits unnecessary re-routing which helps to save energy.	Extended Dijkstra’s algorithm is used to find k disjoint paths in ascending order of cost where the cost function for each link is based on communication distance, nodes residual energy and transmission error rate. Appropriate transmission range for each node is calculated, satisfying time deadline. The method is extended to support sink mobility by employing automatic re-routing approach. If the MS lost connectivity due to its movement, new nodes are directed to connect with the sink to relay packets and routes are updated by adding new hops. If the sink moves close to the source nodes, it over-hears the current transmitting data traffic, reduces unnecessary relaying and end-to-end delay. Re-routing only happens when already established routes become less energy efficient.	This does not support scalability factor. Utilization of class-based, priority queueing mechanism rises complexity and makes the network costlier for the resource-constrained sensors.
2006 [92]	Proposal of a ‘Multi-Path and Multi-Speed (MMSPEED) protocol’ considering timeliness and reliability factor. MMSPEED achieves guaranteed end-to-end delay and reliability requirements in a localized way. Suitable for more scalable and large dynamic WSNs.	This protocol works on two important notions: virtual isolation among the SPEED layer and dynamic compensation of local decisions. Virtual isolation aims to classify incoming packets into an appropriate speed layer based on their speed, this reduces transmission delay of lower speed packets by differentiating them from higher speed packets from the queue. The dynamic compensation is the adjustment in speed at intermediate nodes while transmission to meet end-to-end deadline. Delay is estimated at each node after every hop using elapsed time concept for updation of speed if necessary. To enhance the reliability level, MMSPEED adopts multiple path data forwarding. Number of forwarding path is depending on packet reliability level as low reliability packet is delivered using a single path while a high reliability packet is delivered via multiple paths.	MMSPEED requires more complex calculations for multiple path selection and dynamic compensation, it may lead to more power consumption. This is designed for short-lived network only.
2006 [93]	Proposal of an ‘energy-aware dual path real-time routing scheme’ for delay sensitive data packets. This approach focused on achieving reliability and improving network lifetime. It also reduces congestion at intermediate nodes by minimizing redundant transmission.	Packets are prioritised to deliver on the basis of urgency factor depending on path distance and packet deadline. More is the urgency factor is for a packet at a node, earlier the packet has to deliver. Also on the basis of urgency factor, the value of distance traveled from source node towards the destination node is decided. Intermediate nodes are prioritised using the calculation of energy and delay so that packets can reach final destination on time. This scheme involves dual transmission of data packets from the source node i.e. source transmits same data packets to first priority node as well as second priority node in its surrounding and after that only one path is taken to route data upto final destination. Additionally, Author engaged MAC layer to minimize delay on its part by stopping post backoff for real-time data packets.	This approach ignores nodes energy consumption to achieve QoS and delay requirement.
2007 [94]	Proposal of ‘LR-ENR and HR-ENR’ real-time routing algorithms with the aim to have energy efficient delay constraint routing with achieving optimal lifetime in the connected network.	At first, author implement a LPP objectifying max of min network lifetime avoiding delay factor for energy efficient routing. A distributed algorithm is proposed which iteratively solve this LPP by giving least cost path routing where the cost of each path is either sum or max of cost of the nodes on that path. Due to ignorance of delay factor, author extends the approach further by incorporating delay guarantee which has been named as level restricted energy efficient routing (LR-ENR) and hop restricted energy efficient routing (HR-ENR). LR-ENR limits the number of iterations and HR-ENR limits the length of path by minimizing number of hop counts to meet the deadline.	It does not consider link’s reliability. An unreliable link in any route may cause longer delay because of higher retransmission probability, which in turn increases energy consumption.

Table 1.6: Continued

Reference	Contribution	Description	Limitation/Future scope
2008 [111]	Proposed a new routing algorithm undertaking energy, delay and reliability factors simultaneously. This increases sensors lifetime and works for low mobile sensors also.	Approach is the extension of AODV protocol where nodes discover routes in request-response cycles with the difference of opting energy efficient paths on which transmission delay is lesser than the allowed delay.	Delay is measured on the basis of hop counts only. Interference and distance parameters should be taken into account for transmission delay.
2012 [112]	Introduced a 'Fast Time-Dependent Shortest Path (FTSP) Algorithm' for end-to-end delay reduction in WSN. FTSP algorithm achieved least latency. Moreover, another version of FTSP is also proposed, named as 'FTSP-M' to deal with mobility of sensors. Both the proposed algorithms have polynomial time complexity. FTSP-M runs concurrently at multiple nodes if there are multiple node updates.	FTSP algorithm works similar to Bellman-Ford algorithm with the inclusion of β -synchronizer to bound the message complexity for time-varying network. FTSP consists of two major steps: first is construction of spanning tree and second is computation of shortest path from every node to sink, layer-wise i.e. nodes which are lying in the nearest layer to sink are computed first and so on. Control messages play the task of doing acknowledgements about distance between nodes. This algorithm is further extended to dynamic networking named as FTSP-M. FTSP-M updates shortest path only for updated nodes without disturbing whole network and the process of spanning tree formation accomplished with the help of control messages.	This cannot solve the void path and routing loop problem which affect the real-time communication.
2016 [97]	Proposal of a 'delay-constrained energy multi-hop (DCEM)' algorithm to find balanced solution between optimization of energy consumption and delay.	A new energy-cost function is designed with the delay estimation including link, queuing, transmission and propagation delay to find the least cost route satisfying delay constraint. DCEM does network organization task including clusters, route formation and data transmission i.e. multi-hop inter cluster routing. DCEM operates the same task after every round. Two parameters α , related to CHs' residual energy and β , related to distance between CHs and sink, are engaged to establish the balance between energy consumption and end-to-end delay.	Designed for static network only. Consecutive network organization causes energy consumption.
2016 [113]	Proposal of an 'energy-efficient adaptive routing protocol (EE-ARP)' for real-time communication. EE-ARP ensures energy efficiency, better routing performance and increases network lifetime with least latency and few packets dropping.	This work involves the identification of slow data packets by calculating expected delay whose transmission deadline to reach destination are expired at any intermediate node and removal of those expired or much delayed packets from the queue of that node to eliminate its transmission. In this way, a queue maintains data packets of sufficient residual deadline only. Also nodes can vary their transmission power based on distance using two-hop ngnh information to reduce dropping of data packets and retransmission.	Designed for homogenous, static and dense network only. Disconnectivity and voidness can be a problem in less dense network.
2017 [95]	Proposal of a 'Delay-constraint energy efficient routing protocol based on Lagrange relaxation (DELAR)'. This heuristic algorithm has polynomial time complexity. Reduces avg delay and energy consumption. This method has been extended to find multiple shortest path.	DELAR consists of two phases: cluster setup and routing phase. An optimization problem is formulated in which link cost is taken as an aggregated function of energy consumption and transmission delay to find the least cost path from set of all paths from SNs to sink satisfying the delay constraint. Two heuristic algorithms are proposed, algorithm 1 is to find the optimal multiplier α which belongs to some interval $[a, b]$ which is further engaged in algorithm 2 to find optimal path of least cost satisfying end-to-end delay.	Analysis complexity increases with the increment of number of SNs. Defined for static network only.
2017 [104]	Proposes an energy efficient link delay aware routing algorithm. This approach is very well suited for real-time wireless communication as it reduces end-to-end delay and balances the optimization between energy consumption and delay factor.	The routing mechanism comprises of four phases, i.e., neighbour discovery, metric calculation, score calculation and node selection. Exchange of 'Hello' msg tracks the neighbour list. Residual energy, end-end delay and transmission counts are considered metrics and PRD value is the required score based on the given metrics. PRD demonstrates the predicted remaining counts of successful deliveries within one unit of delay. Next relay node has been selected for routing using PRD results best link quality, shortest end-to-end delay and highest residual energy.	Designed for directed and homogenous network. Authors didn't provide the complexity of the proposed work with the change of network density.

Table 1.6: Continued

Reference	Contribution	Description	Limitation/Future scope
2018 [109]	Proposed a distributed ‘Minimum-Delay Energy-efficient flooding Tree (MDET)’ algorithm to find the optimal solution to the multi-objective problem of minimizing delay and energy consumption in flooding based data transmission. MDET shows a very good balance between flooding delay and energy efficiency with minimization of flooding cost and delay.	MDET is designed to bring out the balance optimization between energy-delay in low duty cycled network with unreliable links by constructing energy-optimal tree with bounding flooding delay. It involves the process of Expected Transmission Count (ETX)-based shortest path formulation and corresponding nodes enhance energy efficiency locally by choosing its parent node with best link quality and loop free transmission. A biased back-off mechanism is adopted in flooding tree construction to reduce control message overhead.	Designed for static network with low duty cycle and unreliable wireless link. Reduction of delay is usually achieved at the price of de-truncate network lifetime.
2019 [102]	Proposed a shortest path strategy for MS for data aggregation while ensuring connectivity in a typically connected/disconnected network. This approach reduces end-to-end delay with load balancing. The proposed work is most suitable for delay-harsh applications where the delivery delay is much more important than energy efficiency.	The methodology consists of two phases: first phase determines the anchor points (APs) where MS visits to collect data. APs are selected such that some are close to network center and some are far from network center. So the target area has been partitioned into grids having APs and joining of 4 grids having distant APs from center results quadrilateral like structure. Second phase adds new APs to the existing structure until there exists an AP in every segment. The new APs must be near the current path as much as possible. DAAP considers disconnected segments of the network in an decreasing order of their minimum joining cost to obtain a final and shortest path for MS.	The proposed solution is localized and approximate one. This approach is more inclined to minimize delay so energy consumption is still persisting issue.
2019 [98]	Proposed a ‘delay aware energy efficient reliable routing (DA-EERR)’ technique for a large scale heterogeneous WSNs. The proposed work deals with hot-spot problem, improves network lifetime and reduces packet loss. This technique achieves a good balance between energy efficiency and fast data delivery in the large densely deployed heterogeneous WSN having mobile sink.	In DA-EERR, the network area has been partitioned into multiple rings with three types of nodes based on energy level i.e. energy of type A > type B > type C. Type B and C nodes sense the data packets and type C nodes collect and forward data while carrying the information of mobile sink node. For quick and easy data transmission, search space towards the destination node has been reduced. Every source node chooses relay node of highest energy level within min distance to transmit its data to mobile sink.	Defined for static network. The proposed work depends on excess beacon messages transmission.
2019 [99]	Proposal of a new ‘Delay Constraint Energy-efficient Multi-hop routing algorithm (DCEMRA)’ involving a new rule-based clustering and routing which builds balanced intelligent clusters and low cost routes. The major advantage of DCEMRA algorithm is the improvement in reliability, packet delivery ratio, network lifetime and reduction in energy consumption and delay.	Clustering process consists of three major steps: CH selection, cluster formation and tradeoff of energy and delay (TED) calculation. Selection of CH is based on nodes energy, RSSI and agent computed value to acknowledge ngh. Updation of CH need to be done when it doesn’t able to satisfy energy threshold. TED factor adjusts mobility parameters over the CHs energy expense and distance of CHs from MNs to bring out the balance between energy consumption and end-end delay. DFS algorithm is utilized to find low cost path satisfying the delay constraint during inter-cluster routing. The whole routing process consists of link and route cost calculation of all possible paths, end-end delay calculation of selected low cost shortest paths and multi-hop data communication.	Designed for static homogeneous network and work in a centralized way by sink node which is located at the centre of the field. Engagement of intelligent agents makes the process costly.
2020 [100]	Proposed a ‘Delay-aware Green Routing Protocol (DGRP)’ to deal with hot-spot problem in WSNs by engaging mobile sink node. Engagement of mobile sink nodes reduces the energy hole problem, end-end packet delivery, energy consumption and outperform general ring routing protocol (RRP) in large WSNs.	DGRP partitions the target area into optimal number of rings, by considering the length of the sensor field and communication range of SNs. The nodes which are closer to sink node in every ring chosen as high tier nodes which keep the location information of sink nodes and then high tier nodes, belonging to the same partition, reshape the ring by joining themselves. Second type of nodes are called low tier nodes which pass their data in one hop to high tier node belonging to the same ring. Thereafter high tier nodes pass the information either directly to sink node or minimum length neighbour high tier node by setting an acute angle towards the direction of sink node.	Homogeneous and static sensor nodes having same radio range limits the application of the proposed work.

Table 1.6: Continued

Reference	Contribution	Description	Limitation/Future scope
2021 [101]	Proposed a ‘Delay-Sensitive, Reliable, Energy-Efficient, Adaptive and Mobility-Aware (DREAM)’ routing protocol, incorporating nodes mobility and network heterogeneity. DREAM is the cluster-based solution to the multi-objective problem which is simultaneously dealing with energy consumption, network lifetime, latency, throughput, reliability and scalability.	In DREAM, target area has been partitioned into sectors where nodes are randomly spread and BS at the center. From each sector, a CH is selected based on the highest power among the competing nodes which are eligible to be CHs on the basis of residual energy and distance factor. MNs from each sector send their data to the CH and CH, depending on its distance from BS, can transmit data directly to BS or use other CH as a relay node to pass the data to BS.	This work utilizes single and two-hop routing in 2-D model of WSNs persisting redundant transmission issue. The work can be extended to 3-D model incorporating multi-hop routing and data redundancy issues.
2021 [106]	Proposed a real-time application based routing scheme to minimize end-to-end delay using two-hop information. This approach establish a good balance between energy efficiency and QoS level.	Every node keeps the record of its hop count from BS as height i.e. node with one hop away from BS has height 1 and so on. Data transmission has been carried out by estimating every two-hop away node’s residual energy, delay and link quality and so a PRI (Potential relay information) function has been formulated to estimate the possible forwarding nodes based on these parameters. After computation of PRI value, each node computes its neighborhood state index (NSI) based on PRI which depicts the quality of its surrounding nodes. Higher the value of NSI, higher the chance of being selected as a relay node during data transmission.	Designed for static and homogenous network. The proposed scheme utilizes two-hop information at the expense of computational complexity.

Chapter 2

Interval graph model and energy efficient routing scheme for a connected topology in WSNs

In this chapter¹ excessive power usage in transmission among the sensors can be reduced by designing an energy efficient routing topology which reduces nodes' energy consumption by abandoning those links which are highly energy expensive. We have introduced a new 'interval graph model' of WSN in which each node has been assigned with an appropriate power interval to hold the communication bidirectionally and obey the properties of being an interval graph. Two new polynomial time algorithms are proposed to optimize energy consumption on the interval graph model subject to constraint that the resultant topology remains strongly connected with minimum number of required links. Both the algorithms work to evaluate an energy optimal path cover whose path components combine to result an energy efficient connected routing structure (spanning path/tree) on interval graph model by reducing power usage over links and nodes in WSN. Optimization of max transmission cost in worst case scenario over a maximum weighted interval graph model and reduction of max power consumption limit of nodes to the min possible value are also discussed.

¹The content of this chapter is based on a research paper "Interval graph based energy efficient routing scheme for a connected topology in wireless sensor networks", *Wireless Networks*, **27**, 5085-5104 (2021). <https://doi.org/10.1007/s11276-021-02782-0> (SCIE, Impact Factor: 3)

2.1 Introduction

Wireless communication exists only when sensors have sufficient amount of power stored in them so that they can receive, sense and transmit data to the required destinations. Since sensors are powered by battery, they have limited power and so their life in WSNs. This requires energy-aware routing topology which can significantly save nodes power. Many authors like Li et al. [115], Panda and Shetty [3] believe that power consumption of nodes increases at least quadratically with distance while transmission, so they have proposed topology control algorithms to conserve energy by abandoning long-range communication links with the adoption of multi-hop data transmission. Li and Halpern [116] have revealed that it is not energy efficient to set every node at maximum transmit power in multi-hop WSN. As maximum transmit power of a node implies longer transmission range of that node which arises more unnecessary communication links with other nodes within its transmission disc hence causes higher interference. Therefore maintaining smaller number of communication links with reduced transmission range provided the whole network remains connected would speedup the routing protocol and simultaneously leads to energy saving [18],[117]. Reduced transmission range omits unnecessary interference since it reduces collisions of data packets along the route which in turn lowers node's energy consumption [4].

Panda and Shetty in [3] have modelled WSN as a complete weighted graph G^c where weight on each edge is the transmission cost, taken equal to the square of the Euclidean distance following the power dissipation theory on each bidirectional link, explained in definition 1.7. So power function r (refer definition 1.8) assigns transmit power to each node of G_c as $r(u) = \max_{uv \in E(G^c)} ||uv||^2$ where $u \in V(G^c)$, $V(G^c)$ is the set of nodes and $E(G^c) = \{uv : c(uv) = ||uv||^2\}$ is a set of bidirectional links/edges of G^c with transmission cost $c(uv)$. This follows that when a node u is assigned with fixed power level $r(u)$, it forms a transmission disc with transmission range $r(u)$ taken as a radius and u as centre. So whenever node u starts transmitting, data is received by all nodes which belong to its transmission disc. Hence this works good in case of broadcasting of data packets but in case of data transmission by node u to some particular nodes v such that $c(uv) \ll r(u)$, then this causes unnecessarily data transmission to the remaining nodes which come inside the disc of radius $r(u) = \max_{uv \in E(G)} c(uv)$. However this problem of unnecessary transferred data can be reduced to some level

by allowing nodes to decrease their transmission range according to the priority/order of data transmission.

In WSN, transmission power level of each node can be tuned between a minimum and a maximum value [3] to establish/demolish communication link with other nodes directly.

Definition 2.1. Let $r' : u \rightarrow [r_{min}, r_{max}]$ be another power function which assigns transmit power in the form of interval to each node u of a WSN G so that node can vary its transmission range between r_{min} to r_{max} where r_{min} is the minimum power required to establish/maintain at least a single connection in the network and r_{max} is the maximum power required to communicate the farthest node directly by u (refer definition 1.8).

Definition 2.2. Power interval PI : When the power assignment to each node has been done in the form of interval so that nodes can vary their transmission range from min to max power level (as given in definition 2.1) depending upon the transmission need. Then power levels are called power interval and denoted as PI . Therefore for a node u , $PI(u) = r'_u$.

The work of this chapter is dealing with WSNs where each sensor is static and assigned with unique PI like $[r_{min}, r_{max}]$ and incident links extract power ranging in the interval. This communication network, formed by wireless sensors, has an interval graph layout when two sensors communicate directly via a link if and only if their PI s have non-empty intersection. Not every WSN forms an interval graph layout, it can be checked in linear time [118]. For a WSN whose underlying graph is complete i.e. each sensor can directly approach to every other sensor by adjusting its transmission range then wireless network can definitely form an interval graph layout. Allowing nodes to adjust their transmission range in an assigned PI is quite helpful in saving energy when large number of nodes are deployed in a region having a complete communication network instead of assigning fixed transmission range to each node. In this chapter, wireless sensors are assigned with interval of transmit power levels forming a bidirectional interval graph model and goal is to find an energy efficient routing topology, covering all nodes with minimum number of required links on any interval graph model of WSN. Problem related to high energy consumption by links while transmission in WSN is under taking by newly proposed algorithms and resulting to minimize max upper limit of PI , minimizing max node's power consumption, max transmission cost and total transmission cost of the bidirectional links where links are using max power available to them i.e. worst case of power consumption. The aim of

proposing three new algorithms is to evaluate an energy efficient spanning path/path cover on an any interval graph model by selectively choosing minimum number of links based on power usage so that energy can be saved and result to an energy efficient connected topology (i.e. no loss of any sensor in the resultant network). First algorithm finds an energy optimal spanning path/path cover on a complete interval graph model and second algorithm is the generalisation of first algorithm, finds an energy optimal path cover on an arbitrary interval graph model. Achieved energy optimal path cover when contains more than one path component, is required to turn into a connected energy efficient routing structure (spanning path/tree) with the joining of distinct path components in an energy efficient way by the third algorithm. When energy optimal path cover contains single path component then path cover is itself a spanning path/tree.

To perform our task, following assumptions are made:

1. WSNs whose graphical layout are connected bidirectional interval graphs have only been considered in this work.
2. Communication links are assumed to be symmetric and bidirectional in behaviour.
3. Each sensor node becomes static after deployment.
4. Whenever interval graph is mentioned it must be understood as connected bidirectional interval graph model of WSN.
5. Sensors are position-aware of their neighbourhood nodes in term of co-ordinates.

The rest of the chapter has been organised as follows, section 2.2 provides the basic study on interval graphs and its properties. Section 2.3 introduces interval graph models of WSNs then section 2.4 provides the study on formation of maximum weighted interval graph in worst case of power consumption. Section 2.5 introduces problem definition on right-end ordered complete interval graph model of WSNs and a new methodology i.e. Algorithm 2.1. Detail discussion on working of Algorithm 2.1 and corresponding results have also been provided in section 2.5. Section 2.6 provides the discussion on the formation of interval weighted complete interval graph and optimization by Algorithm 2.1 in general case. In section 2.7, discussion on arbitrary interval graph model and proposal of Algorithm 2.2 along with its explanation have been provided. Formation of connected routing topology by the proposed Algorithm 2.3 and some corresponding results have been given in subsection 2.7.1. Performance evaluation of proposed algorithms have been given in section 2.8 then

section 2.9 shows the simulation results. Lastly, section 2.10 provides a conclusion of the chapter.

2.2 Interval graph and its properties

A graph G is called an interval graph if there exists an one-to-one mapping between graph vertices and a family of intervals on real line such that adjacency of any two vertices corresponds to a non-empty intersection of their intervals. $IM(G)$ is called an intersection model for G [2]. An interval graph can be obtained by just knowing an intersection model of it and viceversa. Fig. 2.1 and 2.2, redrawn from [2], show

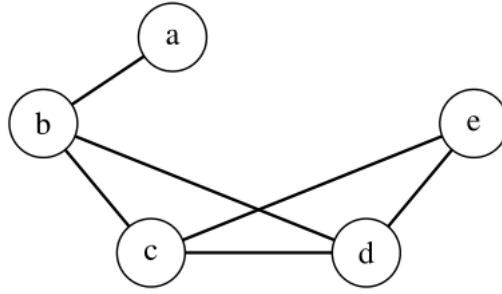


Figure 2.1: Interval graph G

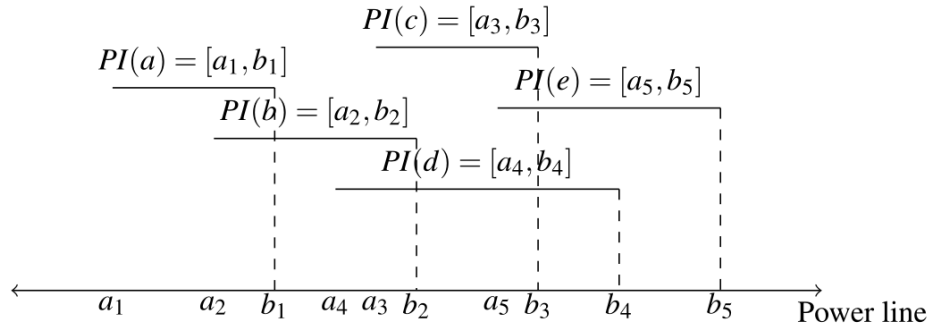


Figure 2.2: Intersection model $IM(G)$ of G

an interval graph G of five vertices a, b, c, d, e and its corresponding intersection model $IM(G)$, where each vertex is weighted with a PI , represented by $PI(a) = [a_1, b_1]$, $PI(b) = [a_2, b_2]$, $PI(c) = [a_3, b_3]$, $PI(d) = [a_4, b_4]$, $PI(e) = [a_5, b_5]$ respectively, has a unique plot on power line ($\mathbb{R}^+$) representing continuous range of power levels. For any adjacent vertices like a and b , their corresponding PI s overlap and for non-adjacent vertices like b and e , their PI s don't overlap, demonstrated in Fig. 2.2. This implies that sensor nodes are directly accessible to each other if and only if they have non-empty intersection of their PI s.

2.3 Interval graph model of WSNs

Let $N = \{u_1, u_2, \dots, u_n\}$ be a set of sensor nodes of a WSN. We model a weighted interval graph $G(V, E)$ with each node is taken as a vertex forming a vertex set $V(G)$ and each communication link is taken as an edge forming an Edge set $E(G)$. Let $\{r'_{u_1}, r'_{u_2}, \dots, r'_{u_n}\}$ be the corresponding power assignment to each node in the form of $r'_{u_1} = [r_{min}(u_1), r_{max}(u_1)]$, likewise each node is assigned with this kind of transmit power interval (or *PI* can be said) such that no two vertices have equal power assignment, at least they must be differ by epsilon(ϵ) quantity. That epsilon difference is very minute does not have any effect on the established network but helps to satisfy an interval graph definition and property. Adjacency of any two vertices implies a non-empty intersection of *PIs* assigned to both the vertices and viceversa, description is given in section 2.2.

In WSNs, each sensor can detect the location of every other sensor either through global positioning system (GPS) or manual pre-programming during deployment [119] and so Euclidean distance among themselves [120]. Since we are dealing with non-mobile sensors deployed in any region to detect and transmit signals simultaneously, knowledge of position and Euclidean distance of neighbour nodes must be preserved by each wireless node and this works enough to design an energy efficient routing structure connecting all sensors.

Let's move towards the right end ordering of an interval graph since right end ordered interval graph has been an important part of this chapter.

2.3.1 Right-end ordered interval graph

A right end ordering of an interval graph $G(V, E)$ can be obtained by sorting intervals in its intersection model at their right ends in time $\mathcal{O}(|V(G)| + |E(G)|)$ [121]. Looking at the intersection model $IM(G)$ in Fig. 2.2 where $PI(a) = [a_1, b_1]$, $PI(b) = [a_2, b_2]$, $PI(c) = [a_3, b_3]$, $PI(d) = [a_4, b_4]$, $PI(e) = [a_5, b_5]$ are the power intervals assigned to nodes a, b, c, d, e of G respectively. Every interval has two extreme values so a right end value is the upper bound of power consumption, therefore each vertical dotted line in $IM(G)$ shows the right end value of its corresponding power interval. Relabelling of nodes of G as $u_1, u_2, \dots, u_5$ according to the fall of right end on the power line going from left to right i.e. node corresponds to first dotted line is relabelled as u_1 , node corresponds to second dotted line is relabelled as u_2 and like so on until all

dotted lines are counted and all nodes get relabelled. Fig. 2.3 represents a right end ordered interval graph of G and Fig. 2.4 shows its corresponding right end ordering.

A right end ordered interval graph makes easy to compare the upper limit of pow-

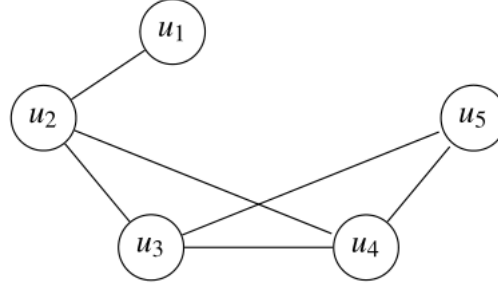


Figure 2.3: Right end ordered interval graph of G

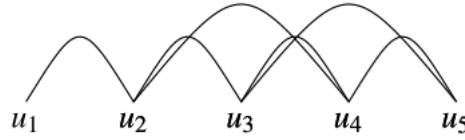


Figure 2.4: Right end ordering of G

er consumption of each node. This is because upper limits/right end value of power intervals are in increasing order with subscript of vertices and it can be seen in Fig. 2.2 that the upper limit of power interval of $u_1 < u_2 < u_3 < u_4 < u_5$. Hence vertices' subscript plays an important role in this chapter.

Definition 2.3. A vertex whose subscript is the minimum number among all vertices' subscript of a right-end ordered interval graph G_R is called *leftmostvertex*(LMV) of G_R and a vertex whose subscript is the maximum number is called *rightmostvertex*(RMV) of G_R .

leftmostvertex and *rightmostvertex* are abbreviated to LMV and RMV respectively.

$$LMV(G_R) = \{u_j | j = \min_{v_i \in V(G_R)}(i)\} \quad (2.1)$$

$$RMV(G_R) = \{u_k | k = \max_{v_i \in V(G_R)}(i)\} \quad (2.2)$$

A right-end ordered interval graph has some properties [2] which help to build a routing tree with an obtained path cover which is having more than one path component. Considering connected graphs only, every path in a right end ordered interval graph must intersect with atleast one other path.

Property 2.1: For every path p on a right end ordering graph, there must be another path $q \neq p$ such that p and q are intersecting.

Property 2.2: Let G_1 and G_2 be two intersecting subgraphs of a right-end ordered interval graph G_R . If $leftmost(G_1) < leftmost(G_2)$, then $leftmost(G_2) < rightmost(G_1)$.

Remark 2.3.1. If an interval graph model of a WSN has labelling in the form of $u_1, u_2, \dots, u_n$ in this chapter, then it should be understood as a right-end ordered interval graph. Also from now onwards in this chapter, mentioned interval graph must be understood as right-end ordered interval graph.

2.4 Formation of maximum weighted interval graph by wireless sensors

Let us consider a WSN with the knowledge of number of nodes and Euclidean distance between pair of nodes where each node can approach to every other node. Euclidean distance between the nodes a and b is taken as $||ab||^2$ in bidirectional topology and $c(ab) = ||ab||^2$ (i.e. $\alpha = 2$) is the symmetric transmission cost for a bidirectional link ab [3]. In this chapter, transmission cost for any link, based on Euclidean distance only, is taken as a minimum guess/approximate value to assign PI to its corresponding end nodes and allow transmission cost to increase in a particular range/power interval available on each link in order to enhance quality and quantity of signals to travel and reach destination successfully. For better understanding, let us consider a complete wireless network of 5 sensor nodes, given in Fig. 2.5(a), redrawn from [3].

Consider node 'a' of Fig. 2.5(a), mention all the links incident on 'a' with Euclidean distance i.e. $||ab||^2 = 252, ||ac||^2 = 14, ||ad||^2 = 589, ||ae||^2 = 34$. Minimum and maximum weight value among all the links incident on 'a' is 14 and 589 respectively. Therefore, $r'_a = [14, 589]$ is the range of power assigned to node 'a' so that all required communication can take place by node 'a' and likewise $r'_b = [75, 252], r'_c = [12, 428], r'_d = [75, 589]$ and $r'_e = [12, 452]$ are computed power intervals, assigned to the corresponding nodes b, c, d and e respectively. Then intersection model provides right-end ordering of PIs . Fig. 2.5(b),(c),(d) have shown the transformation of a weighted bidirectional complete graph into a complete right-end ordered interval graph using intersection model.

Definition 2.4. Maximally assigned power node: A node having power interval with max upper bound means the node has max upper limit of power consumption i.e. $max PI$, is called maximally assigned power node. A maximally assigned power node retains a communication

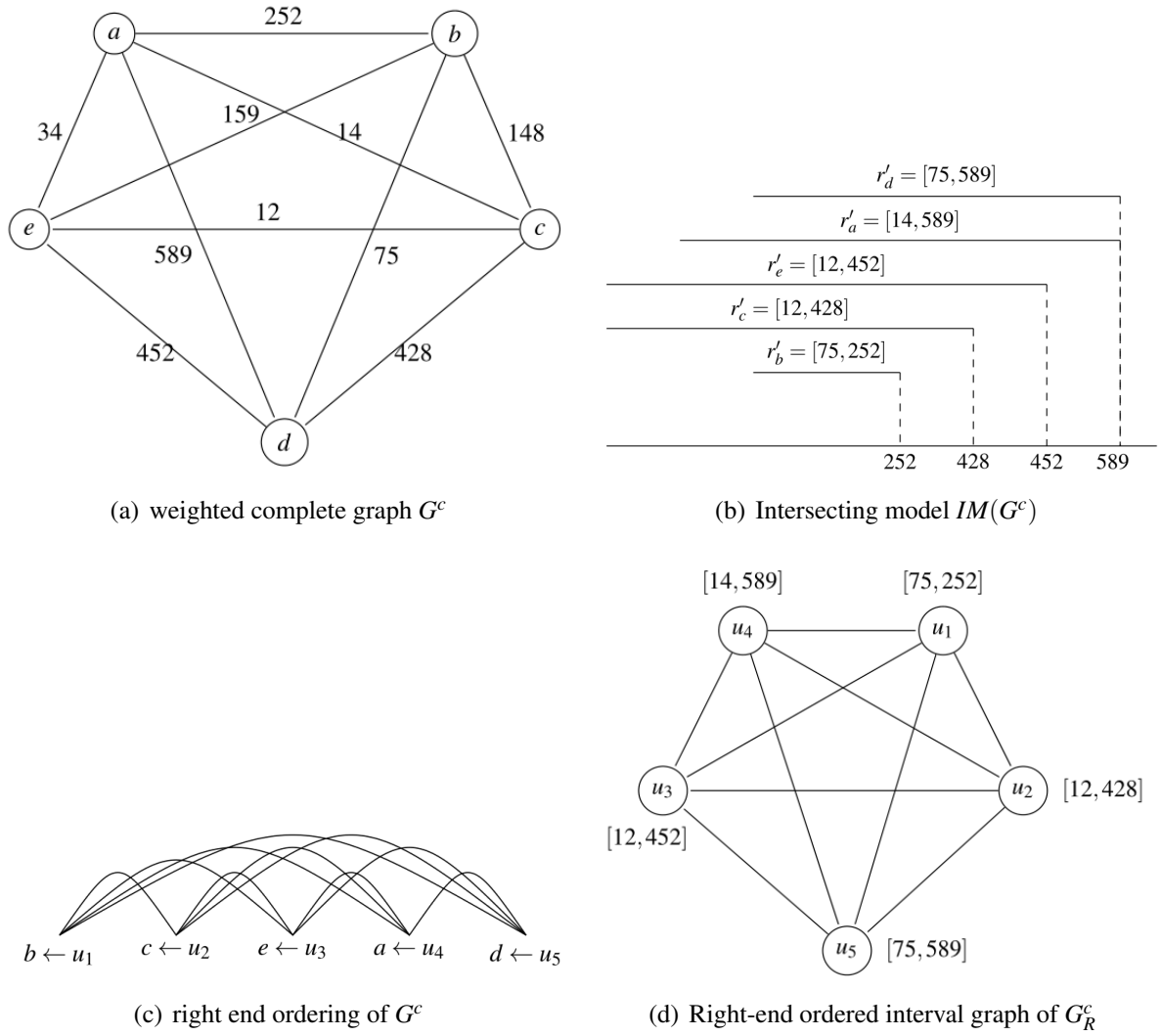


Figure 2.5: Transformation of a complete bidirectional graph into right-end ordered interval graph

link with max transmission cost as in Fig. 2.5(a) nodes a and d retain a communication link of max transmission cost 589 and so their PI s in Fig 2.5(d) have max upper bound. Therefore a and d , relabelled as u_4 and u_5 respectively are called maximally assigned power nodes. One of the maximally assigned power nodes is a *RMV* of a network based on higher subscript as u_5 is a *RMV*.

Definition 2.5. Minimally assigned power node: A node having power interval with min upper bound means the node has min upper limit of power consumption i.e. *min PI*, is called minimally assigned power node as in Fig. 2.5(d) node u_1 is minimally assigned power node. A minimally assigned power node is a *LMV* of a network.

Definition 2.6. Available power interval: All PI s in a complete interval graph are intersecting to each other because every node is bidirectionally approachable to every other via bidirectional links. Common link contributes its weight value to its end nodes' power inter-

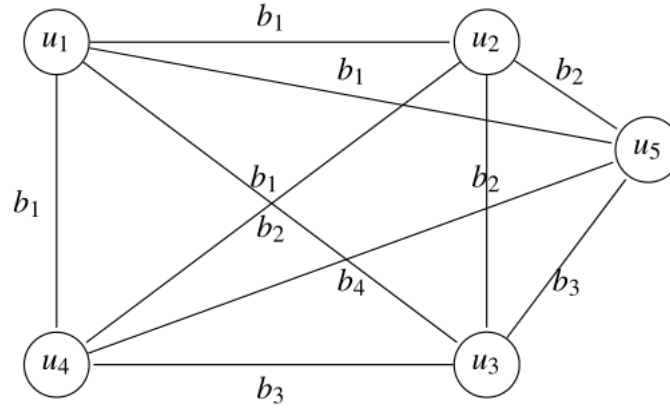


Figure 2.6: Maximum weighted complete interval graph

val so adjacent nodes in a complete interval graph always have non-empty power intersection. This non-empty power intersection contributes weight to the existing links in a form of interval called ‘available PI ’ of each link.

Let us move forward to see maximum weighted complete interval graph where each link is using maximum power from its available power interval during transmission. Let $PI(u_1) = [a_1, b_1], PI(u_2) = [a_2, b_2], PI(u_3) = [a_3, b_3], PI(u_4) = [a_4, b_4]$ and $PI(u_5) = [a_5, b_5]$ be the assigned range of power to the corresponding nodes, forming a complete interval graph G^c .

Since labelling of vertices are subscripted with integers $1, 2, \dots, |V|$ using right end ordering, therefore we have $b_1 \leq b_2 \leq b_3 \leq b_4 \leq b_5$ (more often $b_1 < b_2 < b_3 < b_4 < b_5$). Larger the value of b_i , more is the power emission through that node. Nodes u_1, u_2, u_3, u_4, u_5 are subscripted as increasing order of their power emission capacity in the network and adjacency of every two nodes implies a non-empty PI intersection with each other. Let $PI(u_1u_2)$ be a non-empty PI intersection of adjacent nodes u_1 and u_2 , then $PI(u_1u_2) = PI(u_1) \cap PI(u_2)$ i.e. $[max\{a_1, a_2\}, min\{b_1, b_2\}] = [max\{a_1, a_2\}, b_1]$ ($\because b_1 \leq b_2$). Let us consider the weight of the link u_1u_2 is denoted by $wgt(u_1u_2)$ has one of the value from $PI(u_1u_2)$. Considering a worst case scenario of power consumption where every link is using maximum power available to them (more than they actually need) during transmission i.e. $max wgt(u_1u_2) = supremum\{PI(u_1u_2)\} = b_1$. Likewise, we have computed maximum weight on every link to obtain maximum weighted bidirectional complete interval graph, shown in Fig. 2.6.

b_4 is the amount of max power consumption in the worst case corresponding to the link u_4u_5 over a complete network of 5 vertices, given in Fig. 2.6. Hence without the loss of generality, max transmission cost is always found on link $u_{n-1}u_n$ on any interval

graph of n vertices when worst case happens.

The first goal of this chapter is to minimize max transmission cost of the worst case scenario so that energy could be conserved even when links are using max power available to them during transmission.

2.5 Problem definition and proposed Algorithm 2.1

Min-max TCP (Minimization of maximum transmission cost problem): Given a bidirectional complete right-end ordered interval graph with n nodes, find an energy optimal connected routing structure (spanning path/tree) such that the maximum transmission cost gets minimize over the link in the worst case of power consumption.

To deal with Min-max TCP on a complete right-end ordered interval graph model of WSN, a linear time algorithm has been proposed named as Algorithm 2.1. For the requirement of a connected network i.e. no loss of node in the resultant graph, Algorithm 2.1 computes a path cover (covering all the vertices of a complete interval graph) which optimizes network transmission cost of worst case. Algorithm 2.1 is applicable to complete interval graph and the path cover obtained on complete interval graph is a spanning path since a single path component spans all the nodes.

Algorithm 2.1 Find an energy optimal path cover on complete interval graph model of WSN

Input : n , number of vertices of a complete interval graph G^c .

Output : A path cover/spanning path of G^c which minimizes the maximum transmission cost over the link.

```

1: Define  $V = \{u_1, u_2, \dots, u_n\}$  ▷ (A right end ordered vertex set of  $G^c$ )
2: Path is empty initially i.e.  $P = \phi$ 
3: Take  $i = 1$  and  $j = n$  ▷ ( $i$  and  $j$  denote the subscript of vertices  $u_i$  and  $u_j$  respectively)
4: while ( $j \geq i$ ) do
5:    $LMV = u_i$ 
6:    $RMV = u_j$ 
7:   if  $LMV \neq RMV$  then
8:      $P = P \cup RMV \cup LMV$  ▷ ( $\cup$  shows the joining of vertices in path  $P$ )
9:   else  $P = P \cup RMV$ 
10:   $i = i + 1$  and  $j = j - 1$ 
11: return  $P$ 

```

The basic theme of Algorithm 2.1 is to first find maximally assigned power node which is the RMV , join it with the minimally assigned power node which is the LMV of G^c , then minimally assigned power node is joined to second maximally assigned power node, second maximally assigned power node to second minimally assigned

power node and so on until all the vertices get covered in path. These joining are nothing but the links which are using max power from their available PI . A link which is a joining of maximally assigned power interval with the minimally assigned power interval has min limit of power consumption in transmission. So joining by this way preserves those links which can form a spanning path with low power consumption by abandoning high power usage links.

In other words, a path computed by Algorithm 2.1 is nothing but a joining from RMV to LMV , LMV to second RMV , second RMV to second LMV , second LMV to third RMV and so on until all vertices get covered in path with minimum number of required edges. But instead of mentioning second RMV , third LMV and all, updating RMV and LMV among the uncovered vertices has been preferred.

Step by step description of Algorithm 2.1

Algorithm 2.1 takes an input of n number of vertices of a right end ordered complete interval graph.

Step 1. Defines V , set of n vertices, in ascending order of their subscript.

Step 2. Initially path P has covered no vertex so it is taken as empty.

Step 3. Two distinct variables i and j are taken to denote the subscripts of vertices, starting with the opposite ends of V i.e i starts denoting from leftmost and j from rightmost of V .

Step 4. Repetition of steps from 5 to 10 until the subscript number j becomes less than subscript number i .

Step 5. Vertex with the subscript i becomes LMV .

Step 6. Vertex with the subscript j becomes RMV .

Step 7. It checks whether LMV is same as RMV if it is not then

Step 8. Path P starts with RMV and join it to LMV .

Step 9. Otherwise path P visits RMV only.

Step 10. i adds one to its subscript number and j subtracts one from its subscript number.

Step 11. Returns the Spanning path P after the termination of step 4.

Example 2.5.1. Working illustration of Algorithm 2.1

Let us consider an interval graph G^c of 5 vertices, given Fig. 2.6 where $PI(u_1) = [a_1, b_1]$, $PI(u_2) = [a_2, b_2]$, $PI(u_3) = [a_3, b_3]$, $PI(u_4) = [a_4, b_4]$ and $PI(u_5) = [a_5, b_5]$. First evaluate LMV and RMV of G^c in Fig. 2.6 so $RMV = u_5$ and $LMV = u_1$. Path is initiated with

$RMV = u_5$, u_5 looks all the way to choose min among the max weighted links which is u_5u_1 , so u_5 maintains single link with u_1 i.e. RMV is joined to LMV . Path becomes u_5u_1 , u_5 and u_1 are covered in path so it is assumed that u_5 and u_1 are eliminated from the vertex set of G^c since covered vertices are not considered in further evaluation. Therefore RMV and LMV of G^c are updated to u_4 and u_2 respectively and path becomes $u_5u_1u_4u_2$. Now only one vertex u_3 is left to cover in path and after updation, $RMV = LMV = u_3$, thus final path becomes $u_5u_1u_4u_2u_3$, by covering all the 5 vertices of G^c . Fig. 2.7 shows the resultant path cover which is a spanning path, obtained by applying Algorithm 2.1 on Fig. 2.6 and max transmission cost minimizes from b_4 to b_2 in worst scenario of transmission.

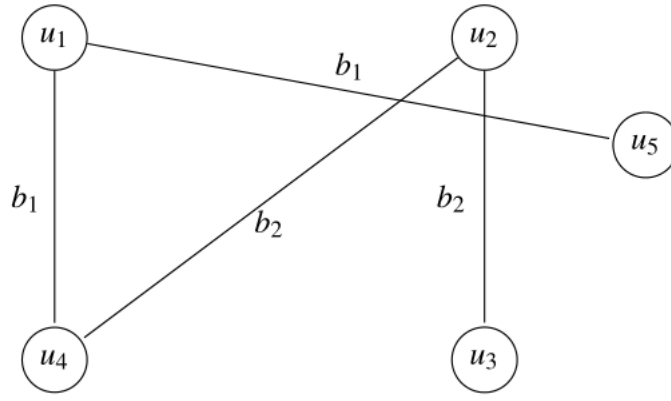


Figure 2.7: Path cover/spanning path by Algorithm 2.1 on G^c given in Fig. 2.6

Theorem 2.5.1. Algorithm 2.1 gives an optimal spanning path in term of minimizing maximum transmission cost where each link is using maximum power available to it i.e. worst case of power consumption during transmission.

Proof. The proof is trivial for $n = 1$ and 2, let's start proving for $n = 3$. Find all possible spanning paths traced in a complete interval graph G^c of 3 vertices. These are as follow:

$$u_1u_2u_3 \rightarrow b_1 + b_2$$

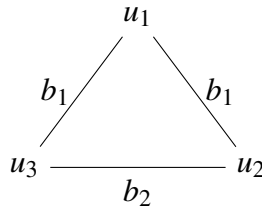
$$u_1u_3u_2 \rightarrow b_1 + b_2$$

$$u_2u_3u_1 \rightarrow b_2 + b_1$$

$$u_2u_1u_3 \rightarrow b_1 + b_1$$

$$u_3u_1u_2 \rightarrow b_1 + b_1$$

$$u_3u_2u_1 \rightarrow b_2 + b_1$$



Algorithm 2.1 provides spanning path ' $u_3u_1u_2$ ' whose maximum transmission cost is b_1 and b_1 is the minimum value can exist in any path of G^c . Hence maximum transmission cost minimizes to the optimal value for $n = 3$.

Let's prove the optimality of Algorithm 2.1 for $|V| = n$ by contradiction method. Spanning

path traced by Algorithm 2.1 for n number of nodes is given by Eq. (2.3) and sum of max transmission cost over each link is given by Eq. (2.4).

$$\begin{cases} u_n u_1 u_{n-1} u_2 \dots u_{\lfloor \frac{n}{2} \rfloor + 2} u_{\lfloor \frac{n}{2} \rfloor} u_{\lfloor \frac{n}{2} \rfloor + 1}, & n \text{ is odd} \\ u_n u_1 u_{n-1} u_2 \dots u_{\lfloor \frac{n}{2} \rfloor - 1} u_{\lfloor \frac{n}{2} \rfloor + 1} u_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is even} \end{cases} \quad (2.3)$$

$$\begin{cases} 2b_1 + 2b_2 + \dots + 2b_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is odd} \\ 2b_1 + 2b_2 + \dots + b_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is even} \end{cases} \quad (2.4)$$

Max transmission cost value over the link is $b_{\lfloor \frac{n}{2} \rfloor}$ for the case $|V| = n$. Let the spanning path given by Eq. (2.3), is not optimal, then there exists atleast one another spanning path whose max value is strictly less than $b_{\lfloor \frac{n}{2} \rfloor}$. Let P' be another spanning path whose max transmission cost over a link is minimum. Therefore no edge in P' has value equal to $b_{\lfloor \frac{n}{2} \rfloor}$ and values over every edge must be less than $b_{\lfloor \frac{n}{2} \rfloor}$. But number of edge count must be equal in both the spanning paths, it means in path P' , $b_i \{1 \leq i < \lfloor \frac{n}{2} \rfloor\}$ repeats more than twice to cover the place of $b_{\lfloor \frac{n}{2} \rfloor}$ in Eq. (2.4) since every $b_i (1 \leq i < \lfloor \frac{n}{2} \rfloor)$ has already contributed twice in P' . Therefore in order to have equal count of edges, one/two of the value in $\{b_i, 1 \leq i < \lfloor \frac{n}{2} \rfloor\}$ has to come thrice in Eq. (2.4), depending on whether n is odd or even, which implies that one of the vertex, belonging to $\{u_i, 1 \leq i < \lfloor \frac{n}{2} \rfloor\}$ repeats more than once in P' which contradicts the definition of path. Hence, our assumption is wrong that P' is optimal spanning path. Algorithm 2.1 provides optimal spanning path in term of minimizing maximum transmission cost over the link in worst case. $\square$

Proposition 2.5.1. Optimal spanning path obtained by Algorithm 2.1 minimizes total of max transmission cost over each link in worst case of power consumption.

2.6 Interval weighted complete interval graph

In previous section, optimization of energy consumption has been done where links are conditionally weighted with maximum power from an available power interval (worst case scenario) and so a maximum weighted complete interval graph has been formed with crisp power value at all links. This is one of a particular case of the general theory followed from this section. This general concept of interval weighted complete interval graph come into picture when links are considered to use any power level ranging in their available power intervals in a complete network according to their necessity. This available power interval is a power consumption interval of links. Links

need to vary their power level while transmission according to different surrounding characteristics, quality and quantity of data packets to deliver and rate of delivery. So allowing wireless links to be weighted with their available power interval, creates an interval weighted interval graph. Proposed Algorithm 2.1 works well in optimizing power consumption in interval weighted interval graph also. For better understanding of this type of graph, let us initiate with an example of complete network and energy optimization by Algorithm 2.1.

Example 2.6.1. Consider a right-end ordered complete interval graph G^c given in Fig. 2.5(d). Each edge of Fig. 2.5(d) is a wireless link, extracting power from its corresponding end nodes, end nodes which are adjacent contributing a power interval to their common link and allow link to use any power level ranging in that available power interval according to its transmission necessity.

Table 2.1 shows the available power interval on each link of G^c . Available power interval on each link is taken as edge weight in G^c . Interval weight on edge u_1u_2

$PI(u_1u_2) = [75, 252]$	$PI(u_2u_4) = [14, 428]$
$PI(u_1u_3) = [75, 252]$	$PI(u_2u_5) = [75, 428]$
$PI(u_1u_4) = [75, 252]$	$PI(u_3u_4) = [14, 452]$
$PI(u_1u_5) = [75, 252]$	$PI(u_3u_5) = [75, 452]$
$PI(u_2u_3) = [12, 428]$	$PI(u_4u_5) = [75, 589]$

Table 2.1: Available power interval to each link of G^c

is a non-empty intersection of power intervals of node u_1 and u_2 i.e. $wgt(u_1u_2) \in PI(u_1u_2) = [\max\{75, 12\}, \min\{252, 428\}] = [75, 252]$, likewise $wgt(u_1u_3) \in PI(u_1u_3) = [\max\{12, 75\}, \min\{252, 452\}] = [75, 252]$ and similarly all the available power intervals are computed and weighted on its corresponding links. In this manner, network given in Fig. 2.5(d) becomes an interval weighted complete interval graph where each link's interval weight is given in Table 2.1.

When Algorithm 2.1 is applied on the network (given in Fig. 2.5(d)), it results to a spanning path $= u_5u_1u_4u_2u_3$ (given in Fig. 2.8) in which interval weight of each edge is shown.

This resultant path has connectivity among all the nodes with minimum number of links. Abandoning unnecessary links lead to conservation of power of each node, hence enhancement to lifetime of wireless devices because more links create more interference and hindrance in networking.

Power interval of a node u must contain intervals of all the links incident on it, therefore resultant power assignment to each node when links are weighted with power interval, has

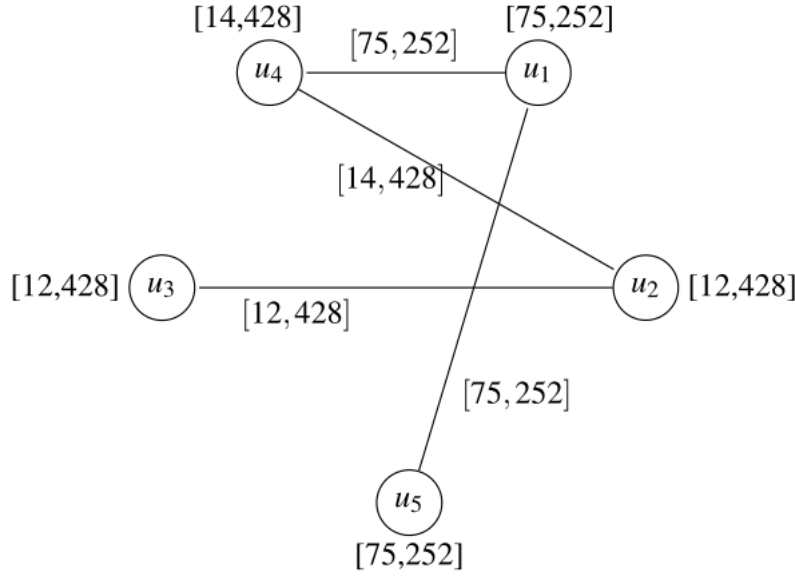


Figure 2.8: Resultant path cover/spanning path by Algorithm 2.1 with interval weight on each edge

been computed using Eq. (2.5)

$$r'_u = [\min\{\text{lower bounds}\}, \max\{\text{upper bounds}\}] \quad (2.5)$$

where lower bounds and upper bounds are taken over every incident link's power interval. Let's see the resultant power consumption interval of each node in the traced spanning path given in Fig. 2.8, for a node u_1 : edges u_1u_4 and u_1u_5 are incident on u_1 and interval weights on the edges are same and equal to $[75,252]$, therefore $r'_{u_1} = [75,252]$. For node u_2 , incident links are u_2u_4 and u_2u_3 having weights $[14,428]$ and $[12,428]$ respectively, therefore $r'_{u_2} = [\min\{12,14\}, \max\{428,428\}] = [12,428]$ similarly $r'_{u_3} = [12,428]$, $r'_{u_4} = [\min\{14,75\}, \max\{252,428\}] = [14,428]$ and $r'_{u_5} = [75,252]$ by Eq. (2.5). u_5 which is maximally assigned power node with $PI = [75,589]$ in the original network (Fig. 2.5(d)) reduces to $[75,252]$ in the spanning path (Fig. 2.8) traced by Algorithm 2.1.

Remark 2.6.1. Maximally assigned power interval of a node is the interval which is having maximum power emission and one of the incident link has that much power usage. It is also observed that right-end value of assigned power interval to each node is the power consumption of one of the incident link in an original interval graph. That's why algorithm works to eliminate those highly power usage links.

Remark 2.6.2. Comparison among the power intervals are done on the basis of each interval maximum power consumption capacity which is the upper bound of each power interval.

This is because nodes' power consumption can be reduced by eliminating those links whose available power interval are maximally upper bounded and consequently reduces power consumption interval of nodes from the right end. Diminished power interval won't lead to high power consumption while transmission. That is why right extreme values of assigned power intervals are compared.

Theorem 2.6.1. *RMV* of a complete interval graph which is a maximum value of power consumption in the network reduces to the minimum possible value, being an upper bound in the resultant path cover obtained by Algorithm 2.1.

Proof. An interval $[a_i, b_i]$ is a range of power emission capacity assigned to a node i . The value of b_i reveals the maximum power emission capacity/limit of power consumption of a node i in transmission. Since in a right end ordered interval graph, *LMV* $u_1 = [a_1, b_1]$ is minimally upper bounded by b_1 and *RMV* $u_{|V|} = [a_{|V|}, b_{|V|}]$ is maximally upper bounded by $b_{|V|}$ which means one of a link incident on $u_{|V|}$ consumes maximum power equal to $b_{|V|}$ but the path traced by Algorithm 2.1, is initiated by a joining from *RMV* to *LMV* to terminate that link from the network and this joining has weight $\in [\max\{a_1, a_{|V|}\}, b_1]$, and since *RMV* has only single link incident on it in the resultant path hence $r(u_{|V|})$ is upper bounded by b_1 which is the minimum among all the upper bounds. Hence maximum upper limit of power consumption in the network reduces to minimum value possible to save the energy. $\square$

Theorem 2.6.2. Path cover obtained by Algorithm 2.1 minimizes maximum node power consumption i.e. $\max PI$ optimally.

Proof. Path cover obtained by Algorithm 2.1 consists of single path component covering all vertices so let P be that spanning/routing path obtained by Algorithm 2.1. Assume that it is not optimal and P' is an optimal routing path. Consider a link with maximum power consumption interval in P , say l . Then link l does not belong to P' (otherwise path P would have been optimal). Also the power consumption intervals of links in P' is less than l 's. This means a connected path can be constructed using links whose power consumption interval is less than the power consumption interval of link l . In that case our Algorithm 2.1 would have terminated before processing l and this is contradiction. $\square$

Theorem 2.6.3. Algorithm 2.1 takes $\mathcal{O}(n)$ time, where n is number of nodes.

Proof. Step 1 takes $\mathcal{O}(n)$ time. Step 2 and 3 can be done in constant time. Steps inside the while loop take constant time since *LMV* and *RMV* can be directly assigned by using subscript and while loop repeats at the most n times. So total time complexity can be $\mathcal{O}(n)$. $\square$

2.7 Arbitrary interval graph and proposed Algorithm 2.2

Till now, discussion has been carried out on complete interval graph only, what if underlying graph of WSNs form an incomplete connected bidirectional interval graph. For a communication network topology whose graphical layout is an incomplete connected interval graph, a methodology in the form of algorithm (say Algorithm 2.2) has been proposed to optimize energy consumption of the established network. Algorithm 2.2 presents a general method for providing an optimal path cover with respect to low power consumption on an induced arbitrary interval graph model of WSN. Obtained path cover results to an energy efficient routing structure connecting all nodes with minimum number of required edges. That routing structure can be a spanning path or tree, depending upon the path components obtained in path cover. If there is a single path component covering all vertices then the routing structure is a spanning path else it can be a spanning tree. When Algorithm 2.2 applies to a complete interval graph, it gives the same optimal path cover/spanning path as Algorithm 2.1 does and some similar results are also seen with a communicating wireless network whose graphical layout need not to be a complete interval graph.

The working of Algorithm 2.2 is started with RMV of an arbitrary interval graph same as Algorithm 2.1 and joined to LMV among its neighbour vertices. Since graph can be incomplete so LMV among the neighbour vertices need not to be a LMV of the whole graph. Hence every vertex keeps two sets, say S and T , consisting of unvisited/uncovered neighbour nodes. Let u be the vertex, first set S contains those unvisited neighbour vertices of u whose power interval is greater than the power interval of u and second set T contains those unvisited neighbour vertices of u whose power interval is less than the power interval of u . Algorithm 2.2 prioritizes set S first and join the vertex u with another vertex of max power interval from S i.e $RMV(S)$. T is considered when S of u is empty, then u selects vertex with minimum power interval from T i.e. $LMV(T)$ to extend the path further. Since initially chosen RMV which is a RMV of the whole graph has its S empty so it is joined to LMV of its T set. This is how two sets S and T play role in computing path cover by Algorithm 2.2 using RMV and LMV on an arbitrary interval graph model. For better understanding, an illustration (Example 2.7.1) has been provided following the steps of Algorithm 2.2.

Algorithm 2.2 Find an energy optimal path cover on any arbitrary interval graph model of WSN

Input : An arbitrary right-end ordered interval graph $G_R = (V, E)$ with $|V| = n$ and $|E| = m$.

Output : A path cover/spanning path of G_R which reduces max power consumption of the network optimally.

```

1: Define an array of n vertices,  $V = \{u_1, u_2, \dots, u_n\}$ 
2: Let path cover as  $opc = \phi$  initially.  $\triangleright$  ( $opc$  is a set of distinct path components, required
   to cover all vertices of  $G_R$ )
3: Let  $uncoverV = V$  initially.  $\triangleright$  ( $uncoverV$  = a set of vertices which are not yet covered in
   any of the distinct path of  $opc$ )
4: Take  $num = 1$ .  $\triangleright$  ( $num$  is the number of distinct paths and  $num = 1$  denotes the first path
   is to be begun)
5: Path is started with  $u_n \in V$  ( $RMV$  of  $G_R$ ) i.e.  $Pnum = u_n$ 
6:  $uncoverV = uncoverV / \{u_n\}$ 
7: while  $uncoverV \neq \phi$  do
8:   calculate  $S = \{u_k \mid u_k \text{ is an unvisited neighbour node of } u_n \text{ and } k > n\}$  and  $T =$ 
      $\{u_k \mid u_k \text{ is an unvisited neighbour node of } u_n \text{ and } k < n\}$ .
9:   if  $S$  and  $T$  of  $u_n$  is empty then
10:     $[v = RMV(uncoverV); num = num + 1; Pnum = v]$ 
11:   if  $S \neq \phi$  then
12:     $[v = RMV(S); Pnum = Pnum \cup v]$   $\triangleright$  ( $\cup$  shows the joining of vertices)
13:   else
14:     $[v = LMV(T); Pnum = Pnum \cup v]$ 
15:    $u_n = v$ 
16:    $uncoverV = uncoverV / \{u_n\}$ 
17: return  $opc = \{P_1, P_2, \dots, Pnum\}$ 

```

Step by step explanation of Algorithm 2.2

Algorithm 2.2 takes an input of an arbitrary right end ordered interval graph G_R of n vertices and m edges.

Step 1. A vertex set V of n vertices is defined in ascending order of their subscript.

Step 2. Initially path cover opc is empty i.e. no vertex is visited/covered by any path component.

Step 3. Define another set $uncoverV$ which consists of unvisited vertices of V so initially $uncoverV$ is taken same as V .

Step 4. Significance of taking num variable is to obtain distinct number of path components of opc as $opc = \{P_1, P_2, \dots, Pnum\}$. So num is starting with 1 to start with P_1 .

Step 5. P_1 starts with the RMV of $uncoverV$ so $P_1 = \{u_n\}$.

Step 6. Since u_n is covered by P_1 so it is removed from $uncoverV$.

Step 7. Repetition of steps from 8 to 16 until all vertices get covered.

Step 8. Computation of set S and T for each u_n . S comprises of those unvisited

neighbour nodes of u_n whose subscript is greater than n and T comprises of those unvisited neighbour nodes of u_n whose subscript is less than n .

Step 9. Checks emptiness of S and T . If it is then

Step 10. P_1 is terminated with the initiation of P_2 which starts with RMV of $uncoverV$ and jumps to step 15.

Step 11. Checks the non-emptiness of set S if the condition of step 9 is not satisfied and P_1 keeps on covering unvisited vertices.

Step 12. If set S is non-empty then P_1 extends to cover v which is a RMV of S by joining it with u_n .

Step 13. Otherwise

Step 14. P_1 extends to cover v which is taken as LMV of T by joining it with u_n .

Step 15. Now variable u_n starts representing vertex v .

Step 16. Since $u_n = v$ is covered it is removed from $uncoverV$ and algorithm jumps to step 7 to repeat the whole procedure again for $u_n = v$.

Step 17. $opc = \{P_1, P_2, \dots, P_{num}\}$ is returned after the termination of step 7.

Example 2.7.1. A path cover computation by Algorithm 2.2 on an arbitrary (incomplete) right-end ordered interval graph given in Fig. 2.9 is as follows.

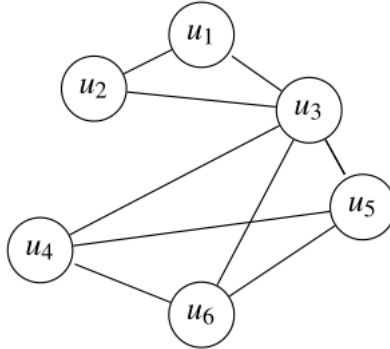


Figure 2.9: An incomplete right-end ordered interval graph G_R

- $opc = \{\}$ initially.
- $uncoverV = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.
- $num = 1$, so first path component is represented as P_1 .
- P_1 is started with u_6 , $\because u_6$ is the RMV .
- $uncoverV = \{u_1, u_2, u_3, u_4, u_5\}$ is updated with the deletion of u_6 .
- Entering in the while loop since $uncoverV$ is non-empty.
- Since $S(u_6) = \{\}$ and $T(u_6) = \{u_3, u_4, u_5\}$, step(14) computes $P_1 = u_6u_3$.

- u_6 is replaced by u_3 and $uncoverV = \{u_1, u_2, u_4, u_5\}$.
- $S(u_3) = \{u_4, u_5\}$ and $T(u_3) = \{u_1, u_2\}$.
- So step 12 extends $P_1 = u_6u_3u_5$.
- u_3 is replaced by u_5 and $uncoverV = \{u_1, u_2, u_4\}$.
- The whole procedure repeats for u_5 and extends $P_1 = u_6u_3u_5u_4$.
- Now u_5 is replaced by u_4 and $uncoverV = \{u_1, u_2\}$.
- S and T of u_4 is empty so step(10) selects u_2 from $uncoverV$, makes $num = 2$ and $P_2 = u_2$.
- Steps inside while loop repeats for u_2 and extends $P_2 = u_2u_1$.
- Then u_2 is replaced by u_1 and $uncoverV$ become empty, hence while loop stops running and path cover is returned as $opc = \{u_6u_3u_5u_4, u_2u_1\}$.

Remark 2.7.1. Path cover obtained by Algorithm 2.2 may contain more than one distinct path components to cover all the vertices on an incomplete interval graph as shown in Example 2.7.1. In order to join distinct path components to form an energy efficient connected routing structure (spanning tree), a quadratic time algorithm (say Algorithm 2.3) has been designed, motivation taken from [2]. Algorithm 2.3 first checks the path cover obtained by Algorithm 2.2 to run after it. When path cover contains single path component covering all vertices then that single path component is itself an energy efficient connected routing path/tree, in that case Algorithm 2.3 doesn't run after Algorithm 2.2.

2.7.1 Formation of connected routing structure by Algorithm 2.3 if the obtained path cover contains more than one distinct path components

Motivated by the methodology of algorithm given by Li et al. in [2], a quadratic time algorithm, say Algorithm 2.3, has been proposed to form an energy efficient connected routing structure (spanning tree) with the joining of distinct path components of path cover obtained by Algorithm 2.2 over any arbitrary right-end ordered interval graph. Algorithm 2.3 runs after Algorithm 2.2 when path cover obtained by Algorithm 2.2 contains more than one distinct path components and results a connected energy optimal spanning tree.

Step by step description of formation of connected routing structure using Algorithm 2.3 with the $opc = \{u_6u_3u_5u_4, u_2u_1\}$ obtained in Example 2.7.1.

Step 1. A path component P_c is chosen from the opc . Let us take $P_c = u_6u_3u_5u_4$, then $opc = \{u_2u_1\}$ and spanning tree $T = u_6u_3u_5u_4$ is updated.

Algorithm 2.3 Connect distinct path components of path cover obtained by Algorithm 2.2

Input : A path cover by Algorithm 2.2, consists of more than one distinct path components i.e. $opc = \{P_1, P_2, \dots, P_{num}\}$ where num is some finite natural number, on any arbitrary interval graph model G_R .

Output : A connected routing structure in the form of spanning tree of G_R .

- 1: $T \leftarrow \{P_c | P_c \text{ is a path component of } opc\}, opc = opc / \{P_c\}$.
 - 2: **while** $opc \neq \emptyset$ **do**
 - 3: choose a path component q from opc , where q is intersecting with T . $\triangleright$ (section 2.3.1, Property 2.1, 2.2)
 - 4: **if** $LMV(q) < LMV(T)$ **then**
 - 5: choose a minimum right-end vertex $w \in V(q)$ adjacent to T , join it to the leftmost adjacent vertex from T and T be the resultant new tree.
 - 6: **if** $LMV(q) > LMV(T)$ **then**
 - 7: choose a minimum right-end vertex $w \in V(T)$ adjacent to q , join it to the leftmost adjacent vertex from q and T be the resultant new tree.
 - 8: $opc = opc / \{q\}$
 - 9: **return** T .
-

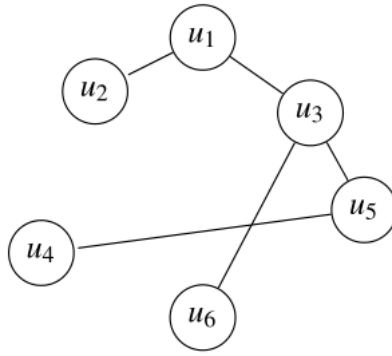


Figure 2.10: Formation of a connected routing structure by Algorithm 2.3 with opc obtained in Example 2.7.1.

Step 2. Repetition of steps from 3 to 8 until opc becomes empty.

Step 3. Let us choose another path component q from opc , we take $q = u_2u_1$.

Step 4. We find $LMV(q) = u_1$, $LMV(T) = u_3$ and $1 < 3$.

Step 5. Choose a minimum right-end vertex from $V(q)$ adjacent to T which is u_1 and joined to u_3 which is the leftmost adjacent vertex from T to u_1 and routing structure of T gets updated as shown in Fig. 2.10.

Step 6. This condition is not satisfied for the given opc , so

Step 7. It is skipped.

Step 8. opc is updated again with the removal of q .

Step 9. Return of connected routing structure can be seen in Fig. 2.10 after the termination of step 2.

Theorem 2.7.1. Path cover obtained by Algorithm 2.2 is energy optimal which result minimum of maximum node power consumption interval on connected topology by Algorithm 2.3.

Proof. Let T be the resultant spanning tree formed with opc , path cover of an interval graph G_R , obtained by Algorithm 2.2. Assume that opc is not optimal and opt is an optimal path cover but with same cardinality as opc . Let T' be the optimal spanning tree formed with opt . Consider a link with maximum power consumption interval in T , say l . Then link l does not belong to T' (otherwise opc and T would have been optimal). Also the power consumption intervals of links in T' is less than l 's (Note: comparison of intervals is based on upper bounds since upper bound is the maximum limit of power consumption for each link). This means a connected tree can be constructed with links whose power consumption interval is less than the power consumption interval of link l with the same cardinality of both the path cover. In that case either Algorithm 2.2 or Algorithm 2.3 would have terminated before processing l and this leads to contradiction. $\square$

Theorem 2.7.2. Algorithm 2.2 reduces max power consumption limit which is the upper bound of maximally assigned power interval on an arbitrary interval graph, to the minimum value possible, being an upper bound in the resultant topology.

Proof. Let's begin by considering the working of Algorithm 2.2 on Fig. 2.9 which results to an optimized routing tree given in Fig. 2.10. Each vertex has power assignment as $u_i = [a_i, b_i], i = 1, \dots, 6$ and $u_6 = [a_6, b_6]$ is joined to $u_3 = [a_3, b_3]$ in the resultant topology (Fig. 2.10). Upper bound of an interval of u_6 reduces from b_6 to b_3 because of $wgt(u_6 u_3) \in [\max\{a_3, a_6\}, \min\{b_3, b_6\}] = [\max\{a_3, a_6\}, b_3]$ and so $r'_{u_6} = [c_{63}, b_3]$ where $c_{63} = \max\{a_3, a_6\}$. Let this reduction is not minimum. Therefore r'_{u_6} can be reduced more by lowering the upper bound of its interval. This means u_3 is not the vertex with minimum subscript/index in u_6 's neighbourhood implies that step(14) of Algorithm 2.2 is not working correct which is not true. Hence our assumption is wrong. $\square$

Proposition 2.7.1. Path cover obtained by Algorithm 2.2 is energy optimal for its path components form an optimal routing topology (spanning tree) which minimizes the maximum transmission cost over link in worst case of power consumption.

Theorem 2.7.3. Algorithm 2.2 has $\mathcal{O}(n+m)$ time complexity where n is number of nodes and m is number of edges.

Proof. Step 1 takes $\mathcal{O}(n)$ time. Step 2-6 can be done in constant time. Steps 8,12 and 14 inside the while loop takes $\mathcal{O}(\text{degree of } u_n)$ time. Step 10 takes $\mathcal{O}(n)$ time and rest of the steps are done in constant time so total time complexity can be $\mathcal{O}(n + m)$. $\square$

Theorem 2.7.4. Algorithm 2.3 has $\mathcal{O}(|V(G)|^2)$ time complexity, where $V(G)$ denotes the number of vertices of interval graph G .

Proof. Step 1 can be done in constant time. While loop repeats at the most $|V(G)|$ times. It takes $\mathcal{O}(|V(q)|)$ and $\mathcal{O}(|V(T)|)$ time to determine LMV or RMV of q and T respectively. So maximum time complexity of steps 4-7 inside the while loop can be $\mathcal{O}(|V(G)|^2)$. Hence total time complexity is $\mathcal{O}(|V(G)|^2)$. $\square$

Remark 2.7.2. Algorithm 2.2 is the general algorithm, can apply to complete interval graph also and it gives the same energy optimal path cover/spanning path as Algorithm 2.1 does.

2.8 Evaluation and Comparison

Li et al. in [2] have proposed a polynomial time algorithm which finds a maximum internal spanning tree by joining distinct path components of an optimal path cover [1] obtained on an arbitrary interval graph. Optimal path cover problem has been solved in linear time by Arikati and Rangan [1] on a right end ordered interval graph and optimality has been taken on the basis of minimum cardinality of the path cover. Path cover, obtained with the algorithm defined in [1] is optimal since it covers all vertices with minimum number of distinct path components, hence minimum cardinality. Path cover obtained by Algorithm 2.2 doesn't guarantee minimum cardinality in incomplete interval graphs yet it is more energy efficient in order to form connected routing structure (spanning path/tree) by Algorithm 2.3 in WSN. Path cover obtained on any complete interval graph by newly proposed algorithms (Algorithm 2.1 and 2.2) has minimum cardinality, equal to the cardinality of optimal path cover obtained by algorithm given in [1], with better optimization of power consumption and load balancing in the resultant network.

Starting with the performance evaluation of Algorithm 2.1 and Algorithm, mentioned in [1]. Both the algorithms work to find path cover on complete interval graph. Path cover obtained on complete interval graph contains single path component which spans all vertices, hence path covers are themselves spanning paths.

Let's consider the example 2.6.1 where path cover is obtained by Algorithm 2.1 gives

min-max node power consumption interval is $r'_{u_4} = [14, 428]$ and u_5 which is maximally assigned with power interval $[75, 589]$ in the original network (Fig. 2.5(d)) reduces to $[75, 252]$ in the resultant topology (Fig. 2.8). Now finding the optimal path cover on the same network (Fig. 2.5(d)) by the Algorithm in [1], results a spanning path ' $u_1u_2u_3u_4u_5$ ' (see Fig. 2.11).

Resultant power consumption interval of each node on the obtained spanning

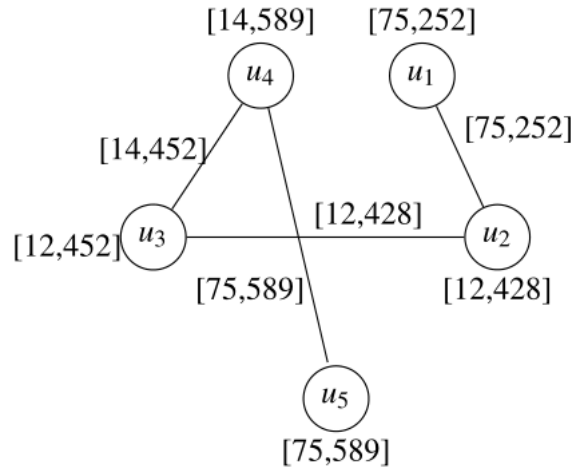


Figure 2.11: Spanning path by Algorithm [1] with edge weights as available power interval on each link and nodes weight as assigned power interval

path (refer Fig. 2.11) is as follows: $r'_{u_1} = [75, 252]$, $r'_{u_2} = [12, 428]$, $r'_{u_3} = [12, 452]$, $r'_{u_4} = [14, 589]$ and $r'_{u_5} = [75, 589]$. Hence min-max node power consumption interval at u_5 is $r'_{u_5} = [75, 589]$, higher than lastly computed value using Algorithm 2.1 at u_4 which is $r'_{u_4} = [14, 428]$ (see Fig. 2.8).

Moving to the worst case and computation of min-max transmission cost and min-total transmission cost over the links of worst case in complete interval graphs.

When Algorithm, mentioned in [1], is applied on the network given in Fig. 2.6, it results to a spanning path ' $u_1u_2u_3u_4u_5$ ' (see Fig. 2.12). In the traced path of Fig. 2.12, min-max transmission cost over the link u_4u_5 is b_4 and min-total transmission cost is $b_1 + b_2 + b_3 + b_4$ while path traced by Algorithm 2.1 (refer Fig. 2.7) results min-max transmission cost is b_2 and min-total transmission cost is $b_1 + b_1 + b_2 + b_2$. Since $b_1 \leq b_2 \leq b_3 \leq b_4 \leq b_5$ hence lesser power consumption over the spanning path obtained by Algorithm 2.1. Therefore spanning path obtained in Fig. 2.11 has min-max transmission cost 589 and min-total transmission cost 1721 while Spanning path (Fig. 2.8) traced by Algorithm 2.1 has min-max transmission cost 428 and min-total transmission cost 1360. This comparison of power optimization can be seen in Table 2.2.

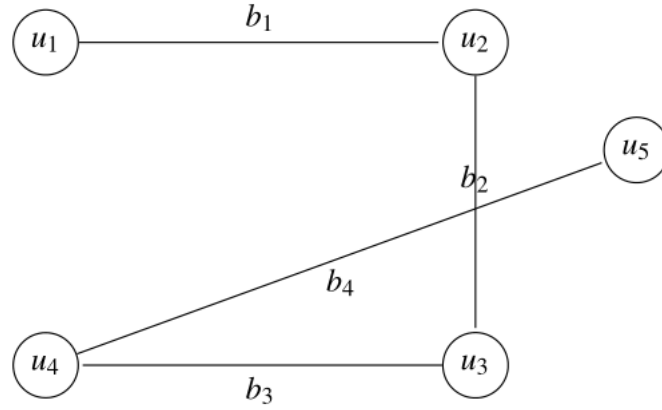


Figure 2.12: Spanning path by Algorithm in [1] on the complete network given in Fig. 2.6

Now coming to the performance evaluation of Algorithm 2.2 and Algorithm 2.3 which

Comparison criteria	Algorithm in [1](refer Fig. 2.11)	Algorithm 2.1(refer Fig. 2.8)
Min-max node's power consumption interval	[75,589]	[75,428]
Min-max transmission cost in worst case	589	428
Min-total transmission cost in worst case	1721	1360
Minimization of upper limit of rightmost vertex	no reduction	minimized from 589 to 252

Table 2.2: Power optimization on a complete interval graph of Fig. 2.5(d)

for convenience written as Algorithm 2.2 and 2.3 (for comparison purpose in the thesis), with Algorithm in [2] on arbitrary interval graphs.

Path cover by Algorithm 2.2 on an arbitrary interval graph given in Fig. 2.9 results $opc = \{u_6u_3u_5u_4, u_2u_1\}$ having two distinct path components, hence after applying Algorithm 2.3, a connected routing structure is obtained (refer Fig. 2.10) gives min-max node power consumption at u_5 whose power consumption interval is upper bounded by b_4 and maximally assigned power interval $PI(u_6) = [a_6, b_6]$ reduces to $[max\{a_3, a_6\}, b_3]$. Now applying the Algorithm of Li et al., mentioned in [2], on the same network (Fig. 2.9). Methodology of the Algorithm in [2] runs with the path cover, obtained in [1] and results to a maximum internal spanning tree ' $u_1u_2u_3u_4u_5u_6$ ' can be seen in Fig. 2.13. This obtains min-max node power consumption interval at $u_6 = [max\{a_5, a_6\}, b_5]$ implies that incident link on u_6 has power consumption equal to b_5 which is greater than b_4 .

Numerical results by both the algorithms has been evaluated on the network given in Fig. 2.9 where each node has been weighted with power intervals as $PI(u_1) = [50, 105]$, $PI(u_2) = [78, 150]$, $PI(u_3) = [85, 203]$, $PI(u_4) = [161, 230]$, $PI(u_5) = [155, 312]$ and $PI(u_6) = [178, 312]$ and shown in Table 2.3 for $n=6$.

Trying to apply Li et al. algorithm, given in [2], to the obtained $opc = \{u_6u_3u_5u_4, u_2u_1\}$,

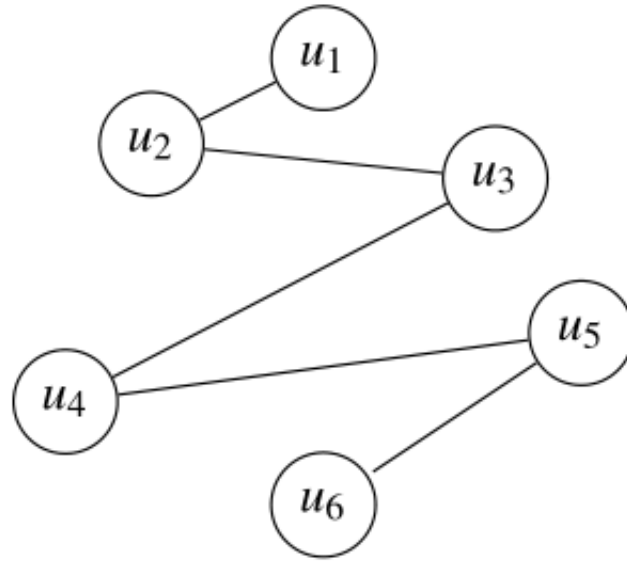


Figure 2.13: Maximum internal spanning tree by Algorithm in [2]

problem comes in choosing relevant energy efficient link to u_3 when there are more than one adjacent vertices, in this case there are u_1, u_2 and also a maximum internal spanning tree doesn't form. Algorithm 2.3 resolves this issue by considering adjacent vertex with minimum subscript since available power interval at u_1u_3 is less than that at u_2u_3 . So selectively joining u_3 to u_1 results to have lesser available power interval at the link and also reduces the power consumption interval at node u_2 .

Moreover, performance of Algorithm 2.2 can be seen on an arbitrary network G_1 given in Fig. 2.14 whose vertices are already right-end ordered according to the assigned power interval. Applying Algorithm 2.2, single path component spans all the vertices and results to an optimized network can be seen in Fig. 2.15. Node u_9 which was maximally assigned with $[115, 140]$ in G_1 (refer Fig. 2.14) minimized to $[115, 125]$ and so min-max node power consumption interval becomes $[115, 125]$ in the resultant topology (refer Fig. 2.15). Also nodes u_8 and u_7 which were assigned with power intervals $[94, 140]$ and $[95, 138]$ reduces to $[95, 112]$ and $[94, 125]$ respectively and upper limit of power consumption of rightmost vertex minimized from 140 to 125. Fig. 2.16 shows the spanning path obtained by applying algorithm, given in [2], on G_1 results min-max node power consumption interval $[115, 140]$ is more than the lastly computed $[115, 125]$ in Fig. 2.15. Performance comparison of both the algorithms including the worst case of power consumption can be seen in Table 2.3 for $n=9$.

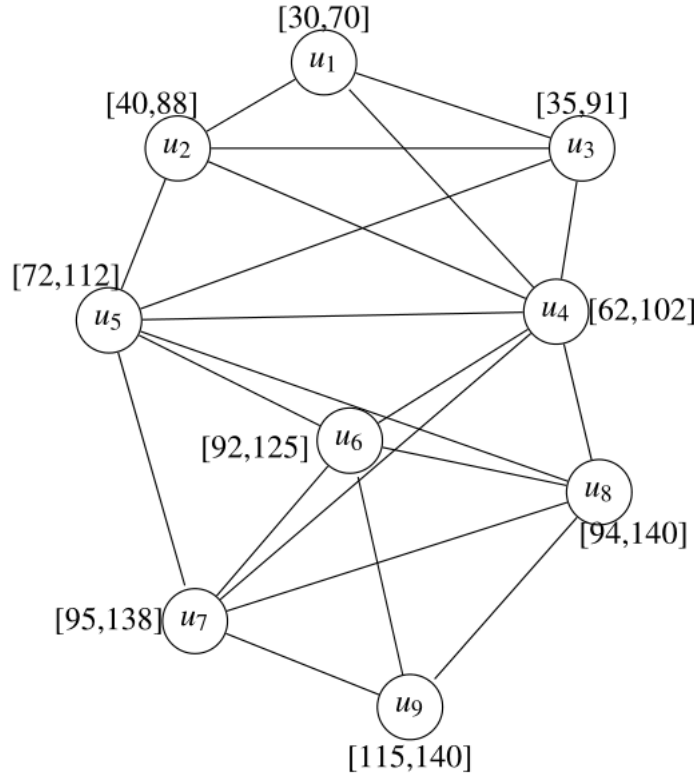


Figure 2.14: Arbitrary interval graph G_1

2.9 Simulation results on arbitrary interval graphs

We run the simulation considering large number of WSNs whose underlying graphs are interval graphs in Spyder (Python 3.7). Input of arbitrary right-end ordered interval graph in term of nodes, edges and data of right end ordered intervals are randomly generated and given to the combined python code of Algorithm 2.2 and 2.3 to evaluate the combined performance of Algorithm 2.2 and 2.3. Simulation is done using Algorithm 2.2 and 2.3 since Algorithm 2.2 is the generalisation of Algorithm 2.1 and applicable on both complete as well as incomplete interval graphs. When obtained path cover, by Algorithm 2.2, contains more than one distinct path components then Algorithm 2.3 results them to a connected routing topology. Therefore python code of Algorithm 2.2 and 2.3 is used for the simulation which takes the input of any arbitrary interval graph in term of number of nodes, number of edges, list of all edges and list of intervals according to right-end ordering of nodes.

The aim of taking input of arbitrary interval graphs is fulfilling the evaluation requirement on complete as well as incomplete interval graphs. Results by Algorithm 2.2 and 2.3 on min-max node power consumption interval and min-max transmission cost in worst case for each input data have been recorded and plotted against the results

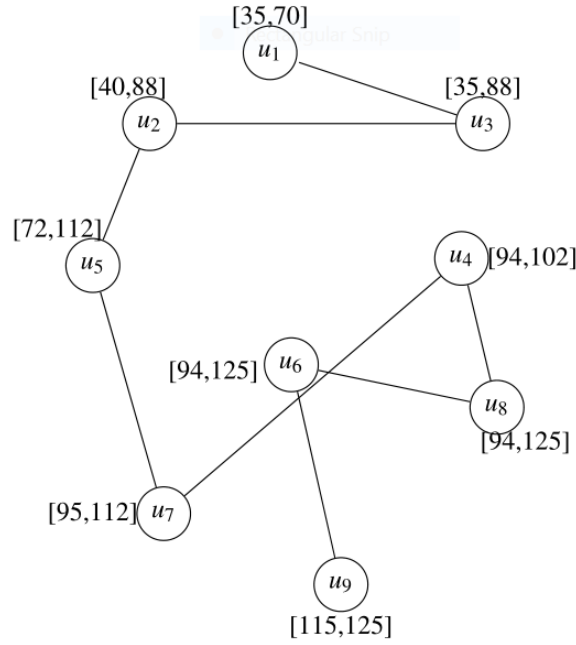


Figure 2.15: Path cover by Algorithm 2.2 with reduced power intervals of G_1

obtained by Algorithm, mentioned in [2] on the same data.

Fig. 2.17 shows the plotting of min-max node power consumption interval on the power line (i.e. nodes transmit power level ranging from 0 to 1000) with the variation of number of nodes from 5 to 50 consisting of both the complete as well as incomplete interval graphs with different edge sets. The range covered by 'blue color' shows the maximum node power consumption interval obtained by Algorithm 2.2 and 2.3 and range with 'red color' shows the maximum node power consumption interval obtained by Algorithm, mention in [2]. It has been observed that blue line terminates before the red line everytime parallel to power line, this implies that routing topology obtained by Algorithm 2.2 and 2.3 results minimum of max node power consumption interval on interval graphs everytime. Fig. 2.18 shows the range of power conserved by Algorithm 2.2 and 2.3 with red color, extracted from Fig. 2.17.

Simulation results of max transmission cost on links in worst case has been plotted in Fig. 2.19. Blue dots whose tracing is shown by blue curve depicting the performance of Algorithm 2.2 and 2.3 and red dots whose tracing is shown by red curve depicting the performance of Algorithm, mentioned in [2]. It can be seen in Fig. 2.19 that blue curve falls below the red curve for every count of nodes which implies that max transmission cost on links in worst case is minimized by Algorithm 2.2 and 2.3 more than the Algorithm in [2].

Summarized simulation results can be seen in Table 2.3, depicting the difference in

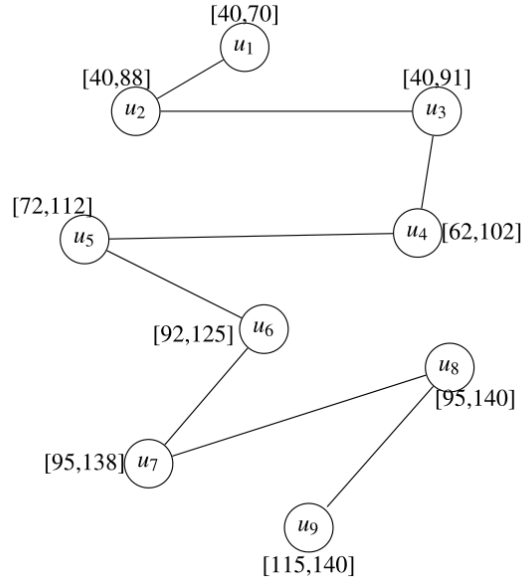


Figure 2.16: Resultant Spanning path after applying Algorithm in [2] on G_1

power consumption with the input of randomly generated right end ordered interval graph whose nodes and edges are mentioned in the table.

Hence the goal of energy saving in topology control of WSNs whose underlying graphs are interval graphs, could be achieved when path cover is obtained by Algorithm 2.1 or 2.2 and distinct path components are joined by Algorithm 2.3.

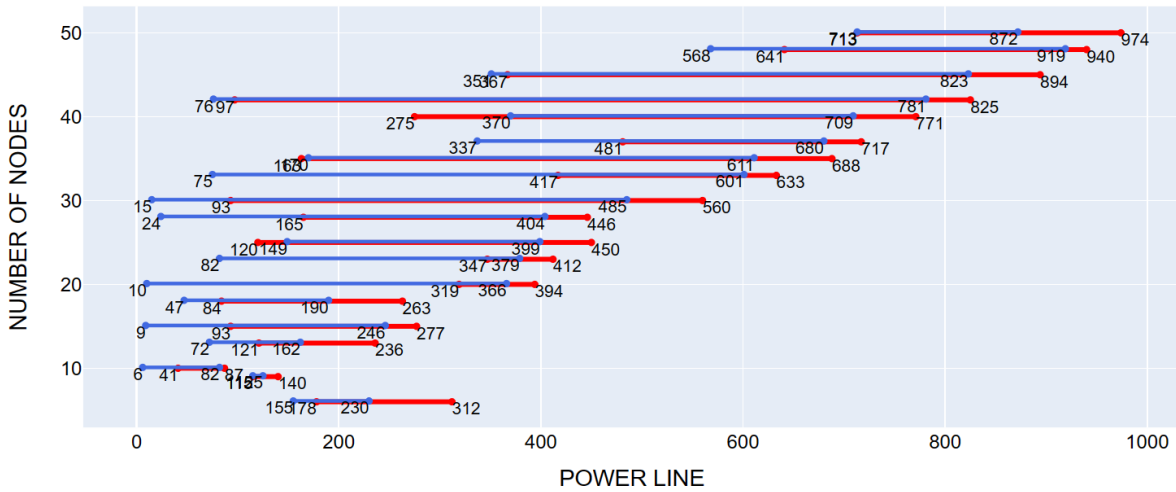


Figure 2.17: Evaluation of maximum node power consumption interval: blue color shows the performance of Algorithm 2.2 and 2.3 and red color shows the performance of Algorithm, given in [2].

Remark 2.9.1. No routing solution exists in literature on heterogeneous WSNs where nodes are explicitly assigned with power intervals to optimize power consumption subject to the connectivity constraint. Proposed methods are the first approach in this respect.

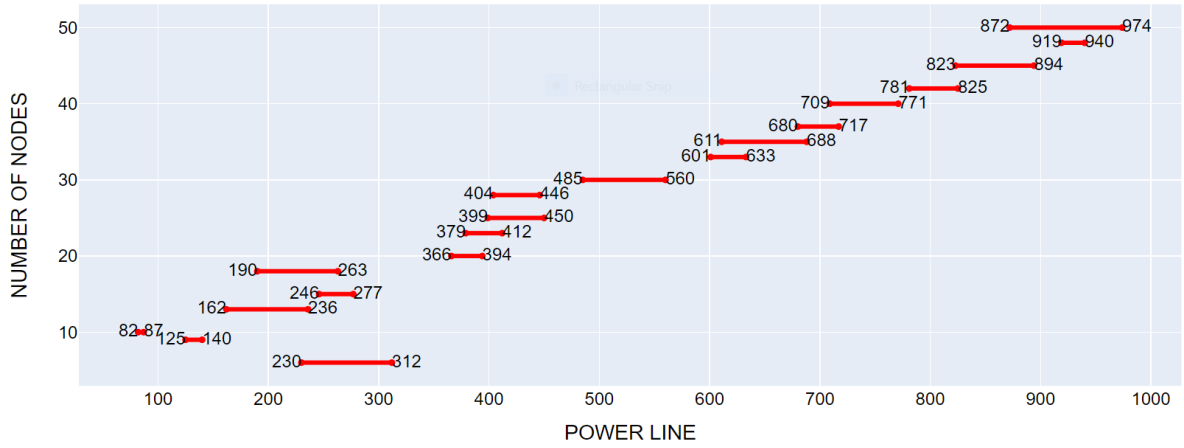


Figure 2.18: Range of Power conserved by Algorithm 2.2 and 2.3

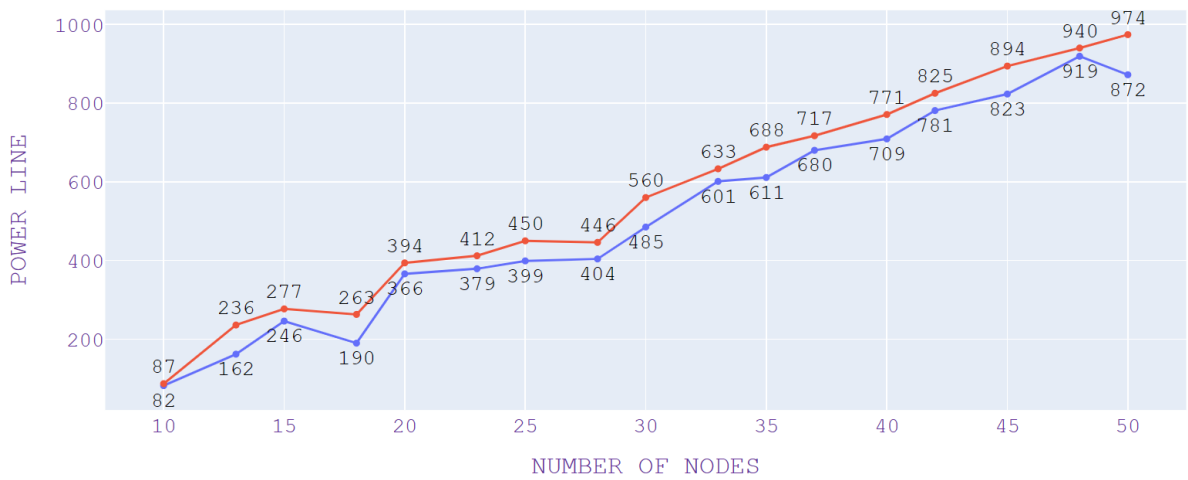


Figure 2.19: Evaluation of maximum transmission cost in worst case: blue color shows the performance of Algorithm 2.2 and 2.3 and red color shows the performance of Algorithm, given in [2].

Remark 2.9.2. There exists no another path cover with cardinality less than or equal to the path cover, obtained by Algorithm 2.2, which can be more energy efficient in general and worst case too. Thus, it is seen that proposed algorithms provide energy optimal path cover/spanning tree on interval graph models of WSNs.

2.10 Conclusion and summary

The work in this chapter is based on a persisting problem of power consumption and network lifetime in WSNs where nodes are allowed to vary their transmission range over a uniquely assigned power interval in accordance to make required links at a particular instant of time. Several works have been done aiming to reduce energy consumption by reducing nodes transmission range via transmission power control

Number of nodes	Number of edges	Min-max node power consumption interval		Min-max transmission cost in worst case	
		Algorithm 2.2 and 2.3	Algorithm in [2]	Algorithm 2.2 and 2.3	Algorithm in [2]
6	9	[155,230]	[178,312]	230	312
9	21	[115,125]	[115,140]	125	140
10	34	[6,82]	[41,87]	82	87
13	38	[72,162]	[121,236]	162	236
15	49	[9,246]	[93,277]	246	277
18	100	[47,190]	[84,263]	190	263
20	120	[10,366]	[319,394]	366	394
23	149	[82,379]	[347,412]	379	412
25	174	[149,399]	[120,455]	399	455
28	202	[24,404]	[165,446]	404	446
30	268	[15,485]	[93,560]	485	560
33	265	[75,601]	[417,633]	601	633
35	380	[170,611]	[163,688]	611	688
37	322	[337,680]	[481,717]	680	717
40	376	[370,709]	[275,771]	709	771
42	448	[76,781]	[97,825]	781	825
45	479	[351,823]	[367,894]	823	894
48	608	[568,919]	[641,940]	919	940
50	632	[713,872]	[713,974]	872	974

Table 2.3: Summarized simulation results

(TPC) approaches, explicitly designed interference model in topology control of WSN, topology control algorithms and routing protocols [3, 12, 14, 122], most of the works are dealing with static transmission range of nodes. The work in this chapter is different in view of modelling WSN as a ‘connected bidirectional interval graph’, assigning transmission range in term of power interval to each node and minimization of maximum power consumption on a connected topology by newly proposed algorithms, based on interval graph theory.

Assigned power interval of each node is ranging from minimum to maximum energy necessarily required for successful transmission of data packets bidirectionally. Hence every node is assigned with an appropriate power interval in which node can vary its transmission range accordingly, forming an interval graph and consequently, a right-end ordered interval graph layout has been formed by nodes. Every WSN whose underlying graph is complete, has an interval graph layout. The problem is to find energy efficient routing topology (spanning path/tree) with connectivity among the nodes i.e. a structure which can be a path/tree, covering all vertices of an interval graph with low power consumption with successful routing among all nodes. Considering this, an idea of energy optimal path cover on an interval graph comes into picture and so three algorithms have been proposed aiming the same for static wireless sensors.

Proposed Algorithm 2.1 is used to find energy optimal path cover on complete bidirectional interval graph models and since network is complete, a single path component covers all vertices and gives an energy optimal spanning path. This means path cover in complete interval graph is itself a connected routing structure which minimizes

maximum node's power consumption interval and optimizes the worst case situation also by minimizing maximum transmission cost over the link and total transmission cost over the resultant network. Algorithm 2.2 is the generalisation of Algorithm 2.1 which computes an energy optimal path cover on an arbitrary interval graph model. Path cover computation on an arbitrary interval graph may result more than one distinct path components so Algorithm 2.3 has been designed to join distinct path components to have an energy efficient connected routing structure which is applied after Algorithm 2.2. If there is only one path component in the path cover obtained by Algorithm 2.2 then that path component is itself an energy optimal routing structure (spanning path). The resultant routing structure by Algorithm 2.2 and 2.3 optimally minimizes maximum node power consumption interval and reduces upper power consumption limit of rightmost vertex to the minimum value possible causing load balancing in the network. Yet Algorithm 2.2 and 2.3 does not guarantee optimal min-total transmission cost in the worst case for incomplete interval graph model of WSN.

The proposed algorithms either have linear or quadratic time complexity. Algorithm 2.1 has linear time complexity i.e. $\mathcal{O}(n)$, Algorithm 2.2 has $\mathcal{O}(n + m)$ time complexity and Algorithm 2.3 has $\mathcal{O}(n^2)$ time complexity where n is the number of nodes and m is number of links.

A comparison has been shown using evaluation and simulation on the basis of power consumption between the performance of Algorithm, given in [2] and the combined performance of Algorithm 2.2 and 2.3 on arbitrary interval graphs reveals that routing topology formed with path cover, obtained by Algorithm 2.2 has been more energy efficient. That's why the path cover and the connected topology obtained in this chapter are called energy optimal path cover and energy optimal topology respectively.

Thus the proposed algorithms work when connected topology formed by wireless sensors has interval graph layout and vertices are represented by right-end ordering. In next chapters, more general algorithms independent of the restriction of right-end ordering would be introduced and discussed under different issue of WSN. Moreover, the proposed solution is centralized in nature. So we intend to explore some distributed solutions for the same problem with the addition of more results based on power optimization, load balancing, fault-tolerance and scalability. A static network is considered in this chapter so we intend to include nodes mobility in our next work as well.

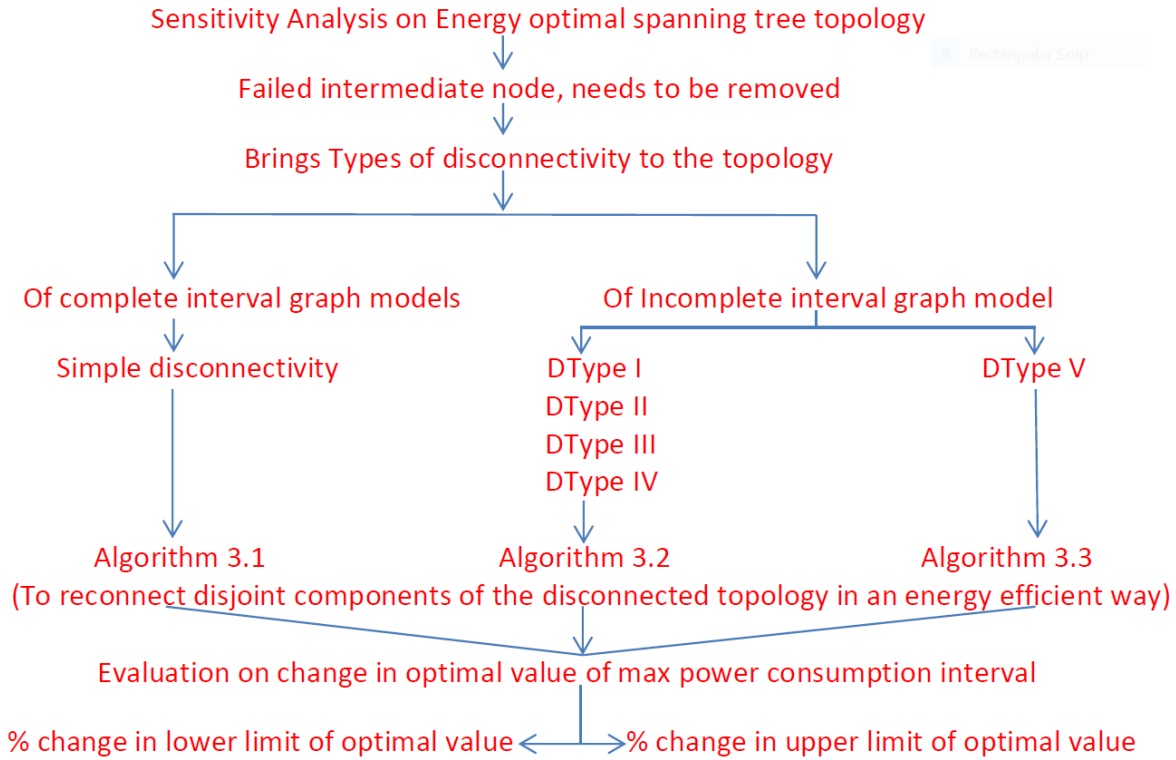
Chapter 3

Sensitivity analysis on energy optimal topology of interval graph model of WSNs with respect to node removal

In this chapter¹, we study many disconnectivity situations on energy optimal tree topologies of interval graph models of WSNs due to failure of any intermediate n -ode as each vertex and edge of tree topology plays a crucial role in maintaining connectivity among the sensors therefore failure or removal of any intermediate n -ode due to its death brings disconnectivity, loss of data during transmission, energy hole and unreliability issue. Hence reorganisation or reconnectivity of a disconnected topology to continue network functionality after any node failure/removal from the existing topology has been a prime concern for routing in WSNs. We have proposed new algorithmic approaches to reconnect the disconnected components under each disconnectivity situation in such a way that change on optimal value of max power consumption should be minimum. Furthermore reconnectivity of disconnected components of a separable interval graph model without its cut-vertex has been a very first attempt in this chapter at the expense of more power consumption of nodes. Sensitivity analysis has been carried out to putforth many analytical results on change in optimal power consumption under distinct situations of disconnectivity.

¹The content of this chapter is based on a research paper “Sensitivity analysis on interval graph based energy optimal spanning tree topology of wireless sensor networks with respect to node removal”, (Submitted).

Graphical abstract of the chapter:



3.1 Introduction

Work on developing new topological models of WSNs have been tremendously growing since past many years. These topological models improve energy efficiency and optimize power consumption by reducing nodes' transmission range or by minimizing interference factor with the maintenance of network connectivity [4, 12, 13, 21, 28, 29, 45]. Many models/methodologies given in [3, 26, 31, 123, 124] build tree-like structures of bidirectional WSNs by optimizing number of links of the network based on Euclidean distance or link interference to minimize nodes' power assignment or transmission range while ensuring connectivity among the sensors. Moreover in the field of WSNs, spanning tree like topology has been preferred over other since it is less complicated, easy to implement with static nodes and results lesser power consumption in the network. The spanning tree-like structures are the most popular approaches in field of topology control in WSNs from history. It has been observed that each intermediate link and node acts as a bridge and cut-vertex respectively in a tree so these structures are more prone to disconnectivity if any node leaves the topology on being die in the network due to drainage of its power completely i.e. node failure.

This therefore resists the network functionality, leads to poor data transmission and many routing issues.

It is obvious that removal of any intermediate node disconnects the tree structure and removal of end node has nothing to do with tree connectivity. Therefore analysis of various disconnectivities occur with the removal of any intermediate node from the tree topology due to its death and effects on optimal value of max power consumption to evaluate the power related sensitivities due to disconnectivity of energy optimal topologies have been our objectives in this chapter. This is just a kick-start towards the sensitivity analysis in the field of topology control in WSNs. This chapter provides thorough sensitivity analysis on the most recent energy optimal spanning tree topology of interval graph models of WSNs, introduced in chapter 2.

The model and methodologies, introduced in chapter 2, are the most recent which result spanning tree-like structure for minimum power consumption while satisfying connectivity constraint with interval data based weights on nodes and links. But since the optimal topology is a tree-like structure, this is more prone to disconnectivity even with a single node's removal if any node gets die in the network. Also the energy optimal topology obtained in chapter 2 has been designed for immobile nodes therefore exit of any node from the optimal topology brings routing issues like energy hole, loss of data or lesser throughput due to topology disconnectivity. Therefore dying/leaving of an existing node from the optimal topology arrives many different and difficult situations for the whole network to cope up with disconnectivity to avoid any routing issue and increment of power consumption.

Following are the prime concerns occur with disconnectivity due to node failure:

1. Is this possible to reconnect the disconnected optimal topology of immobile nodes again? If yes, then will it be reconnected using the existing algorithms or some new methodologies require to introduce?
2. Does the reconnected topology affect the previously obtained optimal value of power consumption of energy optimal tree topology, obtained in chapter 2?
3. If optimal value changes then when and how much does it change?

On the first prime concern, the energy optimal topology, obtained in chapter 2, is no more an interval graph which limits the existing interval graph based algorithms of chapter 2 (Algorithm 2.2 and 2.3) to work further on disconnected components of the optimal topology to reconnect/reorganise because Algorithm 2.2 and 2.3 work only on

right-end ordered interval graphs. This motivates us to develop new approaches, not restricted to right-end ordered interval graph to work on, to reconnect the disconnected topologies in such a way that minimum changes to the optimal power consumption would take place. Thereafter sensitivity analysis has been carried out to provide many theoretical and analytical results on change in optimal value of max power consumption when the disconnected optimal topologies have been reorganized/reconnected by newly proposed algorithms. We have analysed and differentiated types of disconnectivity situations, called DType on large number of energy optimal topologies on interval graph models including separable interval graphs, by assuming that each intermediate node may exit from the optimal topology. We have obtained DType I, II, III, IV and V along with simple disconnectivity.

Evaluation of max power consumption interval (i.e. $\max PI$) on each reconnected topology corresponding to an intermediate node's removal and % change in lower limit as well as upper limit of optimal value have also been provided on many distinct optimal topologies of arbitrary interval graph models.

Briefing the chapter contribution in the following bullets:

1. Providing the sensitivity analysis with respect to disconnectivity and change in optimal value of max power consumption on interval graph based energy optimal topology on removal of intermediate node.
2. Proposed three algorithms to reconnect disjoint components of the disconnected topology such that minimum change on optimal power consumption has been reflected on the reconnected topology.
3. Provided many theoretical and analytical results on change in optimal value of max power consumption under different scenario of disconnectivity in detail on complete as well as incomplete interval graphs.
4. Proposed Algorithm 3.1 handles simple disconnectivity of energy optimal topology on complete interval graph in constant time complexity.
5. Proposed Algorithm 3.2 and Algorithm 3.3 deal DType I, II, III, IV and V of energy optimal topology on incomplete interval graph in quadratic time complexity.
6. Algorithm 3.3 is the very first approach in the field of graph theory to join completely disjoint components of an energy optimal topology of separable interval graph without its cut-vertex when the cut-vertex is failed and removed.

The organisation of the chapter is followed with the introduction of sensitivity analysis with respect to disconnectivity and power consumption on interval graph models in section 3.2 with theoretical aspect of reconnectivity in general. Section 3.3 thoroughly presents the sensitivity analysis on energy optimal topology of complete interval graph models, introduces a new methodology to reconnect the disconnected topology followed by an illustration for better understanding. Also theoretical and analytical results obtained on energy optimal topologies of complete interval graphs after simulation and sensitivity analysis have also been provided in section 3.3. Section 3.4 starts with the discussion on explored DType I,II,III,IV and V on energy optimal topologies of any incomplete interval graph models, presents the newly proposed algorithms followed by illustrations, theoretical results, evaluation and remarks. Finally section 3.5 summarizes and concludes the chapter.

Table 3.1: Description of abbreviations used

PCI	Power consumption interval	IS	Intersecting set
T_o	Energy optimal topology	T_c	Reconnected topology
T_d	Disconnected topology	T_r	Tree

3.2 Sensitivity Analysis

Sensitivity analysis of the energy optimal spanning tree topology T_o , obtained in chapter 2, with respect to removal of any intermediate node from the optimal topology, has been carried out on numerous network and presented thoroughly. Since the optimal topology is a spanning tree-like structure, each node except end nodes, acts as a cut-vertex of the optimal topology hence removal of any intermediate node disconnects the topology in a moment and effects the optimal value of max power consumption in any way.

Following are the undertaking steps to conduct sensitivity analysis on disconnectivity and power consumption on large number of energy optimal topologies of right-end ordered interval graph models.

1. Removal of each intermediate node one by one including optimal node, neighbour nodes of optimal node etc from energy optimal topology T_o are performed

to analyse and differentiate the type of disconnectivity occurs in reorganizing the disconnected topology T_d everytime.

2. To handle each type of disconnectivity with minimum effect on optimal power interval, 3 algorithms have been proposed to reconnect disjoint components of T_d and computation of max PI on each corresponding reconnected topology T_c .
3. Evaluation and analysis of change in max power consumption on T_c in term of % change in lower limit of max PI if lower limit changes and % change in upper limit of max PI if upper limit changes has been carried out thoroughly.

Let us define the disconnected energy optimal topology as T_d consists of two or more disjoint connected components and for convenience it is represented as a set of disjoint components $T_d = \{t_1, t_2, \dots, t_m\}$ where each t_i is a tree-like structure in itself. The way to join disjoint components is by connecting appropriate nodes from each component based on their power intervals. It is obvious that two nodes can communicate each other if they belong to each other's transmission disc. This implies their power intervals must have non-empty intersection.

Theorem 3.2.1. Necessary condition to establish reconnection

Two nodes from distinct components can establish a connection only if their PI have non-empty intersection.

Proof. Let us consider a disconnected topology $T_d = \{t_1, t_2\}$ where nodes $u \in t_1$ and $v \in t_2$ are having $PI(u) = [a_u, b_u]$ and $PI(v) = [a_v, b_v]$. To join u with v , a bidirectional link which is symmetric in nature need to establish between u and v . That bidirectional link must be weighted with the maximum power r_{max} which is evaluated as $r_{max} = \max\{r_u, r_v\}$, where r_u is the power emission from u to v and r_v is the power emission from v to u [18]. This implies that $r_{max} \in PI(u)$ and $r_{max} \in PI(v)$ (refer chapter 2) and so $PI(u) \cap PI(v) \neq \emptyset$. Hence if $PI(u) \cap PI(v) = \emptyset$ then no bidirectional communication can exists between $u \in t_1$ and $v \in t_2$. $\square$

3.3 Sensitivity analysis on energy optimal topology of complete interval graph

Let T_o be an energy optimal topology, formed on right-end ordered complete interval graph in chapter 2. T_o is having spanning path like structure given by Eq. (3.1). Therefore removal of any intermediate node say w from T_o disconnects T_o into two

disjoint path components as $T_d = \{t_1, t_2\}$ where t_1, t_2 are two disjoint path components of T_o after w 's removal.

Theorem 3.3.1. There is always exist non-empty power intersection between adjacent nodes of the leaving node from optimal topology T_o , obtained on right-end ordered complete interval graph.

Proof. Let $t_1 = u_n u_1 u_{n-1} u_2 \dots u_e u_i$ and $t_2 = u_r u_s u_p u_q \dots u_y u_x$ are two disjoint path components with the removal of a node w from T_o where

$$T_o = u_n u_1 u_{n-1} u_2 \dots u_e u_i w u_r u_s u_p u_q \dots u_y u_x. \quad (3.1)$$

To prove: $PI(u_i) \cap PI(u_r) \neq \phi$

Initially u_i, w, u_r belong to a complete interval graph say, $G^c(V, E)$ therefore $PI(u_i) \cap PI(w) \neq \phi$, $PI(w) \cap PI(u_r) \neq \phi$ and $PI(u_r) \cap PI(u_i) \neq \phi$ on $G^c(V, E)$.

Let $PI(u_i) = [a_1, b_1]$, $PI(w) = [a_2, b_2]$ and $PI(u_r) = [a_3, b_3]$ such that $b_1 \leq b_2 \leq b_3$ on $G^c(V, E)$. Therefore

$$PI(u_i w) = [\max\{a_1, a_2\}, \min\{b_1, b_2\}] \Rightarrow PI(u_i w) = [\max\{a_1, a_2\}, b_1] \quad (3.2)$$

$$PI(u_i u_r) = [\max\{a_1, a_3\}, \min\{b_1, b_3\}] \Rightarrow PI(u_i u_r) = [\max\{a_1, a_3\}, b_1] \quad (3.3)$$

$$PI(w u_r) = [\max\{a_2, a_3\}, \min\{b_2, b_3\}] \Rightarrow PI(u_r w) = [\max\{a_2, a_3\}, b_2] \quad (3.4)$$

Eq. (3.2), (3.3), (3.4) $\Rightarrow$ atleast b_1 is common to $PI(u_i w), PI(u_i u_r), PI(u_r w)$. Now T_o , given by Eq. (3.1), preserves link $u_i w$ and $w u_r$ with the PI s given in Eq. (3.2) and (3.4) $\Rightarrow b_1$ common to $PI(u_i), PI(u_r)$ and $PI(w)$ on the optimal topology. Hence $PI(u_i) \cap PI(u_r) \neq \phi$ on T_o as well as on T_d . $\square$

Hence disconnected topology T_d of a complete interval graph can be reorganized by just joining adjacent nodes of a leaving node and this can be done in constant time only.

3.3.1 Proposed methodology

To reconnect disjoint path components of the disconnected energy optimal topology of a complete interval graph model of WSN, a new algorithm say Algorithm 3.1 has been proposed. Algorithm 3.1 works to rejoin each disjoint component by connecting

adjacent nodes of the removed node and results a reconnected topology with minimum changes to optimal value. The proposed Algorithm 3.1 does not need right-end ordered interval graph to run and execute, therefore has constant time complexity.

Algorithm 3.1 Reconnect disconnected energy optimal topology of a complete interval graph model

Input: $T_d = \{t_1, t_2\}$ when any intermediate node w exits from T_o where T_o is given by Eq. (3.1) arbitrarily.

Output: A reconnected topology T_c with $T_c = t_1 \cup t_2$.

- 1: Select $u_i \in t_1$ where $t_1 = u_n u_1 u_{n-1} u_2 \dots u_e u_i$ and $u_r \in t_2$ where $t_2 = u_r u_s u_p u_q \dots u_y u_x$.
 - 2: Join u_i with u_r where $PI(u_i) = [a_{u_i}, b_{u_i}]$ and $PI(u_r) = [a_{u_r}, b_{u_r}]$ have non-empty intersection.
 - 3: Update $PI(u_i u_r) = [a_{u_i}, b_{u_i}] \cap [a_{u_r}, b_{u_r}]$.
 - 4: $T_c = t_1 \cup t_2$ is the required reconnected topology.
-

Proposition 3.3.1. Algorithm 3.1 reconnects disjoint components of a disconnected energy optimal topology in constant time.

Example 3.3.1. Working illustration of Algorithm 3.1 on an energy optimal topology T_o , given in Fig. 2.8, of a complete interval graph.

$T_o = u_5 u_1 u_4 u_2 u_3$ (given in Fig. 2.8) has $\max PCI = [12, 428]$ at u_2 and so u_2 is called an optimal node with optimal value of power consumption i.e. $[12, 428]$.

As adjacent nodes of any leaving node find non-empty intersection of their power intervals with each other where left neighbour node of the leaving node belongs to the first component and right neighbour node of the leaving node belongs to second component of T_d . So last node of first component has been joined to first node of second component by Algorithm 3.1 for each case, shown below.

(i) *Removal of node u_1 from T_o :*

T_d becomes $\{u_5, u_4 u_2 u_3\}$ and it is seen from Fig. 2.8 that $PI(u_5)$ is intersecting with $PI(u_4)$. Therefore u_5 has been joined to u_4 with edge $PI = [75, 252]$ and results $T_c = u_5 u_4 u_2 u_3$. Update $PI(u_4) = [\min\{14, 75\}, \max\{252, 428\}] = [14, 428]$ and $PI(u_5) = [75, 252]$ using Eq. (2.5) results no change in optimal value i.e. optimal $\max PI = [12, 428]$ is found on u_2 .

(ii) *Removal of optimal node u_2 from T_c :*

Now $T_d = \{u_5 u_1 u_4, u_3\}$. Fig. 2.8 shows that $PI(u_4) \cap PI(u_3) \neq \emptyset$ with common link weight is $[14, 428]$. Hence joining of u_4 to u_3 results $T_c = u_5 u_1 u_4 u_3$ updates $PI(u_3) =$

$[14, 428]$ and $PI(u_4) = [14, 428]$ using Eq. (2.5) so $\max PI = [14, 428]$ is found on node u_4 and u_3 . Optimal value changes with the change in lower limit of $\max PI$ only i.e. from 12 to 14.

(iii) *Removal of node u_4 (neighbourhood of optimal node) from T_c :*

Now $T_d = \{u_5u_1, u_2u_3\}$. Fig. 2.8 shows that $PI(u_1) \cap PI(u_2) \neq \emptyset$ with common link weight is $[75, 252]$. Therefore u_1 is joined to u_2 , results $T_c = u_5u_1u_2u_3$ with $\max PI = [12, 428]$ at node u_2 . Hence no change in optimal value.

Remark 3.3.1. In example 3.3.1, we show the results on each reconnected topology obtained with the removal of each intermediate node from T_o , by computing and comparing $\max PI$ with the optimal PI of previously connected optimal topology given in Fig 2.8. This is the pattern, we follow in analysing $\max PCI$ on large number of interval graph models which are complete as well as incomplete and derive many theoretical and analytical results.

To move further in this work let us assume, in general, that $PI(u_n) = [a_{u_n}, b_{u_n}]$ for any node $u_n \in T_o$ and T_d .

Theorem 3.3.2. Reconnected topology by Algorithm 3.1 due to removal of an optimal node from optimal topology of complete interval graph may effect the optimal value either by changing the lower limit or both the lower limit and upper limit of optimal $\max PI$. If upper limit changes, it is reduced to lesser power consumption than the previous one.

Proof. The optimal topology T_o on a complete interval graph of n nodes is given by Eq. (2.3) and the optimal value has been found on node $u_{\lfloor \frac{n}{2} \rfloor}$ with $PI = [a_{\lfloor \frac{n}{2} \rfloor}, b_{\lfloor \frac{n}{2} \rfloor}]$ (refer chapter 2). So let us remove the optimal node $u_{\lfloor \frac{n}{2} \rfloor}$ from T_o when n is odd. Both links $u_{\lfloor \frac{n}{2} \rfloor + 2}u_{\lfloor \frac{n}{2} \rfloor}$ and $u_{\lfloor \frac{n}{2} \rfloor}u_{\lfloor \frac{n}{2} \rfloor + 1}$ incident on $u_{\lfloor \frac{n}{2} \rfloor}$ are having PI s, upper bounded by $b_{\lfloor \frac{n}{2} \rfloor}$, so $PI(u_{\lfloor \frac{n}{2} \rfloor + 2})$ and $PI(u_{\lfloor \frac{n}{2} \rfloor + 1})$ are also upper bounded by $b_{\lfloor \frac{n}{2} \rfloor}$. Therefore after optimal node's removal and corresponding links' removal, Algorithm 3.1 joins $u_{\lfloor \frac{n}{2} \rfloor + 2}$ to $u_{\lfloor \frac{n}{2} \rfloor + 1}$ where the link's PI is again upper bounded by $b_{\lfloor \frac{n}{2} \rfloor}$. Hence if optimal value changes, change is reflected in lower limit only, $\max$ upper limit does not change when n is odd.

Let the optimal node $u_{\lfloor \frac{n}{2} \rfloor + 1}$ with $PI = [a_{\lfloor \frac{n}{2} \rfloor + 1}, b_{\lfloor \frac{n}{2} \rfloor + 1}]$ is removed from T_o when n is even. Links $u_{\lfloor \frac{n}{2} \rfloor - 1}u_{\lfloor \frac{n}{2} \rfloor + 1}$ and $u_{\lfloor \frac{n}{2} \rfloor + 1}u_{\lfloor \frac{n}{2} \rfloor}$ incident on $u_{\lfloor \frac{n}{2} \rfloor + 1}$ are having PI s, upper bounded by $b_{\lfloor \frac{n}{2} \rfloor - 1}$ and $b_{\lfloor \frac{n}{2} \rfloor}$ respectively, so $PI(u_{\lfloor \frac{n}{2} \rfloor - 1})$ is upper bounded by $b_{\lfloor \frac{n}{2} \rfloor - 1}$ and $PI(u_{\lfloor \frac{n}{2} \rfloor})$ is upper bounded by $b_{\lfloor \frac{n}{2} \rfloor}$. Therefore after optimal node's removal and corresponding links' removal, Algorithm 3.1 joins $u_{\lfloor \frac{n}{2} \rfloor - 1}$ to $u_{\lfloor \frac{n}{2} \rfloor}$ where the link's PI is upper bounded by $b_{\lfloor \frac{n}{2} \rfloor - 1}$. Since $b_{\lfloor \frac{n}{2} \rfloor - 1} \leq b_{\lfloor \frac{n}{2} \rfloor}$, then upper limit of optimal value changes when n is even and $b_{\lfloor \frac{n}{2} \rfloor - 1} < b_{\lfloor \frac{n}{2} \rfloor}$.

Hence changed value is lesser than the previous one in this case. If $b_{\lfloor \frac{n}{2}-1 \rfloor} = b_{\lfloor \frac{n}{2} \rfloor}$, then changes are seen in lower limits only if exists. $\square$

Remark 3.3.2. As in example 3.3.1, optimal node removal from T_o of Fig. 2.8 changes the max PCI from $[12,428]$ to $[14,428]$ on the reconnected topology. Hence change has been seen on lower limit of optimal value where $n = 5$ is odd.

Theorem 3.3.3. Reconnected topology by Algorithm 3.1 due to removal of any non-optimal node from energy optimal topology of complete interval graph, does not alter the optimal value.

Proof. T_o of complete interval graph model of n nodes is given by Eq. (2.3) and we are going to prove it under two cases.

Case(i) Removal of a neighbour node of an optimal node from T_o .

For an optimal node $u_{\lfloor \frac{n}{2} \rfloor}$ of T_o when n is odd, $u_{\lfloor \frac{n}{2}+2 \rfloor}$ is one a neighbour node of the optimal node and it is removed from T_o . As a consequence of Theorem 3.3.1 and Algorithm 3.1, $u_{\lfloor \frac{n}{2}-1 \rfloor}$ joins with $u_{\lfloor \frac{n}{2} \rfloor}$ where the link $u_{\lfloor \frac{n}{2}-1 \rfloor}u_{\lfloor \frac{n}{2} \rfloor}$ has $PCI = [\max\{a_{\lfloor \frac{n}{2}-1 \rfloor}, a_{\lfloor \frac{n}{2} \rfloor}\}, b_{\lfloor \frac{n}{2}-1 \rfloor}]$. Let $d = \max\{a_{\lfloor \frac{n}{2}-1 \rfloor}, a_{\lfloor \frac{n}{2} \rfloor}\}$ and updation of $PCI(u_{\lfloor \frac{n}{2} \rfloor}) = [\min\{a_{\lfloor \frac{n}{2} \rfloor}, d\}, \max\{b_{\lfloor \frac{n}{2}-1 \rfloor}, b_{\lfloor \frac{n}{2} \rfloor}\}]$ causes $PCI(u_{\lfloor \frac{n}{2} \rfloor}) = [a_{\lfloor \frac{n}{2} \rfloor}, b_{\lfloor \frac{n}{2} \rfloor}]$. Hence no change in optimal value.

Likewise optimal value does not change with the removal of neighbour node from T_o when n is even.

Case(ii) Removal of any non-neighbour node of an optimal node from T_o .

Any link other than the links incident on optimal node has lesser PI than the max PI of T_o . Let us remove a node u_{n-1} from T_o and T_o gets partitioned into $t_1 = u_n u_1$ and $t_2 = u_2 \dots u_{\lfloor \frac{n}{2}+2 \rfloor} u_{\lfloor \frac{n}{2} \rfloor} u_{\lfloor \frac{n}{2}+1 \rfloor}$. As a consequence of Theorem 3.3.1 and Algorithm 3.1, t_1 and t_2 get connected with the joining of u_1 to u_2 to obtain $T_c = u_n u_1 u_2 \dots u_{\lfloor \frac{n}{2}+2 \rfloor} u_{\lfloor \frac{n}{2} \rfloor} u_{\lfloor \frac{n}{2}+1 \rfloor}$. This shows no link has been added or removed from the optimal node in the process of reconnecting topology which implies no effect on optimal value. $\square$

3.3.2 Simulation and evaluation on change in optimal value

To analyze the changes in optimal value on reconnected topology T_c obtained by proposed Algorithm 3.1, we have done the simulation on large number of hypothetical WSNs whose graphical layouts are complete interval graphs. Evaluation of change in optimal value on T_c has been done in the following steps:

1. For each given network of Table 3.2, optimal value i.e. max PI obtained by Algorithm 2.1 has been noted down.

G^c (Vertices,edges)	Optimal max PI by Algorithm 2.1, given in chapter 2	Max PI by proposed Algorithm 3.1		
		When optimal node exits	When optimal node's neighbour exits	When non-neighbour node exits
(5,10)	[27,79]	[28,79]	[27,79]	[27,79]
(6,15)	[41,101]	[40,82]	[41,101]	[41,101]
(7,21)	[64,95]	[64,95]	[64,95]	[64,95]
(9,36)	[25,126]	[54,126]	[25,126]	[25,126]
(11,55)	[58,155]	[80,155]	[58,155]	[58,155]
(13,78)	[53,175]	[74,175]	[53,175]	[53,175]
(15,105)	[74,178]	[135,178]	[74,178]	[74,178]
(17,136)	[106,251]	[111,251]	[106,251]	[106,251]
(19,171)	[147,286]	[175,286]	[147,286]	[147,286]
(21,210)	[97,261]	[186,261]	[97,261]	[97,261]
(23,253)	[26,368]	[74,368]	[26,368]	[26,368]
(25,300)	[200,363]	[246,363]	[200,363]	[200,363]
(27,351)	[200,431]	[240,431]	[200,431]	[200,431]
(29,406)	[172,429]	[243,429]	[172,429]	[172,429]
(30,435)	[224,460]	[165,451]	[224,460]	[224,460]
(32,496)	[168,475]	[188,473]	[168,475]	[168,475]

Table 3.2: Simulation results on max PI by Algorithm 2.1 and Algorithm 3.1 on complete interval graphs

2. Experimentation of T_o disconnectivity and computation of max PI on T_c , obtained by Algorithm 3.1 with each intermediate node's removal has been accomplished.
3. Evaluation of percentage change in lower limit and upper limit of optimal value of power consumption in each experiment has been tabulated and presented graphically.

Table 3.2 shows all the resultant data on max PI which has been obtained through simulation work on Spyder (Python 3.7). The tabulated data clearly depicts that optimal value of power consumption has only been effected by leaving of optimal node. When optimal topology consists of odd number of nodes, optimal node removal effects the lower limit of optimal max PI and when optimal topology consists of even number of nodes, optimal node removal effects the lower limit as well as upper limit of optimal max PI . Though changed upper limit is lesser than the previous one.

Fig. 3.1 shows the % change in lower limit of max PI only when optimal node exits and no change is seen when any node other than optimal node exits from the optimal topologies. Computed % change in Fig. 3.1 is corresponding to the data given in Table 3.2.

Similarly Fig. 3.2 shows the % change in upper limit of max PI only when optimal node exits from its optimal topology and no change is seen when any node other than optimal node exits from the optimal topology. Computed % change in Fig. 3.2 is corresponding to the data given in Table 3.2.

Moreover, in Fig. 3.1 and 3.2, negative % change depicts that the previous lower limit and upper limit of optimal value is lesser than the new one and positive % change depicts that the new one is lesser than the previous one. Hence upper limit of max PI

on reconnected topology by Algorithm 3.1 is never be greater than the upper limit of optimal max PI by Algorithm 2.1. Fluctuation is majorly seen on lower limit of optimal max PI .

Remark 3.3.3. Proposed Algorithm 3.1 reconnects disconnected energy optimal topology in constant time without limiting itself to right-end ordered interval graph. Also changes in optimal value is seen to be minimum since only optimal node's removal is effecting the optimal solution. Changes are reflected in lower limits more often which does not contribute to max power consumption and changes in upper limits result lesser power consumption than the previous ones.

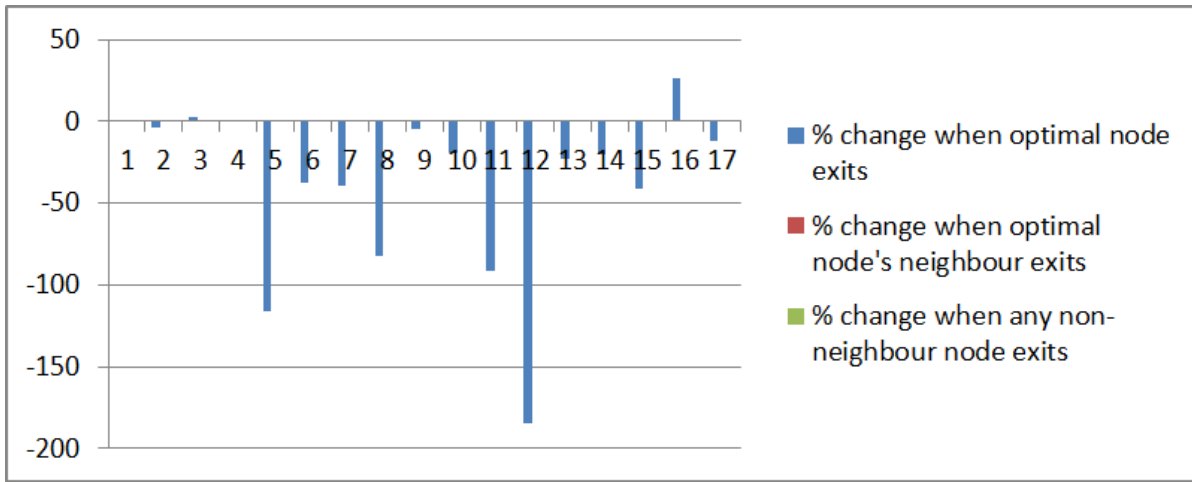


Figure 3.1: Analysis of % change in lower limit of max power consumption on the reconnected topology by Algorithm 3.1

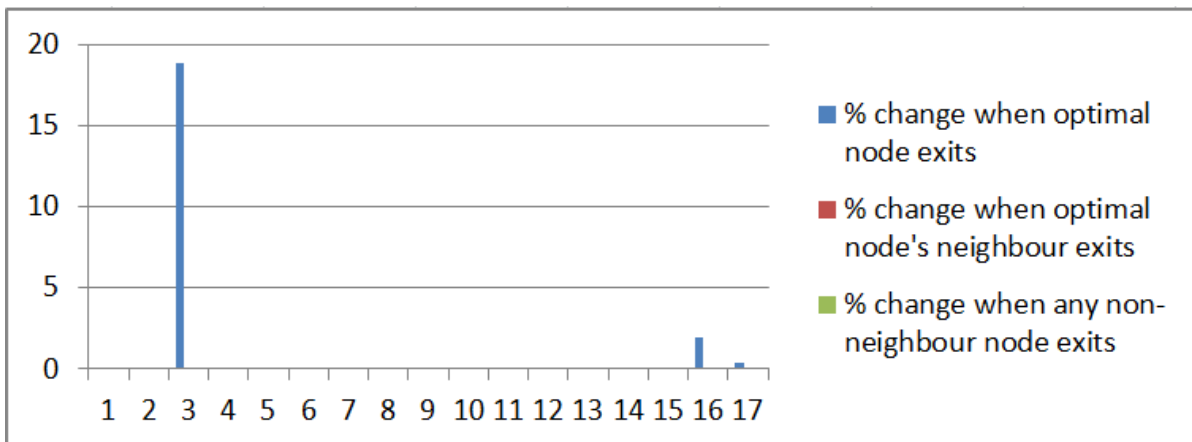


Figure 3.2: Analysis of % change in upper limit of max power consumption on the reconnected topology by Algorithm 3.1

3.4 Sensitivity analysis on energy optimal topology of incomplete interval graph models

Till now, sensitivity analysis has been carried out on energy optimal topologies of complete interval graphs. Extending the analysis to those WSNs whose underlying graphs can be incomplete interval graphs as well. Disconnectivity of energy optimal topology obtained on any incomplete interval graph brings complications in rejoining disjoint components with more effects on optimal max PI . As the disconnected topology of complete interval graph can be reorganized in constant time because of the factor that two nodes which are adjacent to the leaving node find non-empty intersection of their power intervals to establish connection just after the disconnectivity. This does not occur with the energy optimal topologies on incomplete interval graphs. Failure and removal of an intermediate node from the optimal topologies of incomplete interval graphs leads to five different types of disconnectivity which complicates the rejoining of disjoint components of the disconnected topologies. These different types of disconnectivity are called 'DType' and are discussed below. Let an intermediate node w is leaving from an optimal topology T_o of any incomplete interval graph $G(V, E)$, partitions the T_o into $T_d = \{t_1, t_2\}$ where each t_1, t_2 is a tree-like structural component of T_o , $t_1 = u_n u_c u_p \dots u_t$ and $t_2 = v_m v_d \dots v_e v_r$. We now describe the types of disconnectivity (DType), possibly occur in reconnecting the disconnected topology.

DType I: $PI(u_t) \cap PI(v_m) = \phi$ and there exists only one node $\in t_1$, having non-empty power intersection with only one node $\in t_2$.

DType II: $PI(u_t) \cap PI(v_m) = \phi$ and there exists more than one node $\in t_1$, having non-empty power intersection with more than one node $\in t_2$.

DType III: $PI(u_t) \cap PI(v_m) \neq \phi$ and u_t is having non-empty power intersection with more than one node $\in t_2$. Let's say u_t has non-empty power intersection with $\{v_m, v_d, \dots, v_e\}$.

DType IV: None of the node $\in t_1$ is having non-empty power intersection with any node of t_2 although the nodes from t_1 and t_2 were connected in the original network $G(V, E)$.

DType V: Any node $\in t_1$ neither has non-empty power intersection with any node of t_2 nor adjacency in $G(V, E)$. This case occurs when a cut-vertex of $G(V, E)$ has been removed from T_o and $G(V, E)$ is actually a separable interval graph model.

To deal with each DType optimally, we have proposed two more methodologies based on empty as well as non-empty power intersection to establish connectivity between disjoint components with minimum changes to optimal value of power consumption.

3.4.1 Proposed Algorithm 3.2

Removal of any intermediate node from the energy optimal topology of any incomplete interval graph $G(V, E)$, partitions the T_o into two or more disjoint connected components and raises one or combination of the DType I, II, III, IV and V while reconnecting disjoint components.

Algorithm 3.2 has been designed in such a way that it reconnects the disconnected topology under any of the DType I, II, III or IV with the minimum change to optimal value.

Let us consider Fig. 3.3(b) which shows the energy optimal topology T_o by Algorithm 2.2 and 2.3, introduced in chapter 2, obtained on the network G_1 given in Fig. 3.3(a). Table 3.3 shows the *PI* details of each node on G_1 and T_o of G_1 . Removal of a node v_6 from T_o disconnects the T_o into five components, given here as $T_d = \{v_{15}v_8v_{13}v_{14}v_9v_{12}v_{10}v_{11}v_2, v_4, v_5, v_7, v_1v_3\}$. It is seen from Table 3.3 that adjacent nodes of v_6 has empty power intersection with each other i.e. *PI* of nodes v_2, v_4, v_5, v_7 and v_1 are non-intersecting with other. Instead of adjacent nodes of leaving node, nodes v_{11}, v_{13}, v_8 find non-empty power intersection with nodes v_4, v_5 and v_7 (DType II). Nodes v_1 and v_3 are left with empty power intersection with each node though they are having adjacency in G_1 with the nodes of other components (DType IV). This is just an example having DType II and IV in a network with a removal of only one node v_6 . Therefore network would have any of other aforementioned DType with another node's removal.

This shows that joining of disconnected components of an energy optimal topology of an incomplete interval graph is not same as the topology of complete interval graph. Thereafter examining all the possible DType based on power intersection of nodes and adjacency, on large number of distinct networks, Algorithm 3.2 has been proposed to deal with each of the DType I, II, III and IV in such a way that it results reconnected topology of an incomplete interval graph with minimum effect on optimal value.

Algorithm 3.2 Reconnect the disconnected components of energy optimal topology of an incomplete interval graph under DType I, II , III and IV.

Input: A disconnected topology, $T_d = \{t_1, t_2, t_3, \dots, t_m\}$ of an incomplete interval graph $G(V, E)$

Output: A connected topology T_c with $T_c = t_1 \cup t_2 \cup t_3 \cup \dots t_m$.

```

1:  $T_c \leftarrow t_1$  and  $T_d \leftarrow T_d/t_1$ 
2: while  $T_d \neq \phi$  do
3:    $T_r \leftarrow t_2$ 
4:   for  $u \in T_c$ , in reverse order, where  $PI(u) = [a_u, b_u]$  do
5:     Compute  $IS(u) = \{v \mid v \in T_r \text{ and } PI(v) \cap PI(u) \neq \phi\}$  where  $PI(v) = [a_v, b_v]$ .  $\triangleright IS$ 
       is a set of intersecting nodes.
6:     if  $IS(u) \neq \phi$  then
7:       Select a node  $v \in IS$  such that  $b_v$  is minimum and join  $v$  to  $u \in T_c$ .
8:   if  $IS = \phi \forall u \in T_c$  then
9:     Obtain  $E(T_c, T_r) = \{e_1, e_2, \dots, e_k\}$ , list of all edges existing between the nodes of  $T_c$ 
       and  $T_r$  in its interval graph  $G(V, E)$ .
10:    Select  $e_i \in E(T_c, T_r)$  where  $PI(e_i) = [c_i, d_i]$  is minimum.
11:    Let  $p \in T_c$  and  $q \in T_r$  are the end nodes of  $e_i$  with  $PI(p) = [a_p, b_p]$ ,  $PI(q) = [a_q, b_q]$ 
       and  $b_p < a_q$ .
12:    Updating  $PI(p)$  to  $[min\{c_i, a_p\}, d_i]$  and  $PI(q)$  to  $[min\{c_i, a_q\}, b_q]$ .
13:    Update  $T_c \leftarrow T_c \cup T_r$  and  $T_d \leftarrow T_d/T_r$ .
14: Return  $T_c$ 

```

Theorem 3.4.1. Time complexity of Algorithm 3.2 is $\mathcal{O}(n^2)$ where n is number of nodes of an incomplete interval graph $G(V, E)$.

Proof. Step 1 is done in constant time. While loop repeats till some constant time. Steps 4 and 5 collectively, inside the while loop take $\mathcal{O}(n^2)$ time. Step 7 can be done in $\mathcal{O}(n)$ time. Step 9 takes $\mathcal{O}(n^2)$ and step 10 $\mathcal{O}(n)$. Steps 11,12 and 13 can be done in constant time. Hence the total time complexity of Algorithm 3.2 is $\mathcal{O}(n^2)$ where n is the number of vertices. $\square$

Example 3.4.1. Working illustration of Algorithm 3.2 on an energy optimal topology, given in Fig. 3.3.

Let us consider the removal of node v_6 from T_o , it results $T_d = \{v_{15}v_8v_{13}v_{14}v_9v_{12}v_{10}v_{11}v_2, v_4, v_5, v_7, v_1v_3\}$. Algorithm 3.2 starts by selecting first component from T_d by assigning $T_c = v_{15}v_8v_{13}v_{14}v_9v_{12}v_{10}v_{11}v_2$ and T_d becomes $\{v_4, v_5, v_7, v_1v_3\}$ in step 1. Step 2 initiates while loop as $T_d \neq \phi$ and step 3 starts with next component from T_d to join it with T_c by assigning $T_r = v_4$. Step 4 starts For loop over the nodes of $T_c = v_{15}v_8v_{13}v_{14}v_9v_{12}v_{10}v_{11}v_2$ in reverse order i.e. loop runs from last node to first, therefore $u = v_2$. Step 5 computes the intersecting set for u i.e. $v \in IS(u)$ if $PI(u) \cap PI(v) \neq \phi$. Hence $IS(v_2) = \{\}$, then it computes $IS(v_{11}) = \{v_4\}$ and validates the If condition of step 6. Step 7 joins v_{11} with v_4 . Step 13 merges T_c with T_r and T_d becomes $\{v_5, v_7, v_1v_3\}$. Again the same procedure repeats

for component v_5 and v_7 , connects v_{11} with v_5 and v_7 , merges the components with T_c and so updated $T_d = \{v_1v_3\}$. Previous joining has been done under the condition of $IS \neq \phi$ and DType II. Now when $T_r = v_1v_3$, $IN = \phi$ for all nodes $\in T_c$. Step 9 put forth edges v_2v_3 , v_3v_{11} and v_3v_{13} with $PI(v_2v_3) = [4, 6]$, $PI(v_3v_{11}) = [6, 6]$ and $PI(v_3v_{13}) = [3, 6]$ respectively from original graph. Hence step 10 selects v_3v_{13} to connect T_c and T_r . Finally step 13 empties T_d and step 14 returns the desired reconnected topology, shown in Fig. 3.4.

Remark 3.4.1. Optimal max PI on T_o has been $[9, 246]$ and it becomes $[3, 246]$ on reconnected topology by Algorithm 3.2, hence upper limit of max power consumption is left unchanged with the decrement in lower limit of max PI . This changes occur because of update of PI of v_{13} under DType IV when $IS = \phi$.

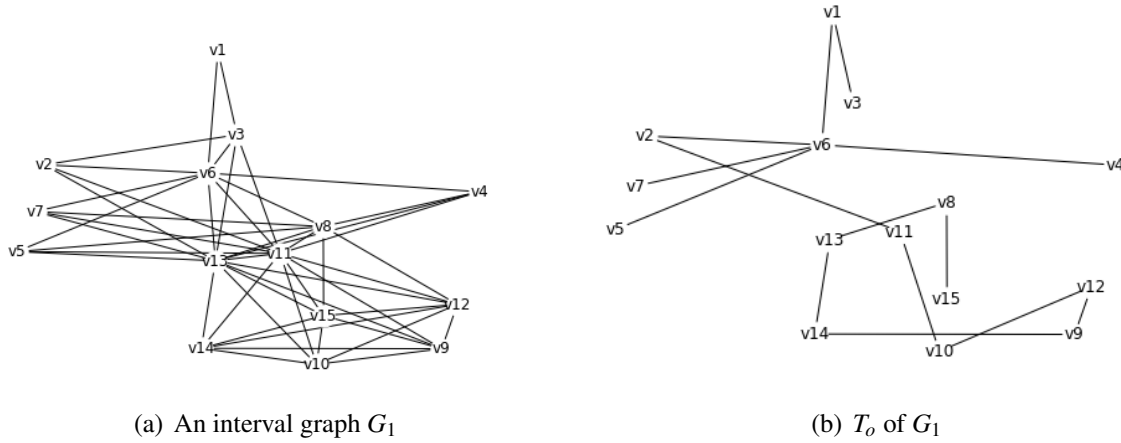


Figure 3.3: An incomplete interval graph and its energy optimal topology by Algorithm 2.2 and 2.3

Node	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
PI on G_1	[1,2]	[4,6]	[1,6]	[12,18]	[20,22]	[2,29]	[23,54]	[9,91]	[95,142]	[121,158]	[6,189]	[78,227]	[3,246]	[93,277]	[68,293]
PI on T_o of G_1	[1,2]	[4,6]	[1,2]	[12,18]	[20,22]	[2,29]	[23,29]	[9,91]	[95,142]	[121,158]	[6,158]	[95,158]	[9,246]	[93,246]	[68,91]

Table 3.3: PI details of Fig. 3.3

3.4.2 Proposed Algorithm 3.3

Algorithm 3.3 is the most innovative approach in the field of graph theory to connect disconnected components of a spanning tree like subgraph of a separable interval graph when the cut-vertex of the graph has been removed from its spanning tree like subgraph. This has been accomplished using triangle inequality based on link length.

When an intermediate vertex say w is leaving from an energy optimal topology T_o of an incomplete interval graph $G'(V, E)$ and result DType V. This implies that w is a

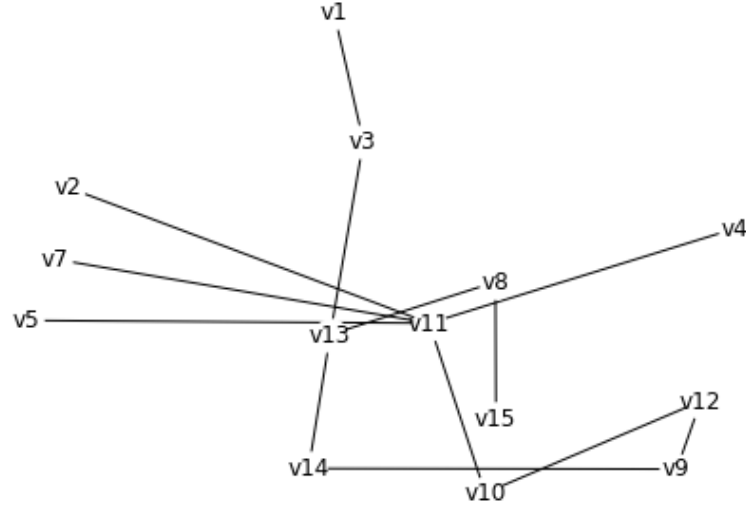


Figure 3.4: Reconnected topology by Algorithm 3.2 when v_6 exits from Fig. 3.3(b)

cut-vertex of $G'(V, E)$ and so $G'(V, E)$ is a separable interval graph causes DType V with respect to its cut-vertex failure and removal.

Leaving of any cut-vertex of a separable interval graph from its energy optimal topology T_o causes worst disconnectivity to T_o with no scope of reconnectivity of disjoint components of T_o . Considering this most complicated situation of disconnectivity (D-type V), a new connectivity algorithm, say Algorithm 3.3 has been designed which works to rejoin disjoint components of T_o by introducing dummy links where dummy links' weight have been computed by applying triangle inequality based on the links' length of $G'(V, E)$. The link with minimum interval weight is selected to join disjoint components under DType V.

As in Fig. 3.5(a), G_2 is a separable interval graph and u_3 is its cut-vertex. Removal of node u_3 from the energy optimal topology of G_2 (shown in Fig. 3.5(b)), disconnects it into two disjoint connected components in such a way that no node from the first component has non-empty power intersection with any node of the second component (shown in Fig. 3.6). Also nodes belong to distinct components do not have adjacency even in its original interval graph G_2 (Dtype V).

Example 3.4.2. Working illustration of Algorithm 3.3 on the networks given in Fig. 3.5.

Fig. 3.5(a), (b) and 3.6 show the separable interval graph G_2 , energy optimal topology T_o on G_2 by Algorithm 2.2 and 2.3 and disconnected topology of T_o after u_3 removal, respectively. Removal of u_3 , which is a cut-vertex of G_2 , from $T_o = u_6u_4u_5u_3u_1u_2$, partitions the T_o into two components say $t_1 = u_1u_2$ and $t_2 = u_6u_4u_5$ as shown in Fig. 3.6. Therefore input to Algorithm 3.3 is $T_d = \{t_1, t_2\}$. Step 1 assigns $T_c = u_1u_2$ and update $T_d = \{t_2\}$. Since $T_d \neq \emptyset$, step 2 initiates while loop. Step 3 assigns $T_r \leftarrow t_2$ from T_d so $T_r = u_6u_4u_5$. In step 4, $nghT_c$

Algorithm 3.3 Reconnect the disjoint components of energy optimal topology of a separable interval graph $G'(V, E)$ under DType V.

Input: $T_d = \{t_1, t_2, t_3, \dots, t_m\}$ when cut-vertex w of $G'(V, E)$ exits from T_o .

Output: A reconnected topology T_c with $T_c = t_1 \cup t_2 \cup t_3 \cup \dots t_m$.

- 1: $T_c \leftarrow t_1$ and $T_d \leftarrow T_d/t_1$
 - 2: **while** $T_d \neq \emptyset$ **do**
 - 3: $T_r \leftarrow t_2$
 - 4: Compute $nghT_c = \{v \mid v \in T_c \text{ and } wv \in E(G')\}$ and $nghT_r = \{u \mid u \in T_r \text{ and } wu \in E(G')\}$
 - 5: Obtain $LS = \{d_{vw} \mid d_{vw} = ||vw||^2\}$ for all $v \in nghT_c$ $\triangleright LS$ is a set of link's length
 - 6: Select $v \in nghT_c$ whose $d_{vw} = \min\{LS\}$
 - 7: **for** $u \in nghT_r$ **do**
 - 8: Compute $L_{max}(uv) = d_{uw} + d_{wv}$ and $L_{min}(uv) = \max\{d_{uw}, d_{wv}\} - \min\{d_{uw}, d_{wv}\}$ $\triangleright$ Triangle inequality
 - 9: Update $PI(uv) = [L_{min}(uv), L_{max}(uv)]$
 - 10: Select uv as a connecting edge between T_r and T_c whose $L_{max}(uv)$ is minimum
 - 11: Update $PI(u) = [a_u, b_u]$ to $[\min\{a_u, L_{min}(uv)\}, L_{max}(uv)]$ and $PI(v) = [a_v, b_v]$ to $[L_{min}, \max\{b_v, L_{max}(uv)\}]$ where $b_u < a_v$
 - 12: Update $T_c \leftarrow T_c \cup T_r$ and $T_d \leftarrow T_d/T_r$
 - 13: **Return** T_c
-

computes the set of nodes from T_c which have been neighbour nodes to u_3 in G_2 . Likewise $nghT_r$ computes the set of nodes from T_r which have been neighbour nodes to u_3 in G_2 . Hence $nghT_c = \{u_1, u_2\}$ and $nghT_r = \{u_4, u_5\}$. Step 5 obtains LS , set of links' weight for links u_1u_3 and u_2u_3 in term of Euclidean distance from G_2 . Then $LS = \{60, 90\}$ where $d_{u_1u_3} = 60$ and $d_{u_2u_3} = 90$. Step 6 selects $u_1 \in nghT_c$ as $d_{u_1u_3} = \min\{LS\}$. For u_4 and $u_5 \in T_r$, step 8 assumes new links u_4u_1 and u_5u_1 as shown in Fig. 3.7(a), called them 'dummy links'. Existence of these links require computation of their length to approximate their transmission range and power consumption. It is not possible to compute exact link length in the graph given in Fig. 3.7(a), so Triangle inequality has been used to approximate length of dummy links in term of interval. Therefore step 8 computes L_{max} and L_{min} for each dummy links u_1u_4 and u_1u_5 , using the links' weight given in Fig. 3.5(a). $L_{max}(u_1u_5) = 60 + 105 = 165$ and $L_{min}(u_1u_5) = 105 - 60 = 45 \Rightarrow PI(u_1u_5) = [45, 165]$, likewise $L_{max}(u_1u_4) = 60 + 94 = 154$ and $L_{min}(u_1u_4) = 94 - 60 = 34 \Rightarrow PI(u_1u_4) = [34, 154]$. Step 10 selects dummy link u_1u_4 with $PI = [34, 154]$ whose upper limit of power consumption is lesser than the link u_1u_5 to join t_1 and t_2 . Step 12 updates $PI(u_1)$ to $[34, 154]$ and $PI(u_4)$ to $[34, 154]$ due to addition of dummy link u_1u_4 using Eq. (2.5). Step 13 updates T_c and empties T_d . Step 14 returns the reconnected topology, shown in Fig. 3.7(b).

Remark 3.4.2. Max PI on the reconnected topology T_c comes out to be $[34, 154]$, shown in Fig. 3.7(b), previously it was $[104, 110]$ on T_o of Fig. 3.5(b). Hence cut-vertex removal from T_o of a separable interval graph brings reconnectedness on the expense of more power consumption.

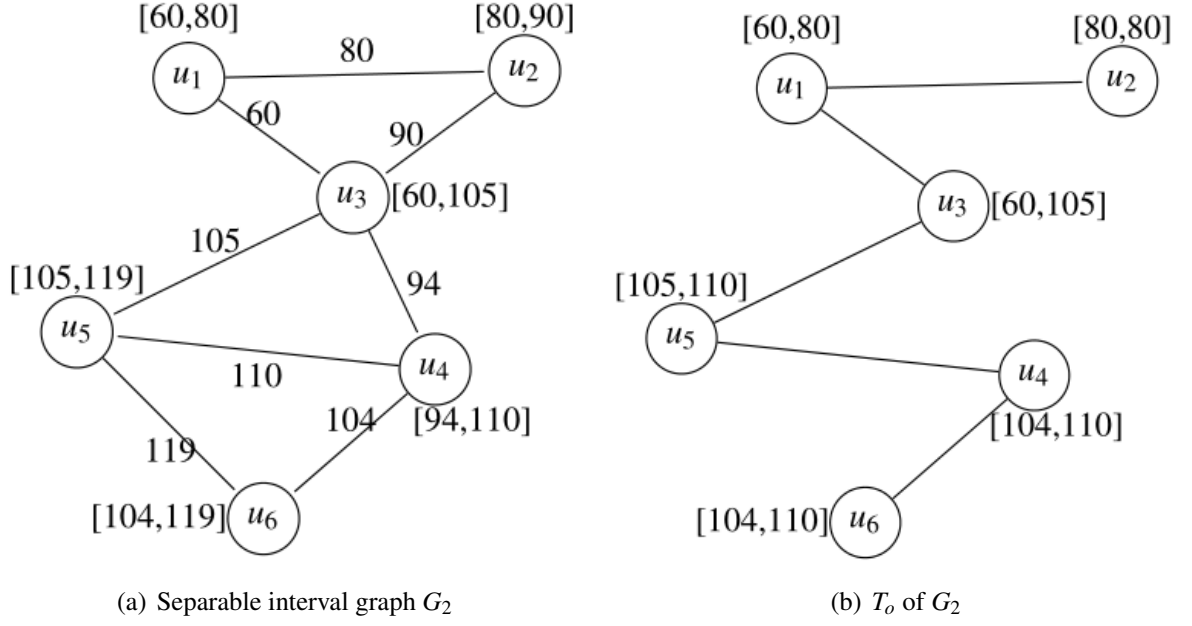


Figure 3.5: Separable graph and its energy optimal topology by Algorithm 2.2 and 2.3

Theorem 3.4.2. Time complexity of Algorithm 3.3 is $\mathcal{O}(n^2)$ where n is the number of vertices of a separable interval graph $G'(V, E)$.

Proof. Step 1 is done in constant time. While loop repeats at the most n times. Steps 4 and 5 inside the while loop take $\mathcal{O}(n)$ time. Step 7 initiates for loop with the iterations at the most n times inside the while loop. Rest of the steps can be done in constant time. Hence the total time complexity of Algorithm 3.3 is $\mathcal{O}(n^2)$ where n is the number of vertices of $G'(V, E)$. $\square$

3.4.3 Results on change in optimal value of power consumption

Theorem 3.4.3. Removal of an optimal node from the energy optimal topology of an arbitrary interval graph $G'(V, E)$ may change the optimal max PI on the reconnected topology. Changed max PI has upper limit lesser and equal to the previous one provided the removing node is not a cut-vertex of $G'(V, E)$.

Proof. Let us prove it by assuming an energy optimal topology $T_o = u_n u_m \dots u_r u_s w v_q v_p v_r \dots v_t$ on any arbitrary interval graph and w is an optimal node with $\max PI = [a_w, b_w]$. Let w is leaving T_o which partitions T_o into $t_1 = u_n u_m \dots u_r u_s$ and $t_2 = v_q v_p v_r \dots v_t$.

Case(I) When optimal node is unique.

Optimal node is $w \Rightarrow \max PI$ exists on $w \Rightarrow$ atleast one of the incident links on w has max power consumption. Therefore removal of w terminates max power consuming link with itself. Let us consider randomly that u_m and v_r is the joining of t_1 and t_2 to obtain T_c where $PI(u_m) =$

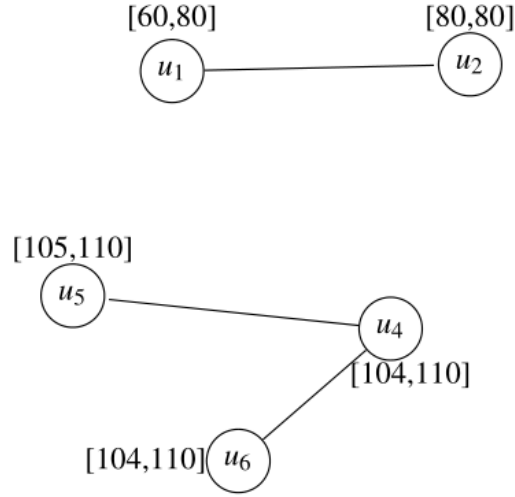


Figure 3.6: An illustration of DType V when u_3 leaves from Fig. 3.5(b)

$[a_{u_m}, b_{u_m}]$ and $PI(v_r) = [a_{v_r}, b_{v_r}]$.

Situation(i) $b_{u_m} < b_{v_r} < \dots < b_{v_p} < b_w$.

Max upper limit after b_w is b_{v_p} hence optimal value changes to $\max PI = [a_{v_p}, b_{v_p}]$ and v_p becomes the optimal node on T_c . It is obvious $PI(v_p) < PI(w)$.

Situation(ii) $b_{u_m} < b_{v_r} < \dots \leq b_{u_r} \leq b_{v_p} \leq b_w$.

Let us consider randomly that nodes u_r, v_p and w are having same value of upper limit with different value of lower limits i.e. $PI(u_r) = [a_{u_r}, b_w], PI(v_p) = [a_{v_p}, b_w]$ and $PI(w) = [a_w, b_w]$ such that $a_w < a_r < a_p$. Hence removal of w and joining of u_r and v_p to obtain T_c , shifts the optimal value to u_r with $\max PI = [a_{u_r}, b_w]$ i.e. changes in optimal max PI has been reflected through lower limit only.

Case(II) When there exists two optimal nodes with same optimal value.

Let w and v_q are two optimal nodes with same $PI = [a_w, b_w]$. This implies link wv_q is having weight equal to $[a_w, b_w]$. Hence removal of any optimal node either w or v_q , terminates wv_q and brings both the situation of *case(I)*. Hence optimal value changes either by changing lower limit or upper limit or both.

Case(III) When there exists more than two optimal nodes with same optimal value.

Let u_s, w and v_q are three optimal nodes with $\max PI = [a_w, b_w]$. Links $u_s w$ and wv_q are having max weight equal to $[a_w, b_w]$. Hence removal of any one optimal node, u_s, w or v_q , terminates only one link at a time, hence optimal value doesn't change. $\square$

Theorem 3.4.4. Removal of a neighbour node of an optimal node from the optimal topology of an arbitrary interval graph $G'(V, E)$, may change the optimal max PI on the reconnected topology. Changed max PI has upper limit lesser and equal to the previous one provided the removing node is not a cut-vertex of $G'(V, E)$.

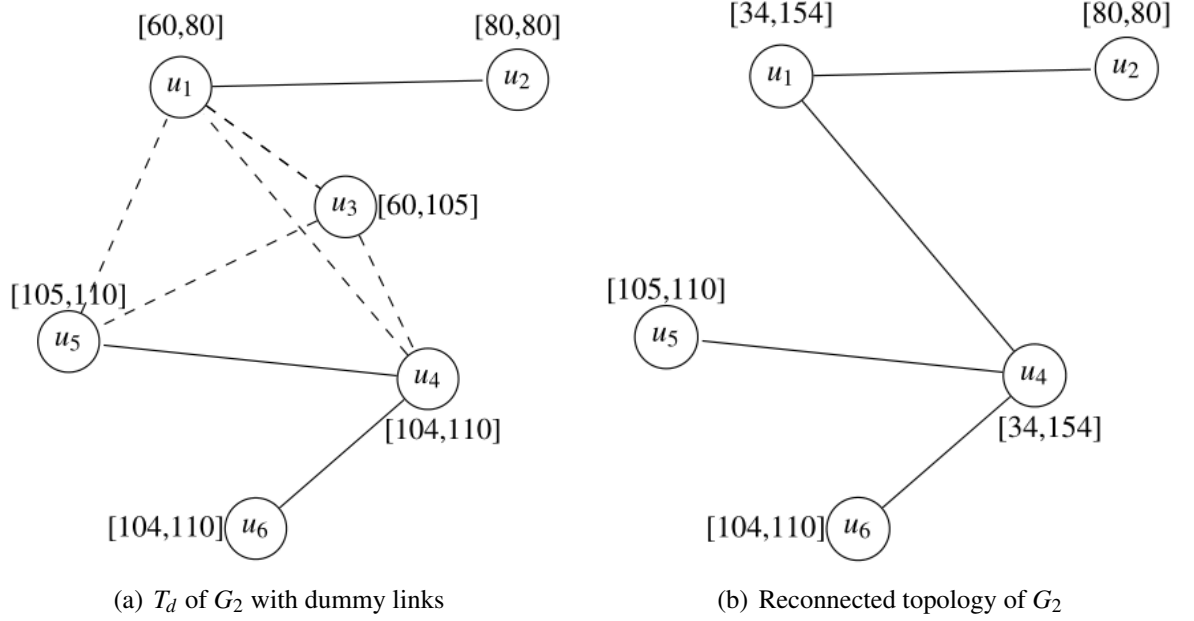


Figure 3.7: Working illustration of Algorithm 3.3 on Fig. 3.6

Proof. Let us prove it by assuming an optimal topology $T_o = u_n u_m \dots u_r u_s w v_q v_p v_r \dots v_t$ on any incomplete interval graph and w is an optimal node. Let u_s , being a neighbour node of w , is leaving T_o , therefore it partitions T_o into $t_1 = u_n u_m \dots u_r$ and $t_2 = w v_q v_p v_r \dots v_t$.

Let $PI(u_s w) = [a_1, b_1]$ and $PI(w v_q) = [a_2, b_2] \Rightarrow PI(w) = [\min\{a_1, a_2\}, \max\{b_1, b_2\}]$ on T_o .

Case (I) If the joining of t_1 and t_2 is made on w .

Let us consider randomly that node $u_r \in t_1$ joins with $w \in t_2$ with $PI(w u_r) = [a_3, b_3]$ to obtain T_c . It is obvious $b_3 < \max\{b_1, b_2\}$.

Situation(i) $b_2 > b_1 > b_3$ and $a_1 < a_2$.

$PI(w) = [a_1, b_2]$ on T_o , exit of u_s and joining of $w u_r$ to w updates $PI(w)$ to $[\min\{a_2, a_3\}, b_2]$ on T_c . Hence optimal value changes with the change in lower limit only.

Situation(ii) $b_1 > b_2 > b_3$ and $a_1 < a_2$.

$PI(w) = [a_1, b_1]$ on T_o , exit of u_s and joining of $w u_r$ to w , updates $PI(w)$ to $[\min\{a_2, a_3\}, b_2]$ on T_c . Hence optimal value changes with the change in lower as well as upper limit.

Situation(iii) $b_1 > b_2 > b_3$ and $a_2 < a_1 < a_3$.

$PI(w) = [a_2, b_1]$ on T_o , exit of u_s and joining of $w u_r$ to w , updates $PI(w)$ to $[a_2, b_2]$ on T_c . Hence optimal value changes with the change in upper limit only.

Situation(iv) $b_1 > b_2 > b_3$ and $a_3 < a_1 < a_2$.

$PI(w) = [a_1, b_1]$ on T_o , exit of u_s and joining of $w u_r$ to w , updates $PI(w)$ to $[a_3, b_2]$ on T_c . Hence optimal value changes with the change in lower as well as upper limit.

Situation(v) $b_1 > b_2 > b_3$ and $a_3 < a_2 < a_1$.

$PI(w) = [a_2, b_1]$ on T_o , exit of u_s and joining of $w u_r$ to w , updates $PI(w)$ to $[a_3, b_2]$ on T_c .

Hence optimal value changes with the change in lower as well as upper limit.

Situation(vi) $b_2 > b_1 > b_3$ and $a_3 < a_2 < a_1$.

$PI(w) = [a_2, b_1]$ on T_o , exit of u_s and joining of wu_r to w , updates $PI(w)$ to $[a_3, b_2]$ on T_c .

Hence optimal value changes with the change in lower as well as upper limit.

Situation(vii) $b_2 = b_1 > b_3$ and $a_3 < a_2 < a_1$.

$PI(w) = [a_2, b_2]$ on T_o , exit of u_s and joining of wu_r to w , updates $PI(w)$ to $[a_3, b_2]$ on T_c .

Hence optimal value changes with the change in lower limit only.

Situation(viii) $b_2 = b_1 > b_3$ and $a_1 < a_2 < a_3$.

$PI(w) = [a_1, b_2]$ on T_o , exit of u_s and joining of wu_r to w , updates $PI(w)$ to $[a_2, b_2]$ on T_c .

Hence optimal value changes with the change in lower limit only.

Situation(ix) $b_2 = b_1 > b_3$ and $a_2 < a_1 < a_3$.

$PI(w) = [a_2, b_2]$ on T_o , exit of u_s and joining of wu_r to w , updates $PI(w)$ to $[a_2, b_2]$ on T_c .

Hence no change in optimal value.

Situation(x) $b_2 = b_1 > b_3$ and $a_2 = a_1 < a_3$.

$PI(w) = [a_2, b_2]$ on T_o , exit of u_s and joining of wu_r to w , updates $PI(w)$ to $[a_2, b_2]$ on T_c .

Hence no change in optimal value.

Case (II) If the joining of t_1 and t_2 is not made on w .

Let $u_m \in t_1$ joins with $v_r \in t_2$ to obtain T_c with $PI(u_mv_r) = [a_3, b_3]$. It is obvious that $b_3 < \max\{b_1, b_2\}$ and $PI(w)$ will only be affected by links $u_s w$ and $w v_q$.

Situation(i) $b_2 > b_1$ and $a_1 < a_2$.

$PI(w) = [a_1, b_2]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence optimal value changes with the change in lower limit only.

Situation(ii) $b_1 > b_2$ and $a_1 > a_2$.

$PI(w) = [a_2, b_1]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence optimal value changes with the change in upper limit.

Situation(iii) $b_1 > b_2$ and $a_2 > a_1$.

$PI(w) = [a_1, b_1]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence optimal value changes with the change in lower as well as upper limit.

Situation(iv) $b_1 > b_2$ and $a_1 = a_2$.

$PI(w) = [a_2, b_1]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence optimal value changes with the change in upper limit only.

Situation(v) $b_2 > b_1$ and $a_1 = a_2$.

$PI(w) = [a_2, b_2]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence no change in optimal value.

Situation(vi) $b_2 > b_1$ and $a_2 < a_1$.

$PI(w) = [a_2, b_2]$ on T_o and removal of u_s updates $PI(w) = [a_2, b_2]$ on T_c . Hence no change in optimal value. $\square$

Theorem 3.4.5. Removal of any non-neighbour node of an optimal node under DType I, II or III, does not effect the optimal max PI on the reconnected topology by Algorithm 3.2.

Proof. Let w be any intermediate node, neither optimal node nor neighbour node of optimal node, removed from $T_o = u_n u_m \dots u_t u_s w v_d v_p v_q \dots v_x v_y$ and T_o gets partitioned into $t_1 = u_n u_m \dots u_t u_s$ and $t_2 = v_d v_p v_q \dots v_x v_y$ under any of the DType I, II and III. According to DType I, II and III, atleast one node $\in t_1$ finds non-empty power intersection.

Let v_p be an optimal node with optimal value $PI(v_p) = [a_{v_p}, b_{v_p}]$ where b_{v_p} is the maximum i.e. $b_{v_p} \geq b_{u_i}$ and $b_{v_p} \geq b_{v_j} \forall u_i \in t_1$ and $v_j \in t_2$ respectively.

Case(i) When connection is made on optimal node v_p .

Let $u_t \in t_1$ finds $IS(u_t) = \{v_p\}$ from t_2 . Therefore Algorithm 3.2 joins u_t to v_p with $PI(u_t v_p) = [\max\{a_{u_t}, a_{v_p}\}, b_{u_t}] = [d1, b_{u_t}]$ where $d1 = \max\{a_{u_t}, a_{u_t}\}$. Again updation of $PI(v_p) \Rightarrow [\min\{a_{v_p}, d1\}, \max\{b_{u_t}, b_{v_p}\}] \Rightarrow PI(v_p) = [a_{v_p}, b_{v_p}]$. Likewise updation of $PI(u_t) \Rightarrow [\min\{a_{u_t}, d1\}, b_{u_t}] \Rightarrow PI(u_t) = [a_{u_t}, b_{v_p}]$. Hence no change in optimal value.

Case(ii) When connection is made on any non-optimal node.

Let's say $u_s \in t_1$ finds $IS(u_s) = \{v_d, v_q, v_x\}$ such that $b_{v_x} < b_{v_d} \leq b_{v_q}$, hence Algorithm 3.2 joins u_s with v_x to connect t_1 and t_2 . So it does not disturb any link incident on optimal node v_p . Also $PI(u_s v_x) = [\max\{a_{u_s}, a_{v_x}\}, \min\{b_{u_s}, b_{v_x}\}]$ is always lesser than $PI(v_p)$. Hence no change in optimal value. $\square$

Theorem 3.4.6. Reconnectivity by Algorithm 3.2 under DType IV effects the optimal max PI under the following conditions:

- (i) when optimal node leaves and reconnection is made between non-optimal nodes.
- (ii) when non-optimal node leaves and reconnection is made using optimal node.

Proof. (i) It is obvious that removal of an optimal node from T_o may effect the previous optimal value of max power consumption by Theorem 3.4.2 and since optimal node exits from T_o , then only non-optimal nodes will join together to establish connection. Hence this is proved.

(ii) Let u and v be the end nodes of the connecting edge uv , selected by Algorithm 3.2 under DType IV. Let if $PI(u) = [a_u, b_u]$ and $PI(v) = [a_v, b_v]$, then $PI(u) \cap PI(v) = \emptyset$ under DType IV on energy optimal topology T_o .

Let the joining is made on an optimal node, this implies that one of the node either u or v is an optimal node of edge uv . Let v be an optimal node and $b_u < a_v < b_v$. Algorithm 3.2 updates $PI(uv)$ to $[b_u, a_v]$ which therefore updates $PI(u)$ to $[a_u, a_v]$ and $PI(v)$ to $[b_u, b_v]$. Hence optimal

value changes from $[a_v, b_v]$ to $[b_u, b_v]$ i.e. only lower limit of max PI changes.

Let if joining is not made on an optimal node then neither u nor v is an optimal node of the connecting link uv . Let another node w is an optimal node with $PI(w) = [a_w, b_w]$ such that b_w is maximum among the upper limit of power consumption $\Rightarrow PI(u) = [a_u, a_v] < PI(w)$ and $PI(v) = [b_u, b_v] < PI(w)$. Hence no change in optimal max PI . $\square$

Theorem 3.4.7. Removal of a cut-vertex of a separable interval graph from its energy optimal topology may effect the optimal max PI by increasing the upper limit and decreasing the lower limit of maximum power consumption on the reconnected topology by Algorithm 3.3.

Proof. Let the removal of a cut-vertex w partitions the T_o into t_1 and t_2 . Algorithm 3.3 makes a dummy link say uv where $u \in t_1$ and $v \in t_2$ with $PI(u) = [a_u, b_u]$ and $PI(v) = [a_v, b_v]$ have empty intersection. Let $d_{uw} = ||uw||^2$ and $d_{wv} = ||wv||^2$ where $d_{uw} \in PI(u)$, $d_{wv} \in PI(v)$ and $d_{wv} > d_{uw}$. Now $PI(uv) = [L_{min}(uv), L_{max}(uv)]$ where $L_{min}(uv) = d_{wv} - d_{uw}$ and $L_{max}(uv) = d_{uw} + d_{wv}$, updates $PI(u)$ from $[a_u, b_u]$ to $[min\{a_u, L_{min}(uv)\}, L_{max}(uv)]$ and $PI(v)$ from $[a_v, b_v]$ to $[L_{min}, max\{b_v, L_{max}(uv)\}]$ where $b_u < a_v < b_v$.

Let if v is an optimal node with optimal value $[a_v, b_v]$, then if $L_{max}(uv) > b_v$ then optimal max PI changes with the decrement of lower limit from a_v to L_{min} and increment of upper limit from b_v to L_{max} .

Let another node s is an optimal node with optimal solution $[a_s, b_s]$ where $b_u < a_v < b_v < \dots < b_s$. Then if $L_{max}(uv) > b_s$ and $L_{min} < a_s$, hence optimal max PI changes with the decrement of lower limit from a_s to L_{min} and increment of upper limit from b_s to L_{max} . $\square$

3.4.4 Simulation and evaluation of change in optimal value

Evaluation to analyse changes in optimal max PI of energy optimal topologies when any intermediate node exits from it, partitions the topology into two or more connected components and reorganisation of the disconnected topology by the proposed algorithms, have been done on large number of hypothetical WSNs whose underlying graphs result incomplete as well as separable interval graphs. Codes of the proposed Algorithm 3.2 and 3.3 are written in Spyder (Python 3.7) and simulations have been done to obtain max PI on each reconnected topology corresponding to exit of each intermediate node from the optimal topology.

Table 3.4 shows the simulation result on max PI obtained by energy optimal Algorithm 2.2 and 2.3, given in chapter 2 and proposed Algorithm 3.2 under the DType I, II and III when $IS \neq \phi$. Max PI obtained by energy optimal Algorithm 2.2 and 2.3, is called

optimal value and max PI obtained by proposed Algorithm 3.2, depicts the variation if exists, from the optimal value. It has been analyzed from Table 3.4 that optimal value changes on the reconnected topology only when either optimal node or neighbour node of optimal node leaves from the optimal topology. No change in optimal value is seen when rest of any intermediate node exists. Also if the optimal max PI has been effected with the change in upper limit then the new upper limit of max power consumption is lesser than the previous one, on the reconnected topology by Algorithm 3.2 under DType I, II and III. Fig. 3.8 and 3.9 show the plotting of % change in lower limit and % change in upper limit of max power consumption respectively corresponding to the data given in Table 3.4. % change on positive axis reveals that new value of max power consumption is lesser than the previous one and negative axis reveals that new value of max power consumption is more than the previous one. From Fig. 3.8, it can be said that changed lower limit of max PI is sometime lesser and sometime more than the previous values and from Fig. 3.9 one can see that changed upper limit of max PI is always be lesser or equal to the previous optimal value.

Max PI evaluation on reconnected topology by Algorithm 3.2 under DType IV when $IS = \phi$, has been noted down in Table 3.5. Data given in Table 3.5 clearly depicts that changes to optimal max PI are occurred when optimal node exits or when joining is made on optimal node. Plots in Fig. 3.10 and 3.11 correspond to the data of Table 3.5 reveal that if optimal max PI changes with the change in upper limit then upper limit of max PI is always lesser than the previous optimal value, although lower limit of max PI may increase or decrease on the reconnected topology.

Furthermore data on max PI , obtained by energy optimal Algorithm 2.2 and 2.3 and proposed Algorithm 3.3 under DType V has been given in Table 3.6. It has been observed from Table 3.6 that if optimal value changes then changes are reflected from both the lower and upper end of max PI . Changed value of lower limits of max PI is lesser than the previous one and upper limits of new max PI is more than the previous optimal value. Fig. 3.12 shows the % change in lower limit as well as upper limit of optimal max PI under DType V corresponding to the data given in Table 3.6. Hence it is clearly depicted that changes in lower limit of max PI are approaching to 98% and changes in upper limits are approaching to -82% due to removal of a cut-vertex of a separable interval graph from its energy optimal topology. Therefore failure and exit of a cut-vertex of any separable interval graph from its energy optimal topology results reconnected topology in expense of more power than previous.

Remark 3.4.3. The simulation data in Table 3.4, 3.5 and 3.6 verify the results given in section 3.4.3. Changes to optimal value on the reconnected topology by Algorithm 3.2 under DType I, II, III and/or IV never increase the upper limit of optimal max PI . Fluctuation over lower limit of optimal value never enhance the max power consumption.

Remark 3.4.4. Proposed Algorithm 3.3 brings reconnectedness to the disconnected topology of the separable interval graph without its cut-vertex on the expense of more power. Though this is the very first attempt in the field of graph theory to provide connectivity to the completely disjoint components of a subgraph of a separable graph without its cut-vertex in quadratic time complexity using triangle inequality.

Graph network (Vertices,edges)	Max PI by energy optimal Algorithm 2.2 and 2.3	Max PI by proposed Algorithm 3.2 under DType I, II and III i.e. when $IS \neq \emptyset$.			
		When optimal node exits	When optimal node's left neighbour exits	When optimal node's right neighbour exits	Rest of any intermediate node exits
(6,8)	[95,110]	[95,110]	[95,110]	[95,110]	[95,110]
(7,16)	[65,120]	[90,115]	[90,115]	[65,120]	[65,120]
(9,21)	[94,125]	[94,125]	[94,125]	[94,125]	[94,125]
(10,32)	[37,82]	[30,70]	[41,82]	[30,70]	[37,82]
(13,27)	[128,145]	[128,145]	[128,145]	[128,145]	[128,145]
(15,49)	[9,246]	[6,158]	[93,246]	[6,158]	[9,246]
(18,100)	[47,190]	[82,162]	[47,190]	[82,162]	[47,190]
(23,149)	[82,379]	[6,362]	[347,379]	[6,363]	[82,379]
(25,174)	[149,399]	[120,351]	[151,399]	[149,399]	[149,399]
(28,202)	[24,404]	[29,375]	[165,404]	[29,375]	[24,404]
(30,268)	[15,485]	[49,477]	[26,485]	[49,477]	[15,485]
(33,265)	[75,601]	[557,584]	[417,601]	[557,584]	[75,601]
(35,325)	[194,625]	[271,544]	[468,625]	[271,544]	[194,625]
(38,353)	[454,688]	[111,669]	[454,688]	[454,688]	[454,688]

Table 3.4: Evaluation of max PI by Algorithm 2.2 and 2.3 and Algorithm 3.2 under DType I,II and III when $IS \neq \emptyset$.

Graph network (Vertices,edges)	Max PI by energy optimal Algorithm 2.2 and 2.3	Max PI by proposed Algorithm 3.2 under DType IV i.e. when $IS = \emptyset$.		
		When optimal node exits, reconnection is made on non-optimal node	Rest of any intermediate node exits reconnection is made by optimal node	Rest of any intermediate node exits reconnection is made on non-optimal node
(6,8)	[95,110]	NA	[60,110]	NA
(6,10)	[148,230]	[105,203]	[155,230]	NA
(8,11)	[71,90]	[71,90]	[57,90]	[71,90]
(9,14)	[95,105]	[95,105]	NA	[95,105]
(11,21)	[122,128]	[112,128]	[112,128]	NA
(13,27)	[128,145]	NA	NA	[128,145]
(13,38)	[72,162]	[121,162]	NA	[72,162]
(25,174)	[149,399]	[120,351]	NA	NA
(30,268)	[15,485]	[49,477]	NA	NA
(40,358)	[64,748]	[394,748]	[4,748]	NA

Table 3.5: Evaluation of max PI by Algorithm 2.2 and 2.3 and Algorithm 3.2 under DType IV when $IS = \emptyset$.

Graph network (Vertices,edges)	Max PI by energy optimal Algorithm 2.2 and 2.3	Max PI by proposed Algorithm 3.3 under DType V when cut vertex leaves T_o .
(6,8)	[95,110]	[34,154]
(8,11)	[71,90]	[8,122]
(9,14)	[95,105]	[19,161]
(11,21)	[122,128]	[21,197]
(13,27)	[128,145]	[7,237]
(13,27)	[128,145]	[32,152]
(15,30)	[98,110]	[98,110]
(17,32)	[97,99]	[2,196]
(17,32)	[97,99]	[4,180]
(17,32)	[97,99]	[21,121]

Table 3.6: Evaluation of max PI by Algorithm 2.2 and 2.3 and Algorithm 3.3 under DType V.

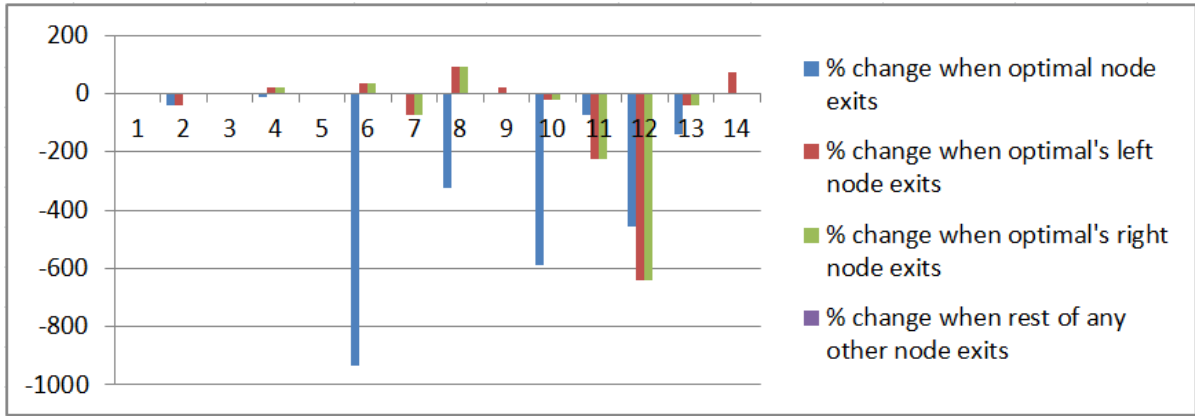


Figure 3.8: % change in lower limit of optimal value on reconnected topology by Algorithm 3.2 under DType I,II and III, when $IS \neq \phi$ (corresponding to data of Table 3.4).

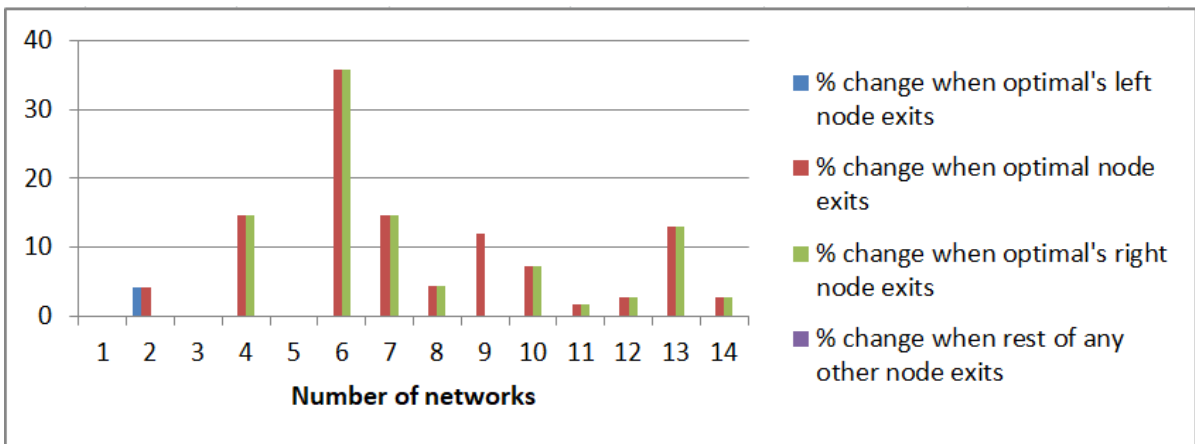


Figure 3.9: % change in upper limit of optimal value on reconnected topology by Algorithm 3.2 under DType I,II and III, when $IS \neq \phi$ (corresponding to data of Table 3.4).

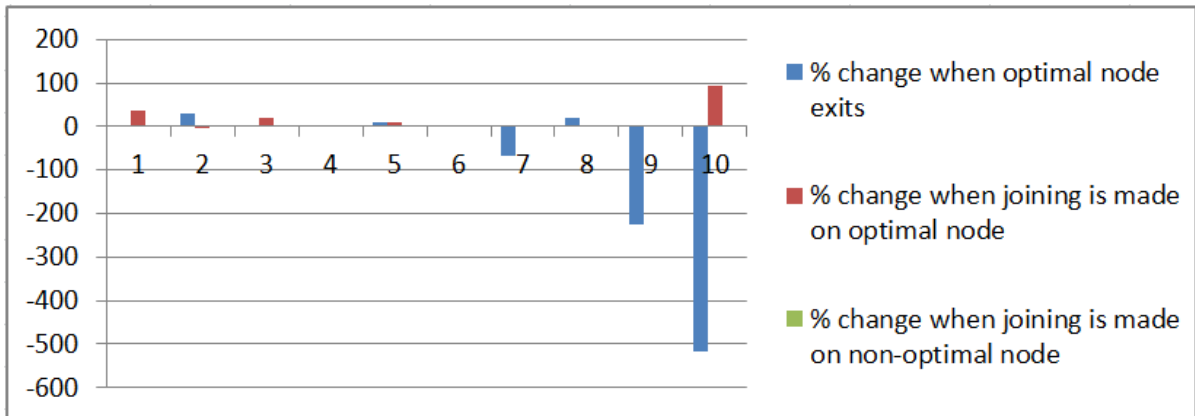


Figure 3.10: % change in lower limit of optimal value on reconnected topology by Algorithm 3.2 under DType IV, when $IS = \phi$ (corresponding to data of Table 3.5).

3.5 Summary and conclusion

This chapter presents sensitivity analysis on the interval graph based energy optimal spanning tree topology, designed for a connected and bidirectional WSNs where

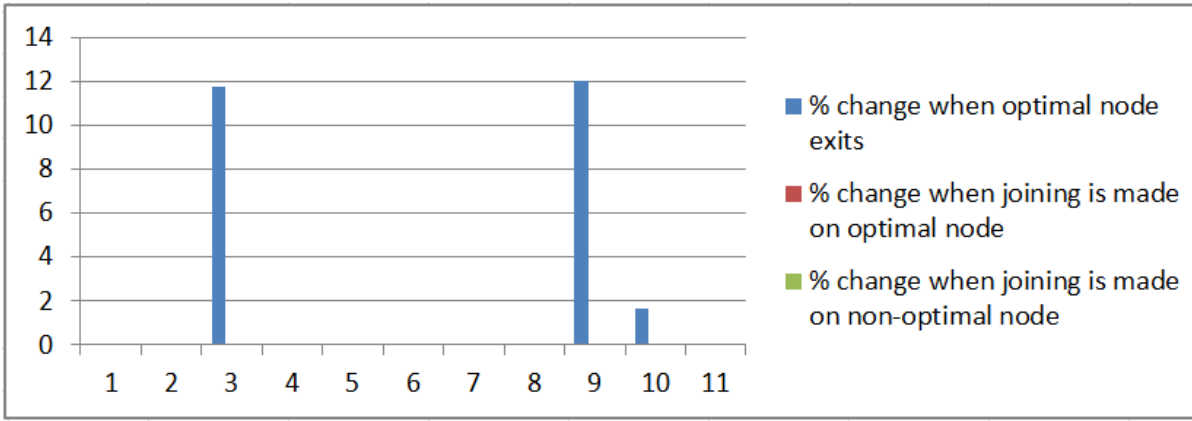


Figure 3.11: % change in upper limit of optimal value on reconnected topology by Algorithm 3.2 under DType IV, when $IS = \phi$ (corresponding to data of Table 3.5).

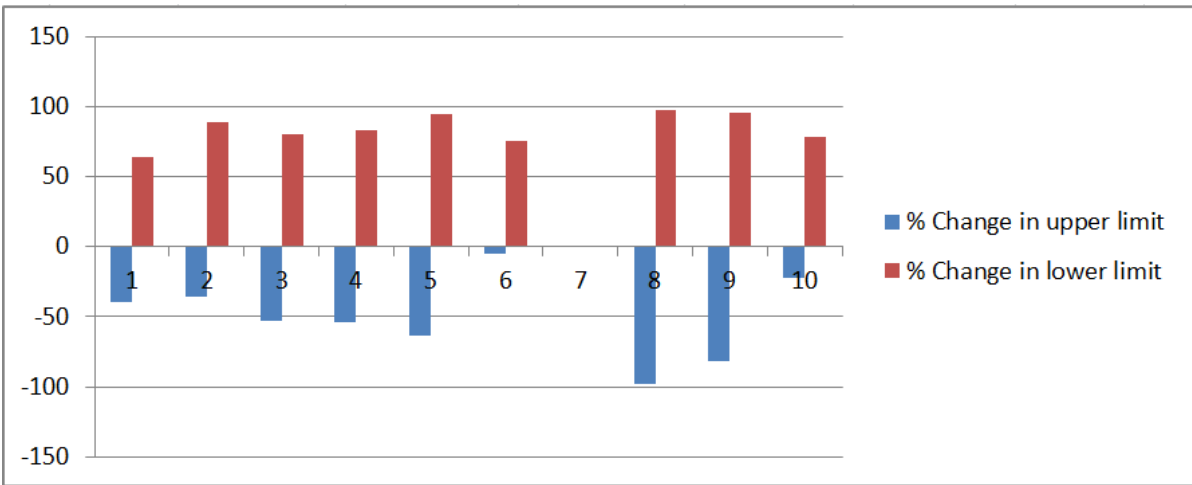


Figure 3.12: % change in lower limit and upper limit of optimal value on reconnected topology by Algorithm 3.3 under DType V (corresponding to data of Table 3.6).

each node is carrying weight in term of power interval. As tree topologies are prone to disconnectivity with the removal of even a single node hence if any node gets died due to drainage of its power in the network then its removal from the optimal topology, partitions the optimal topology into two or more disjoint components which therefore causes loss of data during transmission, improper routing, unreliability and energy hole problem. To handle these issues related to tree topologies due to disconnectivity, new algorithms have been proposed to reconnect/rejoin the disjoint components of the disconnected topology to continue the network functionality. we have conducted vast analysis on large number of interval graph models, with respect to the removal of each intermediate from their energy optimal topologies and evaluated the sensitivity related to max power consumption on the reconnected topology by the proposed algorithms. We put forth five different types of disconnectivity, called DType (DType I, II, III, IV and V) which have brought complications in reconnecting the disjoint compo-

nents of the disconnected topology on any arbitrary interval graph models of WSNs. The existing interval graph based algorithms lack the ability to rejoin the disjoint components as the disjoint components neither have right-end ordering nor satisfy interval graph properties, so 3 new algorithms have been proposed in this respect. Proposed Algorithm 3.1 reconnects the disconnected topology of complete interval graph in constant time, Algorithm 3.2 and Algorithm 3.3 reconnect disconnected topology of any arbitrary interval graph in quadratic time complexity under any of the DType.

Moreover reconnected by Algorithm 3.1 effects the optimal max PI only when the optimal node exits from the optimal topology and the effected value never enhance the upper limit of max power consumption. Reconnected by Algorithm 3.2 effects the optimal value with the removal of optimal node and its neighbour nodes, rest of any node does not effect the optimal max PI under DType I, II and/or III. DType IV may effect the optimal value of power consumption if its applicable with the removal of any intermediate node. Though the effected value never enhance the upper limit of max power consumption under DType I, II, III and/or IV i.e. if optimal value changes via changes in lower limits and if changes occur via upper limits then new upper limits of max power consumption are always lesser than the previous optimal values. Hence Algorithm 3.1 and 3.2 result minimum changes to the optimal max PI on the reconnected topologies.

Reconnected under DType V by Algorithm 3.3 when the cut-vertex of a separable interval graph is removed from its energy optimal topology is the most complicated situation of disconnectivity, results more power expense by increasing the upper limit of optimal power consumption interval on the reconnected topology. The proposed Algorithm 3.3 is the very first attempt in the field of graph theory to bring the connectivity between the disjoint connected components of a subgraph of a separable interval graph without its cut-vertex.

In next chapter, we will come-up with the sensitivity analysis and new approaches to welcome/connect one/more new nodes to the existing energy optimal topology in such a way that it effects optimal value of power consumption to the minimum as this makes the static topology adaptable and dynamic to the node's addition.

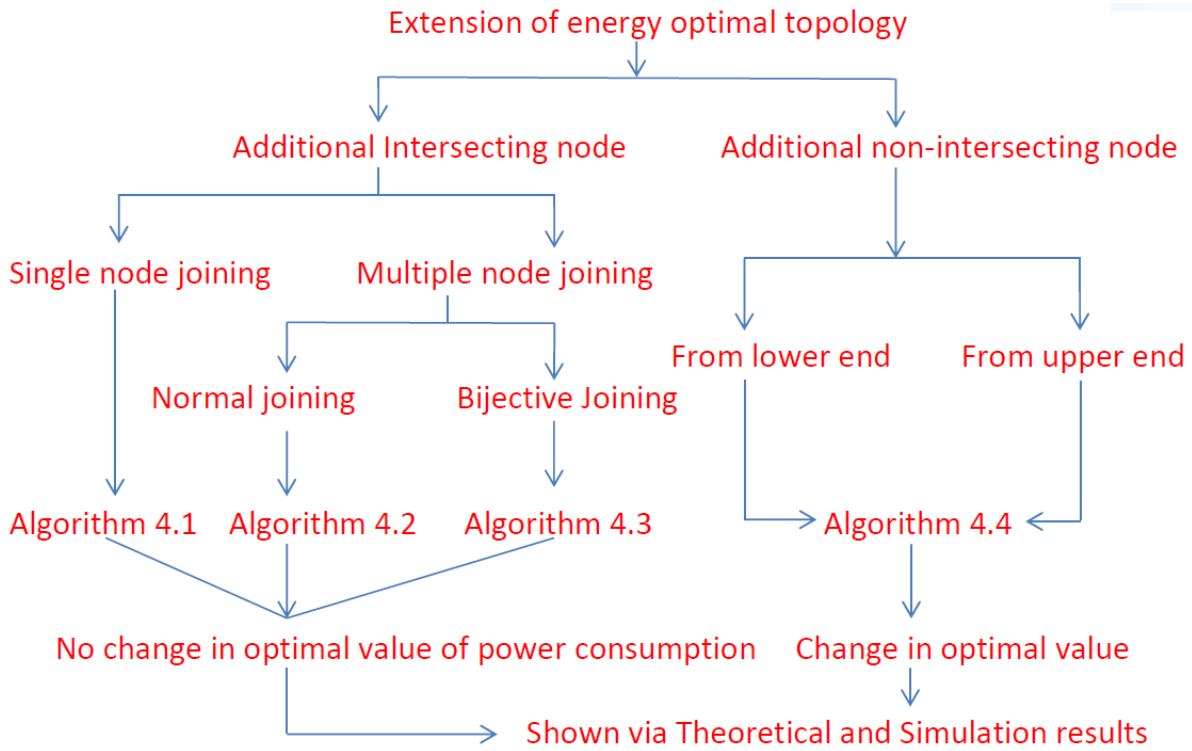
Chapter 4

New approaches of extending energy optimal topology on interval graph model of WSNs with sensitivity analysis

In this chapter¹, we have worked to extend the applicability of the energy optimal tree topology, obtained on interval graph model of WSNs with respect to addition of one or more new nodes to the existing topology. As topology control in WSNs has been greatly influenced by energy efficient tree topologies, engaged with immobile sensor nodes. These tree topologies lack dynamic applicability of WSNs due to nodes immobility and less stable with the removal of old nodes or addition of new nodes. To cope up with nodes immobility in lesser time complexity in reorganizing the energy optimal topology on addition of new nodes, new algorithmic approaches have been proposed to provide the extension to the optimal topology in such a way that no or minimum change in optimal value of power consumption would take place on the extended topology. Sensitivity analysis on addition of one and multiple new nodes to various energy optimal topologies of interval graphs and evaluation of change in optimal value on every extended topology in compare to its energy optimal topology have been provided in detail with all the possible results.

¹The content of this chapter is based on a research paper “New approaches of extending energy optimal topology on interval graph model of wireless sensor networks with sensitivity analysis”, (Submitted).

Graphical abstract of the chapter:



4.1 Introduction

Topology control in WSNs has been a prime goal to maintain nodes' power consumption with the monitoring of established topology among the sensors. Many existing topology control methods work towards the connectivity among the sensors with minimum usage of their power [4, 21, 28, 30, 31, 123, 124, 125, 126]. Most of them are based on fixed transmission range of static nodes i.e. nodes are assigned with max transmission range, required to communicate their farthest node in the network [3, 30, 31, 39, 127]. Whenever a node wants to communicate in the network, it has to use its max transmission range irrespective of the fact that not everytime node has to communicate its farthest node. So we have come-up with a new topological model of WSNs in chapter 2 where nodes can vary their transmission range within an assigned power interval in accordance to the distance between the nodes. We have introduced 'Interval graph models of WSNs' and polynomial time algorithms to establish a bidirectional spanning tree like topology on any arbitrary interval graph model. The resultant topology brings optimal max power consumption i.e. max PI in normal as well as worst case of power consumption. The models and algorithms, introduced in chapter 2 are the most recent with respect to variable

transmission range of each node and optimization of max power consumption interval in the network.

The algorithms and tree topologies, obtained in chapter 2, have been energy optimal and interval graph dependent result optimal max PI only when the network has been modelled as a right-end ordered interval graph. So what if some application of WSNs require new nodes to enter and engage with the energy optimal topologies of interval graph models on urgent basis.

The optimal topologies, obtained on interval graph models of WSNs, cannot adapt themselves due to nodes immobility and do not always satisfy the interval graph property so new node cannot be joined using Algorithm 2.2 and 2.3, introduced in chapter 2. If any chance an energy optimal topology of interval graph model, satisfies the interval graph property then Algorithm 2.2 and 2.3 can work to enter new nodes to the optimal topology only after reorganising the whole optimal topology in right-end ordered form. Right-end ordering of the optimal topology takes $\mathcal{O}(n+m)$ time where n is number of nodes and m is number of edges of the optimal topology. Thereafter execution of Algorithm 2.2 and 2.3 again take $\mathcal{O}(n^2)$ time. Hence total time complexity of adding one and multiple new nodes to the energy optimal topology by Algorithm 2.2 and 2.3, given in chapter 2 becomes $\mathcal{O}(n^3)$.

Hence proposal of new approaches to insert new nodes to the existing topology with lesser time complexity than $\mathcal{O}(n^3)$ such that the value of optimal power consumption remains maintained even on the extended topology has been a prime motivation behind this work.

This chapter introduces four new algorithms to extend an energy optimal topology of an interval graph model of WSN, with no restriction of right-end ordering and interval graph properties when one or multiple new nodes are required to enter to the optimal topology with time complexity lesser than $\mathcal{O}(n^3)$ where n is the total number of nodes of energy optimal topology. Extended topologies by the proposed algorithms maintain the optimal value of power consumption until an additional node with non-intersecting power interval comes to the topology.

An optimal topology after addition of new node is called '**extended topology**' and when optimal solution does not change on the extended topology, it can be said '**extended optimal topology**'. Evaluation of max power interval (i.e. max PI) on extended topologies of many distinct energy optimal topologies and sensitivity analysis to evaluate the change in optimal value due to addition of one and multiple nodes to the optimal topologies, have been carried out in the chapter. It is noted that

the optimal value is the max PI obtained by algorithms, introduced in chapter 2.

Following are the contributions of the present chapter:

1. Extending the applicability of immobile energy optimal topology of interval graph model of WSNs by making it adaptable and stable on addition of one or more new nodes to the existing topology with no or minimum change in optimal value.
2. New algorithms of lesser time complexity have been proposed which focus on joining new node to the existing topology without disturbing the whole topology.
3. Discussion on sensitivity analysis on change in optimal value of power consumption when one and more new nodes are added to many energy optimal of interval graph models have been provided in detail.

The outline of the chapter is as follows, section 4.2 introduces the definition of additional nodes and their types along with the discussion on necessary condition to join new nodes to the existing topology. Section 4.3 provides the concept on entry of a single intersecting node along with the proposed technique then section 4.4 provides the study on entry of multiple new nodes and proposed algorithms. Section 4.5 presents the concept on entry of a non-intersecting node, proposed technique and theoretical results. Section 4.6 presents the simulation and sensitivity analysis and lastly section 4.7 concludes the chapter.

Table 4.1: Abbreviations used in the chapter

T_x	Extended topology
IN	Intersecting node
NIN	Non-intersecting node
$N - IS$	Non-intersecting set
U_u	Upper bound of u
L_u	Lower bound of u

4.2 Additional/New nodes

For an energy optimal topology T_o , introduced in chapter 2 where each node is carrying weight in term of power interval, any new node having crisp power level or power interval as its weight has been considered for addition to extend T_o . As

any additional node with crisp value can be written as an interval, let's say a_{new} has crisp power level 254 then it is equivalently written as $PI(a_{new}) = [254, 254]$ in our work. Therefore new nodes are assumed to have weights in term of power interval where lower limit of power interval can be same as upper limit of power interval.

4.2.1 Types of additional node

Each node whether it is carrying crisp or interval weight, can be written as an interval form. So when there is data in interval form, then there may exist intersection between the intervals as well. This implies that PI of a new node may or may not have power intersection with any node of the existing topology.

Based on power intersection of new node with the nodes of the energy optimal topology T_o , additional node can be of two types. First type is 'Intersecting node IN ' and second is 'Non-intersecting node NIN ', discussed below.

1. **Intersecting node IN :** If a new/additional node has non-empty power intersection with atleast one node of T_o , then the new node constitutes the 'Intersecting set IS ' and the node is called intersecting node IN .
2. **Non-intersecting node NIN :** If a new/additional node is having empty power intersection with each node of T_o , then the new node constitutes a 'Non-intersecting set $N - IS$ ' and the node is called non-intersecting node NIN .

NIN has been further categorized into two kinds based on the possible way of empty power intersection, discussed below.

Let $PI(v_i) = [L_{v_i}, U_{v_i}] \forall v_i \in T_o$ and

(i) NIN from lower end: Let $PI(a_{new}) = [L_{new}, U_{new}]$ and $L_{min} = \min\{L_{v_i}\}$ then $U_{new} < L_{min}$.

(ii) NIN from upper end: Let $PI(a_{new}) = [L_{new}, U_{new}]$ and $U_{max} = \max\{U_{v_i}\}$ then $L_{new} > U_{max}$.

4.2.2 Necessary condition to establish connection between T_o and new nodes

Let a new node, say a_{new} is required to enter T_o with $PI(a_{new}) = [L_{new}, U_{new}]$. Then T_o and a_{new} together form a disconnected topology T_d , denoted as a set $T_d = \{T_o, a_{new}\}$, consists of two disjoint connected components. The way to join disjoint components is

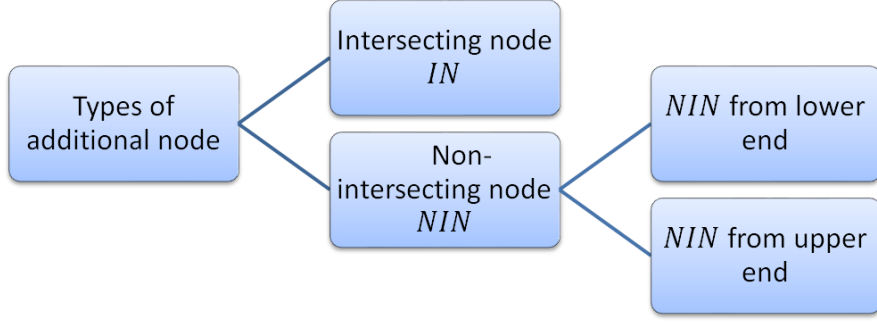


Figure 4.1: Types of additional nodes based on power intersection

by connecting appropriate nodes from each component based on their power intervals because when two nodes communicate bidirectionally, they belong to each other's transmission disc [3] and so their power intervals have non-empty intersection. Hence two nodes from disjoint components can establish a connection only when their power intervals have non-empty intersection (refer Theorem 3.2.1).

It is clear that if the additional node a_{new} belongs to $N - IS$, then communication between the nodes of T_o and a_{new} cannot possible. It can only possible when possible changes to $PI(a_{new})$ brings non-empty power intersection with atleast one node of T_o . Detail discussion on addition of IN and NIN additional nodes with proposed approaches have been followed in the next section.

4.3 Addition of a new intersecting node to T_o

Let us consider a graphical layout of a hypothetical WSN, shown in Fig. 4.2(a), which is actually a right-end ordered interval graph [1, 2]. Fig. 4.2(b) shows an energy optimal topology T_o obtained on the network by the Algorithm 2.2 and 2.3, given in chapter 2. The optimal solution obtained on T_o is $\max PI = [94, 110]$. Let a_{new} be any new intersecting node with $PI(a_{new}) = [85, 94]$, is required to enter/add to T_o of Fig. 4.2(b). It is seen in Fig. 4.2(b) that $PI(a_{new})$ is intersecting with $PI(u_3)$ and $PI(u_4)$. Then a_{new} could be joined to any of the mentioned node, still we prioritize a node with greater PI to join with a_{new} due to the fact of load balancing and network lifetime. Since joining of an extra node to a node of T_o may cause unbalanced power consumption or more power consumption of the node, so if a_{new} is joined to a node with lesser PI , then the node with lesser PI dies soon in the network. Therefore to join a new intersecting node to T_o in this way, Algorithm 4.1 has been proposed. Algorithm 4.1

works to extend T_o of any arbitrary interval graph model of WSN by adding a new intersecting node such that the value of $\max PI$ obtained on the extended topology T_x is same as on T_o i.e. optimal solution remains intact on T_x .

Considering T_o of Fig. 4.2(b), Algorithm 4.1 joins a_{new} with u_4 and results an extended topology T_x , shown in Fig. 4.3. The link $(u_4 a_{new})$ on T_x is having $PI = [94, 94]$, updates $PI(a_{new}) = [94, 94]$ and $PI(u_4) = [94, 110]$ on T_x . Therefore $\max PI$ on T_x is still $[94, 110]$, hence no change in optimal value of $\max PI$.

Algorithm 4.1 Extension of an energy optimal topology by joining a new intersecting node to the topology.

Input: A new intersecting node a_{new} and optimal topology T_o .

Output: An extended topology T_x with $T_x = T_o \cup a_{new}$.

- 1: Compute $IS(a_{new}) = \{u \in T_o \mid PI(u) \cap PI(a_{new}) \neq \emptyset\}$
 - 2: **for** $u \in IS(a_{new})$ **do**
 - 3: Obtain $U = \{U_u\}$
 - 4: Compute $U_{max} = \max\{U\}$
 - 5: Select $u \in IS(a_{new})$ whose $U_u = U_{max}$
 - 6: Join a_{new} to u and update $T_x = T_o \cup a_{new}$.
 - 7: Return T_x
-

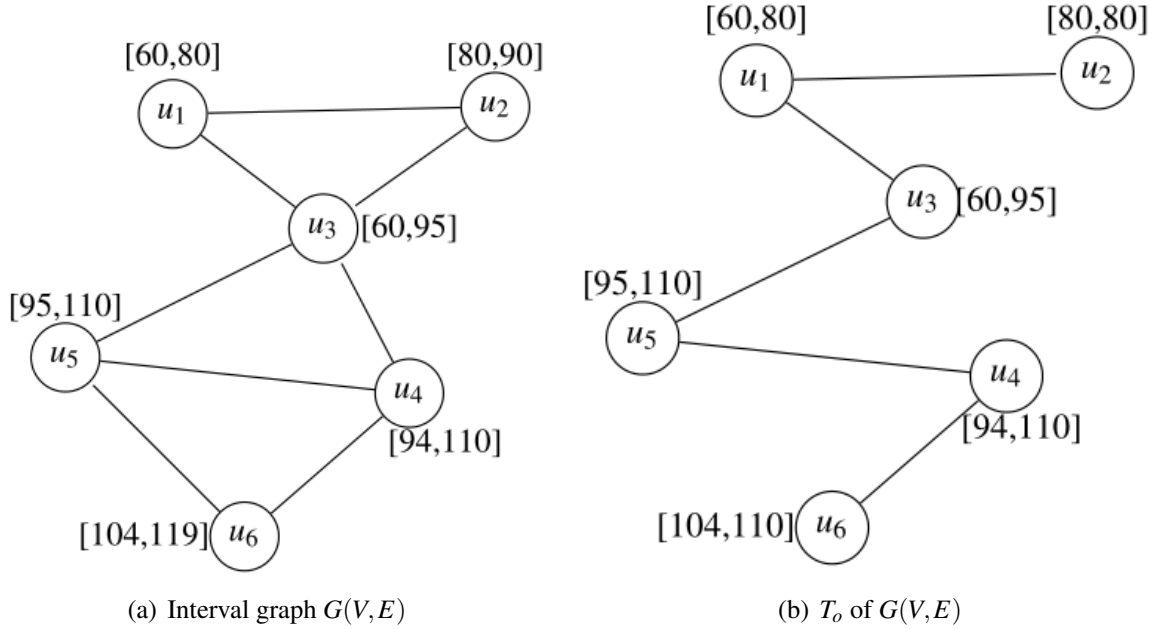


Figure 4.2: Right-end ordered interval graph and its energy optimal topology

Theorem 4.3.1. Time complexity of Algorithm 4.1 is $\mathcal{O}(n)$ where n is the total number of nodes.

Proof. Step 1 can be done in $\mathcal{O}(n)$ time. Steps 2 and 3 collectively take $\mathcal{O}(n)$ time. Steps 4

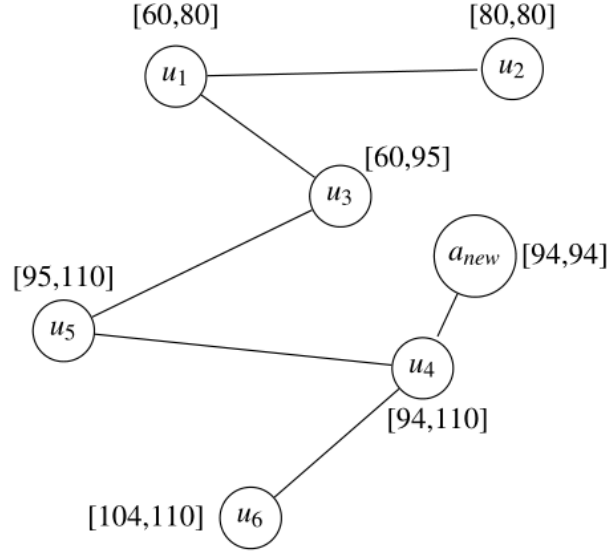


Figure 4.3: Extended topology T_x by proposed Algorithm 4.1

and 5 also take $\mathcal{O}(n)$ time. Step 6 takes constant time. Hence the total time complexity of Algorithm 4.1 is $\mathcal{O}(n)$. $\square$

Discussion on addition of a new intersecting node has been extended to multiple intersecting nodes to energy optimal topologies of interval graph models and is followed in the next section.

4.4 Addition of multiple new intersecting nodes to T_o

Addition of multiple intersecting nodes to T_o is the extended or general concept of addition of single intersecting node to T_o with the concern of no change in optimal value on the extended topology. If $|T_o| = n$, then number of new intersecting nodes can be $N \leq n$.

An extended algorithm say Algorithm 4.2, has been proposed to join multiple new intersecting nodes to T_o with time complexity of $\mathcal{O}(n^2)$ where $n = |T_o|$ such that the extended topology results no change in optimal value of power consumption.

Theorem 4.4.1. Time complexity of Algorithm 4.2 is $\mathcal{O}(n^2)$ where n is the total number of nodes.

Proof. Step 1 can be done in constant time, then **for** loop in step 2 repeats at the most n times. Steps 3 and 4, inside the **for** loop, take $\mathcal{O}(n)$ time. Steps 5 and 6 can be done in constant time.

Algorithm 4.2 Extension of an energy optimal topology T_o by joining N new intersecting nodes to T_o where $N \leq |T_o|$.

Input: N new intersecting nodes and T_o .

Output: An extended topology T_x with $T_x = T_o \cup a_1 \cup a_2 \cup \dots \cup a_N$.

- 1: Obtain $SN = \{a_1, a_2, \dots, a_N\}$ $\triangleright SN$ is a set of new nodes
 - 2: **for** $a_i \in SN, i = 1 \dots N$ **do**
 - 3: Compute $IS(a_i) = \{u \in T_o | PI(a_i) \cap PI(u) \neq \emptyset\}$
 - 4: Obtain $U_{max}(a_i) = \max\{U_u\}$ for each $u \in IS(a_i)$
 - 5: Join a_i to u whose $U_u = U_{max}(a_i)$ and update $PI(a_i u) = PI(a_i) \cap PI(u)$.
 - 6: update $T_x = T_o \cup a_i$ and $PI(a_i) = PI(a_i u)$.
 - 7: **Return** T_x
-

Hence the total time complexity of Algorithm 4.2 is $\mathcal{O}(n^2)$. □

Example 4.4.1. Working illustration of Algorithm 4.2.

Consider the network given in Fig. 4.2. Let 5 new intersecting nodes are required to add to T_o (given in Fig. 4.2(b)) those are $a_1 = [50, 86]$, $a_2 = [73, 103]$, $a_3 = [93, 98]$, $a_4 = [102, 112]$ and $a_5 = [105, 119]$. Step 1 obtains the set of additional nodes $SN = \{a_1, a_2, a_3, a_4, a_5\}$. Step 2 starts the **for** loop for each node in SN . Step 3 computes $IS(a_1) = \{u_1, u_2, u_3\}$, step 4 obtains $U_{max}(a_1) = 95$ since $95 = \max\{80, 95\}$ and it is the upper bound of u_3 . Hence step 5 joins a_1 with u_3 as $U_{u_3} = 95$. Next $IS(a_2) = \{u_1, u_2, u_3, u_4, u_5\}$, step 4 obtains $U_{max}(a_2) = 110$ where 110 is the upper bound of u_4 and u_5 both, so a_2 can be joined any one of them. Likewise $IS(a_3) = \{u_3, u_4, u_5\}$ and a_3 can be joined either with u_4 or u_5 . $IS(a_4) = \{u_4, u_5, u_6\}$ and a_4 has same possibility to join with any of the three. Finally $IS(a_5) = \{u_6\}$ and a_5 is joined with u_6 . The possible formation of the resultant topology after addition is given in Fig. 4.4.

Fig. 4.4(a) shows the topology when a_2 , a_3 and a_4 , all the three have chosen to connect with u_4 and a_5 with u_6 . Therefore updated $PI(a_1) = [60, 86]$, $PI(a_2) = [94, 103]$, $PI(a_3) = [94, 98]$, $PI(a_4) = [102, 110]$ and $PI(a_5) = [105, 110]$ on T_x , brings no change in optimal value of $\max PI$ on T_x , given in Fig. 4.4(a). Fig. 4.4(b) shows the topology when a_2 connects with u_4 , a_3 with u_5 , a_4 and a_5 choose to connect with u_6 . Therefore updated $PI(a_1) = [60, 86]$, $PI(a_2) = [94, 103]$, $PI(a_3) = [95, 98]$, $PI(a_4) = [104, 110]$ and $PI(a_5) = [105, 110]$ brings no change in optimal value of $\max PI$ on T_x , given in Fig. 4.4(b). Moreover $\max PI$ of a_5 among additional nodes has been reduced from $[105, 119]$ to $[105, 110]$ on both the extended topologies, obtained by Algorithm 4.2.

Remark 4.4.1. Joining of multiple new intersecting nodes to the existing nodes of the optimal topology by Algorithm 4.2 as given in example 4.4.1, shows that two or more nodes are joined to a single node. So when there is a large network of n nodes and n number of additional nodes need to enter to the network, Algorithm 4.2 may connect many nodes to a single node,

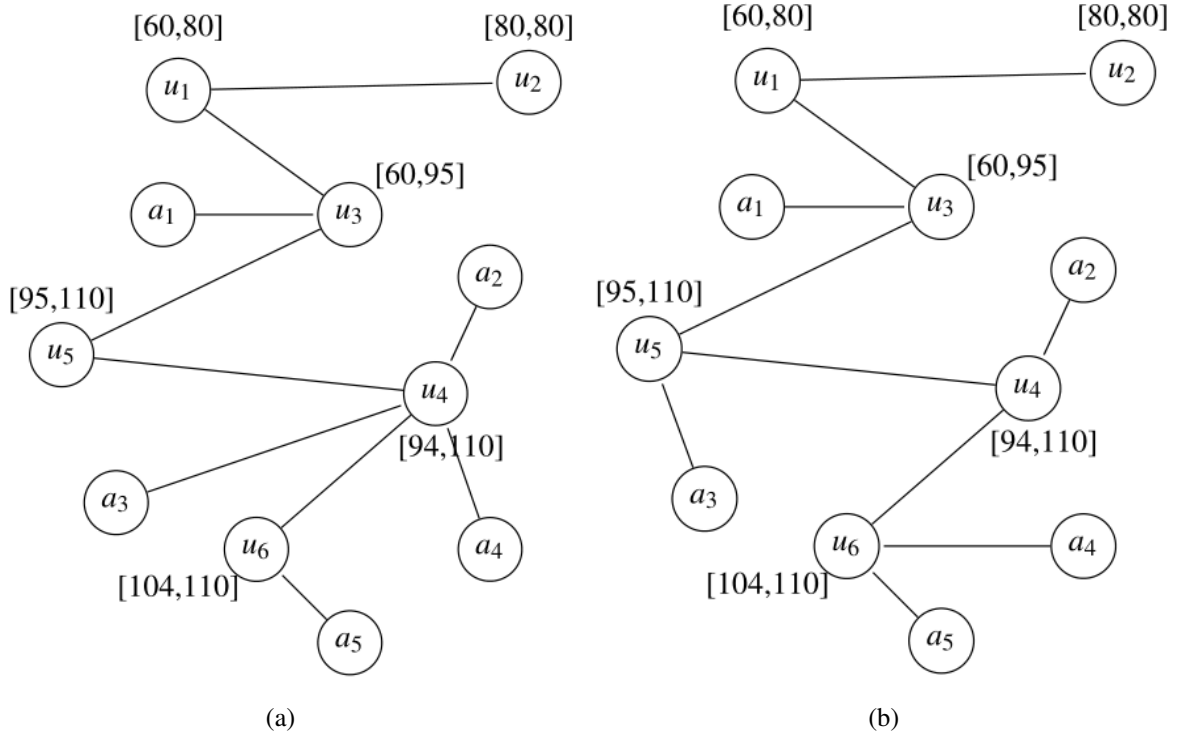


Figure 4.4: The extended topologies by Algorithm 4.2 after addition of 5 INs to T_o of Fig. 4.2(b)

causes faster power depletion of that node in the network irrespective of the fact that each node can be joined to the existing node in such a way that balanced power consumption may occur. Considering the fact of balanced power consumption, we are going to discuss bijective joining of N new intersecting nodes to T_o where $N \leq n$ and $n = |T_o|$.

4.4.1 Bijective joining to T_o

Let $|T_o| = n$ and $N \leq n$ new intersecting nodes are required to add to T_o in such a way that no node of T_o is possibly joined by more than one new node, then the joining is called 'bijective joining' in the field of topology control of WSNs. The purpose of bringing bijective joining of N new intersecting nodes to T_o is the load balancing, by possibly restricting the joining of two or more nodes to a single node. As the joining of multiple nodes to the same node may cause unbalanced power consumption in the network and it decreases the network lifetime.

For the purpose of bijective joining between new intersecting nodes and existing nodes, Algorithm 4.3 has been proposed. Algorithm 4.3 obtains extended bijective topology T_x when N new intersecting nodes are joined to T_o where $N \leq n$ and $n = |T_o|$ with time complexity of $\mathcal{O}(n^2)$ such that optimal max PI of T_o remains intact on T_x .

Algorithm 4.3 Bijective joining of N new intersecting node to the energy optimal topology T_o where $N \leq n$ and $|T_o| = n$.

Input: N new intersecting nodes $a_1, a_2, \dots, a_N$ and T_o .

Output: An extended bijective topology T_x as $T_x = T_o \cup a_1 \cup a_2 \cup \dots \cup a_N$.

- 1: Compute $IS(a_i) = \{v_i \in T_o \mid PI(v_i) \cap PI(a_i) \neq \emptyset\}$ for each a_i .
 - 2: Relabel a_i 's as $\{A_1, A_2, \dots, A_N\}$ such that $IS(A_1) \leq IS(A_2) \leq IS(A_3) \leq \dots IS(A_N)$.
 - 3: $SN \leftarrow \{A_1, A_2, \dots, A_N\}$
 - 4: **for** $A_i \in SN$ **do**
 - 5: Join A_i to v_i where $v_i \in IS(A_i)$ and update $PI(A_i v_i) = PI(A_i) \cap PI(v_i)$.
 - 6: Update $T_x = T_o \cup A_i$ and $T_o \leftarrow T_o / \{v_i\}$.
 - 7: Update $PI(A_i) \leftarrow PI(A_i v_i)$.
 - 8: **Return** T_x
-

Remark 4.4.2. Bijective joining by Algorithm 4.3 can happen only when N^{th} node has atleast N number of intersecting nodes from T_o .

Example 4.4.2. Working illustration of Algorithm 4.3.

Let us consider 6 new intersecting nodes entering to the topology T_o , given in Fig. 4.2(b). Let $a_1 = [50, 86]$, $a_2 = [73, 103]$, $a_3 = [93, 98]$, $a_4 = [102, 112]$, $a_5 = [105, 119]$ and $a_6 = [89, 130]$ are new intersecting nodes. Step 1 computes $IS(a_1) = \{u_1, u_2, u_3\}$, $IS(a_2) = \{u_1, u_2, u_3, u_4, u_5\}$, $IS(a_3) = \{u_3, u_4, u_5\}$, $IS(a_4) = \{u_4, u_5, u_6\}$, $IS(a_5) = \{u_6\}$, $IS(a_6) = \{u_3, u_4, u_5, u_6\}$. Since $IS(a_5) < IS(a_1) \leq IS(a_3) \leq IS(a_4) < IS(a_6) < IS(a_2)$, therefore $A_1 \leftarrow a_5$, $A_2 \leftarrow a_1$, $A_3 \leftarrow a_3$, $A_4 \leftarrow a_4$, $A_5 \leftarrow a_6$ and $A_6 \leftarrow a_2$. So all a_i 's are relabelled and arranged as $\{A_1, A_2, \dots, A_6\}$ where $IS(A_1) < IS(A_2) \leq IS(A_3) \leq IS(A_4) < IS(A_5) < IS(A_6)$ in step 2. Step 4, 5 and 6 join A_1 with u_6 , A_2 with u_1 , A_3 with u_3 , A_4 with u_4 , A_5 with u_5 and A_6 with u_2 . Step 7 updates $PI(A_1)$ to $[105, 110]$, $PI(A_2)$ to $[60, 80]$, $PI(A_3)$ to $[93, 95]$, $PI(A_4)$ to $[102, 110]$, $PI(A_5)$ to $[95, 110]$ and $PI(A_6)$ to $[80, 80]$. Step 8 returns the extended bijective topology T_x , shown in Fig. 4.5.

Max PI on T_x is $[94, 110]$, reveals that optimal value of power consumption does not change after bijective joining of 6 new intersecting nodes to T_o (refer Fig. 4.2(b)) by Algorithm 4.3. Also max PI among the additional nodes has been reduced from $[89, 130]$ to $[95, 110]$ on T_x .

Theorem 4.4.2. Time complexity of Algorithm 4.3 is $\mathcal{O}(n^2)$ where n is total number of nodes.

Proof. Step 1 takes $\mathcal{O}(n^2)$. Step 2 takes $\mathcal{O}(n \log n)$ time. Steps 4 and 5 jointly can be done in linear time. Step 3, 6 and 7 take constant time. Hence the total time complexity of Algorithm 4.3 is $\mathcal{O}(n^2)$. $\square$

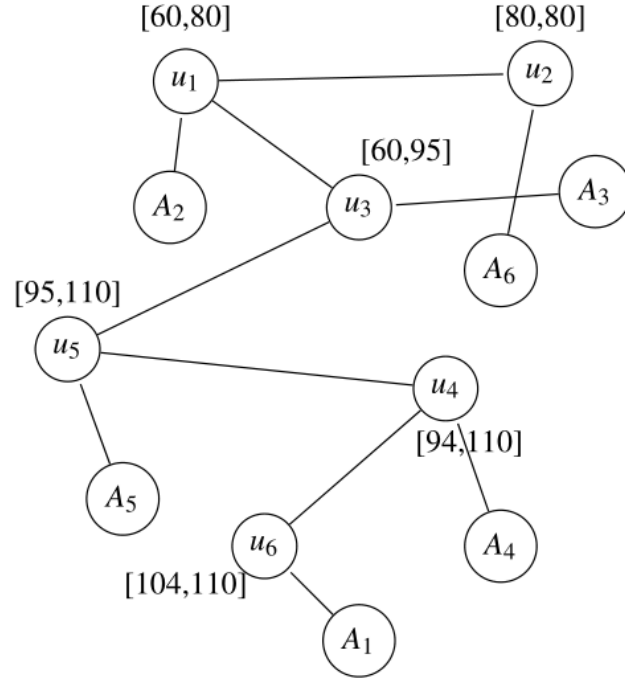


Figure 4.5: Bijective joining of 6 new intersecting nodes by Algorithm 4.3 to T_o given in Fig. 4.2(b)

Theorem 4.4.3. Addition of one or multiple new intersecting nodes to energy optimal topology T_o by proposed Algorithms 4.1, 4.2 or 4.3, does not effect the optimal value of power consumption.

Proof. Let a_{new} is a new intersecting node with $PI(a_{new}) = [l_{new}, U_{new}]$, added to an optimal topology T_o whose optimal max PI is given as $[l_{opt}, U_{opt}]$. Let $w \in T_o$ has non-empty power intersection with a_{new} i.e. $PI(a_{new}) \cap PI(w) \neq \phi$.

Case(i): w is an optimal node of T_o .

This implies that link (wa_{new}) has $PI = [\max\{l_{opt}, l_{new}\}, \min\{U_{opt}, U_{new}\}] = [l_{max}, U_{min}]$ where $l_{max} = \max\{l_{opt}, l_{new}\}$ and $U_{min} = \min\{U_{opt}, U_{new}\}$. Addition of link (wa_{new}) to a_{new} and w updates $PI(a_{new}) = [l_{max}, U_{min}]$ and $PI(w) = [\min\{l_{opt}, l_{max}\}, \max\{U_{opt}, U_{min}\}] = [l_{opt}, U_{opt}]$ using Eq. (2.5). Hence no effect is seen on optimal value.

Case(ii): w is not an optimal node of T_o .

Let $PI(w) = [l_w, U_w]$ and $U_w < U_{opt}$. Joining of a_{new} with $w \in T_o$ implies that $PI(a_{new}) \cap [l_{opt}, U_{opt}] = \phi$. Link (wa_{new}) would have $PI(wa_{new}) = [\max\{l_w, l_{new}\}, \min\{U_w, U_{new}\}] = [l_{max}, U_{min}]$ where $U_{min} < U_{opt}$. Addition of link (wa_{new}) , updates $PI(a_{new}) = [l_{max}, U_{min}]$ and $PI(w) = [\min\{l_w, l_{max}\}, \max\{U_w, U_{min}\}] = [l_w, U_w]$. Hence no effect is seen on optimal value.

Addition of more than one intersecting node to T_o repeats the same fact as discussed above. $\square$

4.5 Addition of non-intersecting node to T_o

Till now, we have discussed about addition of intersecting nodes to the energy optimal topology of interval graph models. What if the new entering node does not find power intersection with any node of the optimal topology. Hence discussion on entering of a non-intersecting node to the energy optimal topology has been carried out in this section with the theoretic result on change in optimal value of power consumption on the extended topology.

Let us consider an energy optimal topology T_o of an arbitrary interval graph model of WSN and a non-intersecting node (NIN) say a_{new} is required to enter to T_o . As a consequence of Theorem 3.2.1, a_{new} cannot be joined to any node of T_o until $PI(a_{new}) = [L_{new}, U_{new}]$ finds atleast single node to have non-empty power intersection. This implies that $PI(a_{new})$ has to modify somehow so that intersection can be occurred with atleast a node of T_o . Based on this fact, a new algorithmic approach say Algorithm 4.4 has been proposed to join any NIN to T_o in linear time complexity.

Algorithm 4.4 Extension of optimal topology T_o with the addition of a non-intersecting node

Input: A non-intersecting node a_{new} with $PI = [L_{new}, U_{new}]$ and T_o .

Output: An extended topology T_x as $T_x = T_o \cup a_{new}$.

- 1: Obtain $l_{min} = \min\{L_{v_i} \mid v_i \in T_o\}$ and $U_{max} = \max\{U_{v_i} \mid v_i \in T_o\}$.
 - 2: **if** $U_{new} < l_{min}$ **then**
 - 3: Select node $v \in T_o$ whose $L_v = l_{min}$.
 - 4: Join a_{new} to v with $PI(a_{new}v) = [U_{new}, l_{min}]$.
 - 5: Update $PI(a_{new}) = [U_{new}, l_{min}]$ and $PI(v) = [U_{new}, U_v]$.
 - 6: **if** $L_{new} > U_{max}$ **then**
 - 7: Select node $v \in T_o$ whose $U_v = U_{max}$.
 - 8: Join a_{new} to v with $PI(a_{new}v) = [U_{max}, L_{new}]$.
 - 9: Update $PI(a_{new}) = [U_{max}, L_{new}]$ and $PI(v) = [L_v, L_{new}]$.
 - 10: $T_x = T_o \cup a_{new}$.
 - 11: Return T_x
-

Proposed Algorithm 4.4 joins a NIN a_{new} to T_o in two ways, discussed below:

(i) *Addition of a_{new} from lower end:* Let us consider T_o , given in Fig. 4.2(b) and $PI(a_{new}) = [42, 56]$ then $PI(a_{new})$ is non-intersecting with PI of each node belongs to T_o . Here $U_{new} = 56$ and $L_{min} = 60$ which is the lower bound of $PI(u_1)$ and $PI(u_3)$, implies a_{new} addition comes under NIN from lower end since $U_{new} < L_{min}$ (refer section 4.2.1). As L_{min} is the closet value to U_{new} on any T_o , therefore joining of the node of L_{min} with a_{new} brings a new link of $PI = [U_{new}, L_{min}]$ which further updates $PI(a_{new}) = [U_{new}, L_{min}]$

on T_x .

Therefore Algorithm 4.4 joins a_{new} to u_1 (or u_3) with $PI(a_{new}u_1) = [56, 60]$ which further updates $PI(a_{new}) = [56, 60]$ on T_x so that $PI(a_{new}) \cap PI(u_1) \neq \emptyset$ on T_x , shown in Fig. 4.6(a).

(ii) *Addition of a_{new} from upper end*: Again considering T_o of Fig. 4.2(b), if $PI(a_{new}) = [123, 156]$ then $PI(a_{new})$ is non-intersecting with PI of each node belongs to T_o . Here $L_{new} = 123$ and $U_{max} = 110$ which is the upper bound of $PI(u_4), PI(u_5)$ and $PI(u_6)$, implies a_{new} addition comes under *NIN* from upper end since $L_{new} > U_{max}$. As U_{max} is the closet value to L_{new} on any T_o , therefore joining of the node of U_{max} with a_{new} brings a new link of $PI = [U_{max}, L_{new}]$ which further updates $PI(a_{new}) = [U_{max}, L_{new}]$ on T_x .

Hence Algorithm 4.4 joins a_{new} to u_4 (or u_5 or u_6) with $PI(a_{new}u_4) = [110, 123]$ which further updates $PI(a_{new}) = [110, 123]$ on T_x so that $PI(a_{new}) \cap PI(u_4) \neq \emptyset$ on T_x , shown in Fig. 4.6(b).

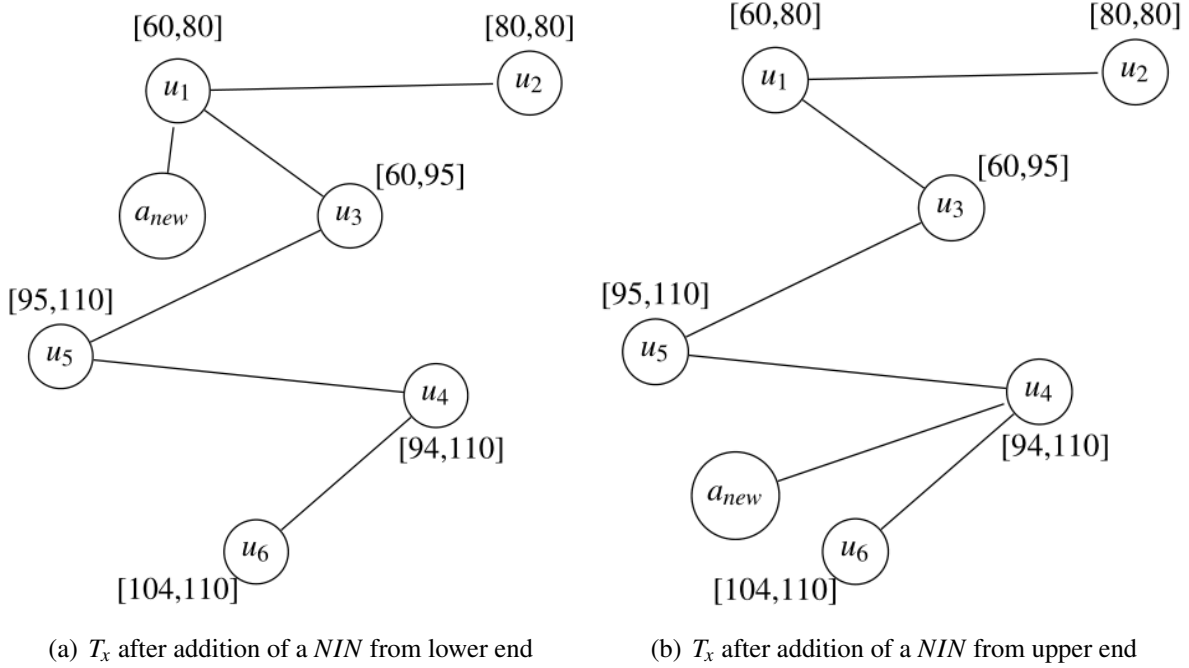


Figure 4.6: Extended topologies by Algorithm 4.4

Remark 4.5.1. Addition of a *NIN* from lower end does not change the optimal max PI and addition of a *NIN* from upper end changes the optimal max $PI = [L_{opt}, U_{opt}]$ to $[L_{opt}, L_{new}]$ on extended topology.

Theorem 4.5.1. Time complexity of Algorithm 4.4 is $\mathcal{O}(n)$ where n is the total number of nodes.

Proof. Step 1 is done in $\mathcal{O}(n)$ time. Steps 2 takes constant time and step 3 takes $\mathcal{O}(n)$ time. Steps 4, 5 and 6 also takes constant time. Step 7 takes $\mathcal{O}(n)$ time. Rest of the steps can be done in constant time as well. Hence the total time complexity of Algorithm 4.4 is $\mathcal{O}(n)$. $\square$

Theorem 4.5.2. Addition of a *NIN* to energy optimal topology T_o by Algorithm 4.4 effects the optimal value by increasing the optimal upper limit of power consumption to the lower limit of the *NIN*'s *PI* only when *NIN* from upper end has been joined to T_o .

Proof. Let us consider a *NIN* i.e. a_{new} with $PI(a_{new}) = [L_{new}, U_{new}]$ such that $PI(a_{new}) \cap PI(v_i) = \emptyset$ for all $v_i \in T_o$. This implies that either $L_{new} > U_{max}$ where $U_{max} = \max\{U_{v_i} | v_i \in T_o\}$ or $U_{new} < L_{min}$ where $L_{min} = \min\{L_{v_i} | v_i \in T_o\}$.

Let w be an optimal node of T_o and being an optimal node $PI(w) = [L_w, U_{max}]$.

Case(i) $L_{new} > U_{max}$ i.e. *NIN* from upper end.

In this case, Algorithm 4.4 joins a_{new} to w with link's $PI(a_{new}w) = [U_{max}, L_{new}]$ and so updated $PI(a_{new}) = [U_{max}, L_{new}]$ and $PI(w) = [L_w, L_{new}]$ would have non-empty power intersection on the extended topology. Hence $PI(w)$ changes from $[L_w, U_{max}]$ to $[L_w, L_{new}]$ after addition of a_{new} to optimal topology.

Case(ii) $U_{new} < L_{min}$ i.e. *NIN* from lower end.

Let $v \in T_o$ be the node corresponding to L_{min} with $PI = [L_{min}, U_v]$ and it is obvious that $U_v < U_{max}$. Therefore joining of a_{new} with v brings a new link $a_{new}v$ with $PI(a_{new}v) = [U_{new}, L_{min}]$ which therefore updates $PI(a_{new}) = [U_{new}, L_{min}]$ and $PI(v) = [U_{new}, U_v]$ on T_x . Max *PI* on T_x is found on w with the same optimal value since no link is added to the optimal node and added link has *PI* lesser than $PI(w)$. Hence no effect is seen on optimal value. $\square$

Remark 4.5.2. We restrict the discussion of addition of non-intersecting node to a single node only. As if a non-intersecting node is added from lower end, it will be always joined to a node of minimum lower limit of power consumption, hence if there are multiple *NIN* from lower end, then all the additional nodes are joined to the same node causes unbalanced power consumption. Same issue happens with the addition of multiple *NIN* from upper end.

4.6 Evaluation and sensitivity analysis

Discussion and evaluation of how much sensitive the energy optimal topology T_o which has been introduced in chapter 2, with respect to addition/joining of new nodes

Graph network (Vertices,edges)	Max PI on T_o by Algorithm 2.2 and 2.3	Addition of an intersecting node to T_o	Max PI on T_x by Algorithm 4.1	Reduction of additional node's PI on T_x
(6,9)	[155,230]	$a_{new}=[218,326]$	[155,230]	[218,326]→[218,230]
		$a_{new}=[38,111]$	[155,230]	[38,111]→[85,111]
(8,14)	[22,121]	$a_{new}=[116,167]$	[22,121]	[116,167]→[116,121]
(10,34)	[6,82]	$a_{new}=[18,79]$	[6,82]	No change
		$a_{new}=[4,98]$	[6,82]	[4,98]→[6,82]
(13,38)	[72,162]	$a_{new}=[90,158]$	[72,162]	No change
		$a_{new}=[18,78]$	[72,162]	[18,78]→[72,78]
(15,49)	[9,246]	$a_{new}=[93,250]$	[9,246]	[93,250]→[93,246]
(18,100)	[47,190]	$a_{new}=[23,98]$	[47,190]	[23,98]→[47,98]
		$a_{new}=[120,296]$	[47,190]	[120,296]→[120,190]
(20,120)	[10,366]	$a_{new}=[2,120]$	[10,366]	[2,120]→[10,120]
(23,149)	[82,379]	$a_{new}=[56,72]$	[82,379]	No change
		$a_{new}=[16,382]$	[82,379]	[16,382]→[82,379]
(25,174)	[149,399]	$a_{new}=[283,406]$	[149,399]	[283,406]→[283,399]
(28,202)	[24,404]	$a_{new}=[8,153]$	[24,404]	[8,153]→[24,153]
		$a_{new}=[394,416]$	[24,404]	[394,416]→[394,404]
(30,265)	[75,601]	$a_{new}=[413,639]$	[75,601]	[413,639]→[413,601]
(35,380)	[170,611]	$a_{new}=[44,138]$	[170,611]	[44,138]→[52,138]
		$a_{new}=[539,640]$	[170,611]	[539,640]→[539,611]

Table 4.2: Resultant data on max PI when a new intersecting node is added to energy optimal topologies

to T_o is provided in this section. Sensitivity analysis, related to change in optimal value of power consumption i.e. optimal max PI , has been conducted on many energy optimal topologies obtained on hypothetical interval graph models of WSNs on addition of new nodes to the topologies.

Addition of new nodes to T_o by proposed algorithms bring extended topology T_x of T_o . Evaluation of max power consumption on extended topology T_x when intersecting nodes as well as non-intersecting node are added to energy optimal topology T_o , have been given in the Tables 4.2, 4.3, 4.4 and 4.5. Each table shows the number of newly added nodes with their PI 's and reduction of max PI of additional nodes on T_x as well.

Table 4.2 shows that the max PI obtained on T_o and T_x when single intersecting node is added to T_o by Algorithm 4.1. It is seen that there is no change in optimal value of power consumption after addition of a intersecting node, also PI of the additional node has been reduced on T_x , obtained by Algorithm 4.1.

Table 4.3 shows that the max PI obtained on T_o and T_x when multiple new intersecting nodes are added to T_o by Algorithm 4.2. It is seen that there is no change in optimal value of power consumption after addition of even multiple intersecting nodes, also max PI among the additional nodes is reduced such that max upper limit of new node is minimized to optimal upper limit on T_x obtained by Algorithm 4.2.

Table 4.4 shows that the max PI obtained on T_o and T_x when $N \leq n$ new intersecting

Graph network (n vertices, m edges)	Max PI on T_o by Algorithm 2.2 and 2.3	Addition of N ($\leq n$) intersecting nodes to T_o	Max PI on T_x by Algorithm 4.2	Reduced max PI of newly added nodes on T_x
(6,9)	[155,230]	$a_1=[75,130]$, $a_2=[159,260]$, $a_3=[133,142]$ $a_4=[190,318]$, $a_5=[38,111]$	[155,230]	[190,318] $\rightarrow$ [190,230]
(7,16)	[65,120]	$a_1=[117,125]$, $a_2=[98,128]$, $a_3=[78,95]$, $a_4=[16,108]$	[65,120]	[98,128] $\rightarrow$ [98,120]
(9,21)	[94,125]	$a_1=[37,145]$, $a_2=[69,170]$, $a_3=[15,90]$ $a_4=[95,102]$, $a_5=[78,126]$, $a_6=[58,86]$	[94,125]	[37,145] $\rightarrow$ [94,125]
(10,34)	[6,82]	$a_1=[3,14]$, $a_2=[29,110]$ $a_3=[19,95]$, $a_4=[64,119]$	[6,82]	[64,119] $\rightarrow$ [64,82]
(12,19)	[102,208]	$a_1=[15,190]$, $a_2=[25,106]$, $a_3=[19,60]$ $a_4=[163,235]$, $a_5=[163,235]$, $a_6=[66,190]$	[102,208]	[163,235] $\rightarrow$ [163,208]
(13,38)	[72,162]	$a_1=[23,45]$, $a_2=[16,83]$, $a_3=[160,195]$, $a_4=[153,189]$ $a_5=[28,145]$, $a_6=[39,126]$, $a_7=[78,150]$, $a_8=[111,198]$	[72,162]	[111,198] $\rightarrow$ [111,162]
(15,49)	[9,246]	$a_1=[11,29]$, $a_2=[50,196]$, $a_3=[89,260]$	[9,246]	[89,260] $\rightarrow$ [89,246]
(17,61)	[14,308]	$a_1=[9,130]$, $a_2=[11,119]$, $a_3=[168,290]$ $a_4=[138,209]$, $a_5=[90,180]$, $a_6=[296,350]$	[14,308]	[296,350] $\rightarrow$ [296,308]
(18,100)	[47,190]	$a_1=[12,60]$, $a_2=[89,138]$, $a_3=[19,65]$, $a_4=[153,200]$ $a_5=[81,119]$, $a_6=[59,193]$, $a_7=[180,269]$	[47,190]	[180,269] $\rightarrow$ [180,190]
(20,120)	[10,366]	$a_1=[8,38]$, $a_2=[16,140]$, $a_3=[286,390]$, $a_4=[316,410]$ $a_5=[78,112]$, $a_6=[93,136]$, $a_7=[38,216]$, $a_8=[43,81]$ $a_9=[102,193]$, $a_{10}=[230,310]$, $a_{11}=[286,393]$, $a_{12}=[180,210]$	[10,366]	[286,393] $\rightarrow$ [286,366]
(23,149)	[82,379]	$a_1=[4,29]$, $a_2=[9,19]$, $a_3=[138,145]$ $a_4=[209,263]$, $a_5=[310,390]$, $a_6=[319,363]$ $a_7=[353,412]$, $a_8=[269,309]$, $a_9=[219,283]$	[82,379]	[353,412] $\rightarrow$ [353,379]
(25,174)	[149,399]	$a_1=[19,39]$, $a_2=[42,58]$, $a_3=[296,316]$, $a_4=[383,412]$ $a_5=[196,206]$, $a_6=[318,410]$, $a_7=[96,148]$	[149,399]	[383,412] $\rightarrow$ [383,399]

Table 4.3: Resultant data on max PI when N ($\leq n$) new intersecting nodes are added to energy optimal topologies

nodes are added bijectively to T_o by Algorithm 4.3 where $n = |T_o|$. It is seen that there is no change in optimal value of power consumption even after bijective joining of n new intersecting nodes to T_o , also max PI among the additional nodes is reduced such that max upper limit of new node is minimized to optimal upper limit on T_x obtained by Algorithm 4.3.

Table 4.5 shows the max PI obtained on T_o and T_x when a non-intersecting node is added to T_o by Algorithm 4.4. It is seen that if NIN is added from lower end there is no change in optimal value of max power consumption even after increment of PI of newly added node on T_x . Addition of a NIN from upper end effects the optimal value by increasing the upper limit of max PI to the new upper limit of power consumption as shown in Fig. 4.7. Blue range in Fig. 4.7 shows the optimal max PI obtained on T_o and red range shows the changed max PI on T_x corresponding to the data obtained in Table 4.5 when a NIN from upper end has been joined to T_o by Algorithm 4.4. Red color range depicts the increment in upper limit of power consumption in each case. In general, optimal value has been changed from $[l_{opt}, U_{opt}]$ to $[l_{opt}, L_{new}]$ where $PI(a_{new}) = [L_{new}, U_{new}]$ and $L_{new} > U_{opt}$. This implies that higher the value of L_{new} more change is reflected on optimal value. Therefore addition of a non-intersecting node from upper end must be restricted by holding some appropriate limit of % change of power consumption like if the difference between L_{new} and U_{opt} is very large, a_{new} cannot be considered to enter into T_o .

Furthermore, plots of Fig. 4.8 shows the reduction of $PI(a_{new})$ from $[L_{new}, U_{new}]$ to

Graph network (n vertices, m edges)	Max PI on T_o by Algorithm 2.2 and 2.3	Bijjective joining of n new intersecting nodes to T_o	Max PI on T_x by Algorithm 4.3	Reduced max PI of newly added nodes on T_x
(6,9)	[155,230]	$a_1=[105,178], a_2=[195,200], a_3=[160,190]$ $a_4=[90,180], a_5=[60,114], a_6=[155,243]$	[155,230]	[155,243] $\rightarrow$ [155,230]
(7,16)	[65,120]	$a_1=[89,119], a_2=[100,113], a_3=[98,107], a_4=[93,150]$ $a_5=[98,118], a_6=[110,123], a_7=[105,125]$	[65,120]	[93,150] $\rightarrow$ [93,120]
(8,14)	[22,121]	$a_1=[1,7], a_2=[9,16], a_3=[9,40], a_4=[11,21]$ $a_5=[68,111], a_6=[98,140], a_7=[12,118], a_8=[10,12]$	[22,121]	[98,140] $\rightarrow$ [107,121]
(9,21)	[94,125]	$a_1=[69,75], a_2=[48,79], a_3=[49,68]$ $a_4=[91,133], a_5=[93,108], a_6=[79,108]$ $a_7=[76,112], a_8=[68,130], a_9=[93,119]$	[94,125]	[91,133] $\rightarrow$ [94,102]
(10,34)	[6,82]	$a_1=[12,20], a_2=[9,16], a_3=[28,59], a_4=[29,68]$ $a_5=[19,95], a_6=[49,60], a_7=[41,56]$ $a_8=[16,80], a_9=[38,79], a_{10}=[4,28]$	[6,82]	[19,95] $\rightarrow$ [37,70]
(11,21)	[122,128]	$a_1=[28,39], a_2=[12,42], a_3=[59,68], a_4=[98,138]$ $a_5=[98,110], a_6=[93,102], a_7=[98,120], a_8=[109,128]$ $a_9=[107,115], a_{10}=[103,200], a_{11}=[111,128]$	[122,128]	[98,138] $\rightarrow$ [98,100]
(12,19)	[102,208]	$a_1=[8,19], a_2=[9,16], a_3=[28,38], a_4=[16,59]$ $a_5=[16,40], a_6=[19,44], a_7=[106,137], a_8=[29,63]$ $a_9=[28,98], a_{10}=[130,198], a_{11}=[81,128], a_{12}=[107,118]$	[102,208]	[130,198] $\rightarrow$ [145,147]
(13,38)	[72,162]	$a_1=[13,43], a_2=[27,69], a_3=[29,58], a_4=[42,68]$ $a_5=[89,96], a_6=[19,50], a_7=[63,108]$ $a_8=[100,133], a_9=[147,173], a_{10}=[112,160]$ $a_{11}=[109,128], a_{12}=[138,183], a_{13}=[18,80]$	[72,162]	[147,173] $\rightarrow$ [147,162]
(14,46)	[90,196]	$a_1=[9,19], a_2=[2,12], a_3=[29,35], a_4=[11,68]$ $a_5=[38,119], a_6=[78,139], a_7=[113,149], a_8=[103,123]$ $a_9=[88,109], a_{10}=[158,179], a_{11}=[109,128]$ $a_{12}=[138,183], a_{13}=[18,80], a_{14}=[188,198]$	[90,196]	[188,198] $\rightarrow$ [190,192]
(15,49)	[9,246]	$a_1=[2,9], a_2=[1,19], a_3=[1,12], a_4=[9,31]$ $a_5=[19,26], a_6=[7,26], a_7=[13,38], a_8=[19,68]$ $a_9=[86,107], a_{10}=[143,169], a_{11}=[13,129], a_{12}=[68,139]$ $a_{13}=[136,249], a_{14}=[190,336], a_{15}=[57,99]$	[9,246]	[190,336] $\rightarrow$ [190,246]

Table 4.4: Resultant data on max PI when n new intersecting nodes are added bijectively to energy optimal topologies

$[U_{opt}, L_{new}]$ on T_x where blue color shows the original upper limit of PI of a NIN and red color shows the reduced upper limit, corresponding to the data given in Table 4.5.

Remark 4.6.1. Addition of new nodes by proposed Algorithms 4.1, 4.2 and 4.3 to energy optimal topologies of interval graph models, shows no effect on optimal value of power consumption. Effect is seen only when addition of a non-intersecting node from upper end is made to the topology by Algorithm 4.4. Addition of a non-intersecting node from lower end by Algorithm 4.4 does not show any effect on optimal value.

Remark 4.6.2. Though NIN from upper end increases the max power consumption of the network still Algorithm 4.4 decreases the max PI among the new node to the possibly minimum value. As initially $PI(a_{new}) = [L_{new}, U_{new}]$, addition of a_{new} by Algorithm 4.4 to the optimal topology having optimal value $[l_{opt}, U_{opt}]$, reduces the $PI(a_{new})$ to $[U_{opt}, L_{new}]$ in each network shown in Table 4.5.

Remark 4.6.3. Change in optimal max PI on addition of NIN from upper end is directly depending on the NIN 's PI as if optimal max $PI = [l_{opt}, U_{opt}]$ and $a_{new} = [L_{new}, U_{new}]$ such that $U_{opt} < L_{new}$, then $[l_{opt}, U_{opt}]$ is changed to $[l_{opt}, L_{new}]$ on the extended topology. This implies that larger the value of L_{new} the more changes is reflected on optimal PI . Hence changes on

Graph network (Vertices,edges)	Max PI on T_o by Algorithm 2.2 and 2.3	Addition of a non-intersecting node to T_o	Max PI on T_x by Algorithm 4.4	Updated PI of a_{new} on T_x
(6,9)	[155,230]	$a_{new}=[52,72]$	[155,230]	[52,72]→[72,78]
		$a_{new}=[265,318]$	[155,265]	[265,318]→[155,265]
(8,14)	[22,121]	$a_{new}=[1,2]$	[22,121]	[1,2]→[2,3]
		$a_{new}=[125,136]$	[22,125]	[125,136]→[121,125]
(10,34)	[6,82]	$a_{new}=[2,5]$	[6,82]	[2,5]→[5,6]
		$a_{new}=[89,98]$	[6,89]	[89,98]→[82,89]
(13,38)	[72,162]	$a_{new}=[20,30]$	[72,162]	[20,30]→[30,32]
		$a_{new}=[169,172]$	[72,169]	[169,172]→[162,169]
(15,49)	[9,246]	$a_{new}=[259,300]$	[9,259]	[259,300]→[246,259]
(18,100)	[47,190]	$a_{new}=[8,18]$	[47,190]	[8,18]→[18,19]
		$a_{new}=[199,210]$	[47,199]	[199,210]→[190,199]
(20,120)	[10,366]	$a_{new}=[2,8]$	[10,366]	[2,8]→[8,10]
		$a_{new}=[390,411]$	[10,390]	[390,411]→[366,390]
(23,149)	[82,379]	$a_{new}=[2,5]$	[82,379]	[2,5]→[5,6]
		$a_{new}=[390,413]$	[82,390]	[390,413]→[379,390]
(25,174)	[149,399]	$a_{new}=[5,19]$	[149,399]	[5,19]→[19,34]
		$a_{new}=[402,460]$	[149,399]	[402,460]→[399,402]
(28,202)	[24,404]	$a_{new}=[3,11]$	[24,404]	[3,11]→[11,12]
		$a_{new}=[410,469]$	[24,410]	[410,469]→[404,410]
(30,265)	[75,601]	$a_{new}=[612,728]$	[75,612]	[612,728]→[75,612]
(35,380)	[170,611]	$a_{new}=[3,5]$	[170,611]	[3,5]→[5,6]
		$a_{new}=[675,690]$	[170,675]	[675,690]→[611,675]

Table 4.5: Resultant data on max PI when a new non-intersecting node is added to energy optimal topology

optimal value can be maintained by imposing a restriction of % change so that only convenient NIN can enter to T_o .

4.7 Conclusion

This chapter introduces new algorithmic approaches to extend energy optimal topologies obtained on any interval graph model of WSNs by adding new nodes to the optimal topology whenever it is required. The extension of optimal topology could not be done with the existing algorithms since the optimal topology is no more an interval graph though each node has been weighted with interval data, to execute the interval graph based algorithms. Also if in anycase existing algorithms work, they can take max $\mathcal{O}(n^3)$ time complexity where n is the total number of nodes whereas the proposed algorithms have constant, linear and quadratic time complexity, lesser than $\mathcal{O}(n^3)$. Work on addition of nodes has been brought here in detail by categorising the types of new nodes based on the intersection of their power interval and four new algorithms have been proposed by considering four different cases of addition of new nodes to the optimal topologies. In addition, sensitivity analysis has been carried out to evaluate the change in optimal value of power consumption of energy optimal topologies due to addition of new nodes and it is revealed that extended

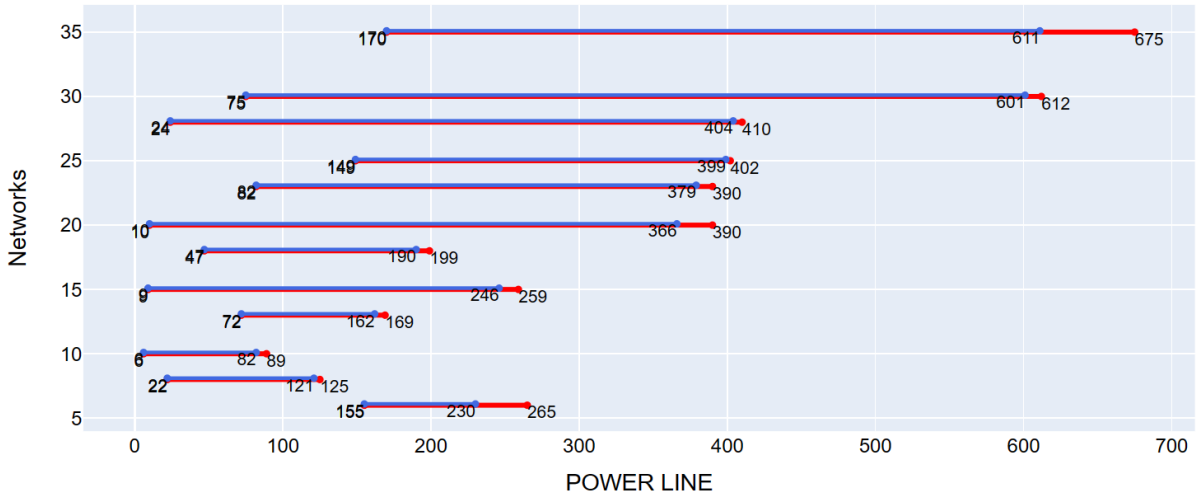


Figure 4.7: Blue color depicts optimal max PI and red color depicts max PI on T_x when a NIN from upper end is added by Algorithm 4.4

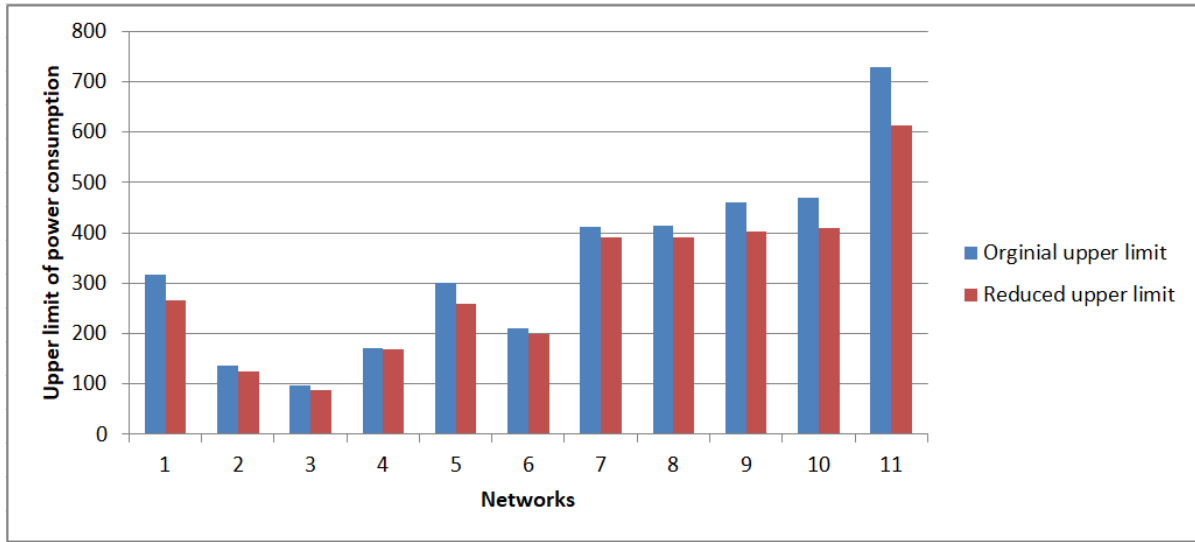


Figure 4.8: Reduction of power interval of a new 'NIN from upper end' on T_x

topologies by proposed algorithms result the same optimal max power interval until the new entering node has completely disjoint power interval with respect to the existing optimal topology. It can be said that the proposed algorithms are the way to make immobile topologies flexible and stable when some application of WSNs require addition of new nodes on the urgent basis to the energy optimal topology of immobile nodes, carrying weight in term of power interval.

Chapter 5

Optimization of energy and delay on interval data based graph model of WSNs

*In this chapter¹, we have provided a study on delay minimum and energy efficient connected routing topology on heterogenous bidirectional WSNs where each node is assigned with a power interval. As emerging applications of WSNs require establishment of such routing topology which can balance energy consumption with delay minimization and despite a plethora research on real-time routing, it has been untouched. Considering this, we have introduced a general ‘Interval data based graph model (IDGM)’ and ‘Sorted-interval data based graph model (S-IDGM)’ of WSNs to explicitly deal with nodes’ power consumption interval and proposed ‘Energy and Delay Optimization (EDO)’ algorithm to optimize S-IDGM such that the max topology delay, total topology delay and max node’s power interval become minimum in quadratic time complexity. A new function has been formulated to estimate topology delay based on link distance and link interference after showing dependency analysis between link distance and link interference towards optimality on numerous networks. Extensive simulation work, graphical and statistical *t*-test analysis have been carried out to show the performance of EDO algorithm in minimizing topology delay and nodes’ power consumption, better than the existing algorithms from similar grounds. *t*-test analysis shows that the proposed EDO algorithm achieves optimal energy sav-*

¹The content of this chapter is based on a published research paper “Optimization of energy and delay on interval data based graph model of wireless sensor networks”, *Wireless Network*, **29**, 2293-2311 (2023). <https://doi.org/10.1007/s11276-023-03292-x> (SCIE, Impact Factor: 3)

ing of nodes at 5% level of significance along with optimal minimization of max and total topology delay at 2% level of significance on S-IDGMs.

5.1 Introduction

It has been observed that routing topologies which aim to minimize nodes' power consumption or focus on saving nodes' power adversely affect delivery of data packets [10, 109]. This means that route with minimum energy consumption causes longer delay in transmitting data packets between the nodes. This is because nodes use smaller transmission range to communicate in order to save their energy and prefer to have hop-by-hop transmission [128, 129]. Delay increases with number of intermediate hops existing between source to destination nodes because data is sensed, processed and transmitted as many time as number of intermediate hops and this contributes processing, propagation, transmission and queuing delay [105, 113]. Therefore delay minimization costs maximization of energy consumption since it requires longer transmission range of every source node to directly transmit data packets to its destination node. Work alone to minimize end-to-end routing delay shortens the network lifetime and work alone to minimize network energy consumption promotes routing delay. This issue in WSNs is called **Energy-Delay Tradeoff** [15, 103, 111, 130, 131] and has been a prime concern from past many years to resolve.

Many authors work towards this Energy-Delay Tradeoff by formulating weighted function of some energy consumption causing factors as well as delay causing factors and use value of their formulated function to achieve desired routing goals. Some authors attempt to minimize delay factor more if the goal is to meet packet delivery deadline and some prioritize low transmission cost path in order to save nodes energy with some delay constraint [99, 101, 130, 132, 133, 134].

Existing approaches on Energy-Delay Tradeoff result delay constraint energy aware path between source to destination node based on hop counts by ignoring rest of the topology. Also existing attempts do not visualise nodes power consumption explicitly while minimizing delay [94, 133].

Some authors estimate queuing, transmission and propagation delay [97, 99] with the

ignorance of transmission failure or retransmission and collision of data packets due to interference factor in large and dense networks. This retransmission and packets collision contribute to more delay in data delivery along with nodes' power consumption [111, 113, 135, 136]. Selvi et al. in [99] estimate end-to-end delay including propagation delay, processing delay, transmission delay and queuing delay, from source S to destination D using Eq. (5.1).

$$Delay_{end-to-end}(S,D) = \sum_{i,j} \left(\left(\frac{1}{\alpha - \beta} \right) + \frac{1}{\phi} + \frac{d(i,j)}{\mu} \right) \quad (5.1)$$

where α, β are constant, ϕ is bandwidth link, μ is the propagation speed and $d(i, j)$ is the distance between intermediate nodes i and j , called hop length or link distance. Here authors have not considered retransmission or interference factor in estimating end-to-end delay.

Moreover solution to Energy-Delay Tradeoff on bidirectional connected heterogenous WSNs where nodes are allowed to vary their transmission range and are assigned with appropriate power interval is still unexplored on topology level.

These existing gaps motivate us to bring out a methodology to obtain delay minimum energy efficient connected routing topology considering interference factor which can simultaneously work to minimize nodes' power consumption and topology delay on bidirectional connected heterogenous WSNs where each sensor node has been allowed to vary its transmission range within a power interval. To estimate delay, hop counts won't be considered since this work is dealt with bidirectional connected topology where each node has to in resultant topology to maintain the connectivity factor therefore hop length (or link distance) between every communicating node is the first factor to estimate delay between every communicating sensor nodes as Eq. (5.1) implies that $Delay_{end-to-end}(S,D) \propto \sum_{i,j} (d(i,j))$ where ϕ and μ are assumed to be same for each sensor nodes in the network.

As delivery time between any two nodes increases with distance but opting a shortest distance doesn't imply fast data delivery between any two nodes until the path carries low interference in a dense network. In view of this, the link-level interference model, introduced by Sun et al. in [38] has been engaged to mitigate interference in the network which in turn reduces topology delay as well as nodes' power consumption, is the new metric and second factor to estimate topology delay in the proposed work.

Briefing the points of motivation and improvements:

1. Proposed methodology attempts to minimize overall topology delay with power consumption while the existing work focuses to minimize end-to-end delay between sender and receiver nodes only.
2. Proposed work shows minimization of nodes power consumption explicitly in the form of power interval while no delay minimization work from literature shows reduction of nodes power consumption explicitly.
3. Our work on developing energy and delay minimum topology on those WSNs whose nodes are assigned with power interval, considers interference factor along with distance factor to estimate topology delay by engaging the link-level interference model introduced in [38] while most of the existing work ignores this retransmission or delay causing factor.

The manifold contributions of the chapter are as follows:

1. Introduction of 'Interval data based graph model (IDGM)' and 'Sorted-interval data based graph model (S-IDGM)' for type of bidirectional connected heterogeneous WSNs where each node is weighted with a power interval so that nodes can vary their transmission range in the assigned power interval according to communication requirement.
2. A new weighted 'Delay estimation (DE) function' based on distance and interference factors has been formulated after showing dependency analysis between link distance (hop length) and link interference towards optimality.
3. A polynomial time 'Energy and Delay Optimization (EDO) algorithm' has been proposed to obtain an energy and delay optimized connected routing topology on S-IDGMs using DE function. EDO algorithm works under an optimal delay constraint in an energy efficient way and so it minimizes node power consumption and topology delay simultaneously.
4. Energy efficiency of EDO algorithm is shown in term of percentage of energy saving by reducing the upper bound of maximally assigned power interval along with minimization of topology delay.
5. Extensive simulation work, graphical and statistical t -test analysis have been carried out to show the performance and comparison of EDO algorithm with delay minimum spanning tree, algorithms introduced in [2] and in chapter 2 on

S-IDGMs in term of maximum and total topology delay, maximum power consumption and percentage of energy saving.

The rest of the chapter has been organized in the following section: section 5.2 introduces new graph models of WSNs. Section 5.3 describes the link-level interference models then section 5.4 shows the dependency analysis of link length and link-level interference model. Delay estimation function has been introduced in section 5.5. Section 5.6 presents the problem definition, proposed techniques along with an explained working illustration and theoretical results. Simulation and performance evaluation have been given in section 5.7 and then section 5.8 concludes the chapter.

5.2 Interval data based graph model (IDGM) and Sorted-interval data based graph model (S-IDGM) of WSNs

Not every WSN can be modelled as an interval graph even having interval weight on each node as their graphical layout do not satisfy interval graph properties [118]. Therefore the type of WSNs whose underlying graphs show interval weight on each node still do not require to satisfy interval graph properties, are called ‘**interval data based graph model (IDGM)**’ of WSNs. Fig. 5.1 is one of an illustration of IDGM. Every interval graph model is an IDGM but converse doesn’t hold always because in an interval graph, two nodes are adjacent if and only if their power intervals intersect [1]. In Fig. 5.1, one can see power intervals of non-adjacent nodes are also intersecting hence this can’t be an interval graph model so it is called an IDGM of WSN.

Sorted-interval data based graph model (S-IDGM) of WSN: Sorting power intervals of an IDGM in an increasing order of their upper bounds results a S-IDGM. Fig. 5.3 shows S-IDGM of Fig. 5.1.

An intersection model, described in [1] has been used for sorting the power intervals of IDGM because of its low complexity. Even though the intersection model, given in [1] has been described for interval graphs, it can work for IDGM in one way only, not viceversa i.e. one can have an intersection model of an IDGM for a sorting purpose but can’t form an IDGM from an intersection model if the IDGM is not an interval graph model. Fig. 5.2 shows the intersection model of Fig. 5.1.

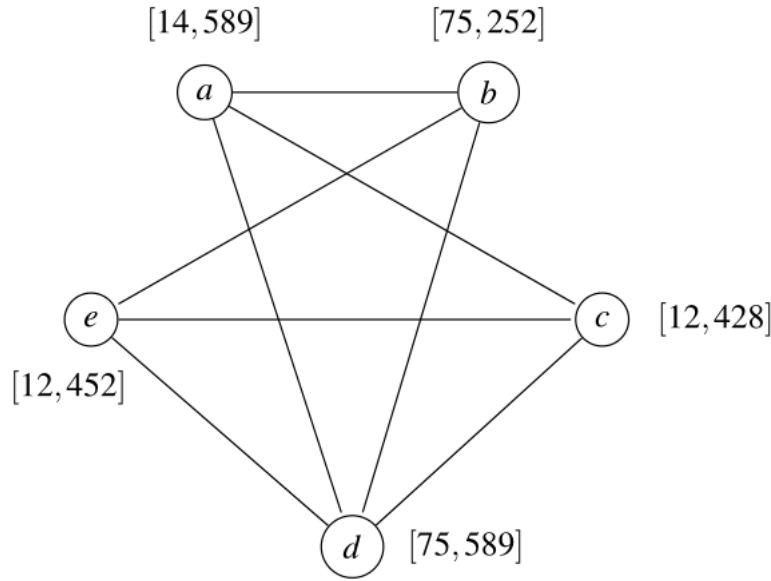


Figure 5.1: Interval data based graph model (IDGM) of a WSN

Sorting and relabelling the nodes as $u_1, u_2, \dots, u_n$ following increasing order of power

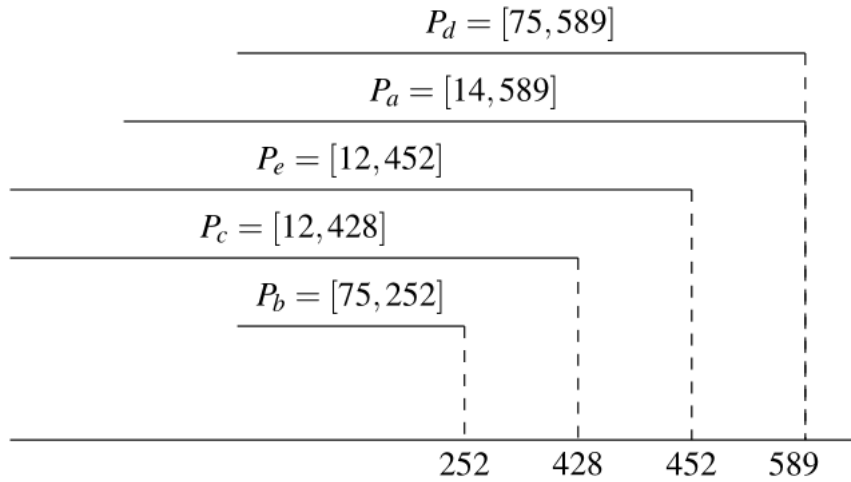


Figure 5.2: Intersection model of IDGM, given in Fig. 5.1

intervals is the very first requirement of our methodology. A node with higher subscript is the node with higher power consumption interval. Fig. 5.3 which is a S-IDGM of Fig. 5.1, visualising that power intervals are increasing with nodes' subscript and viceversa. Although there are two cases, which have been observed during sorting; first is two or more nodes can have same upper limit of power consumption as seen in Fig. 5.1, therefore labelling order depends on the lower limit of their power intervals i.e. node with lesser lower bound of power consumption has been labelled with lesser subscript and the node with higher lower bound of power consumption has been labelled with higher subscript, as seen in Fig. 5.3. Secondly, if two or more nodes have

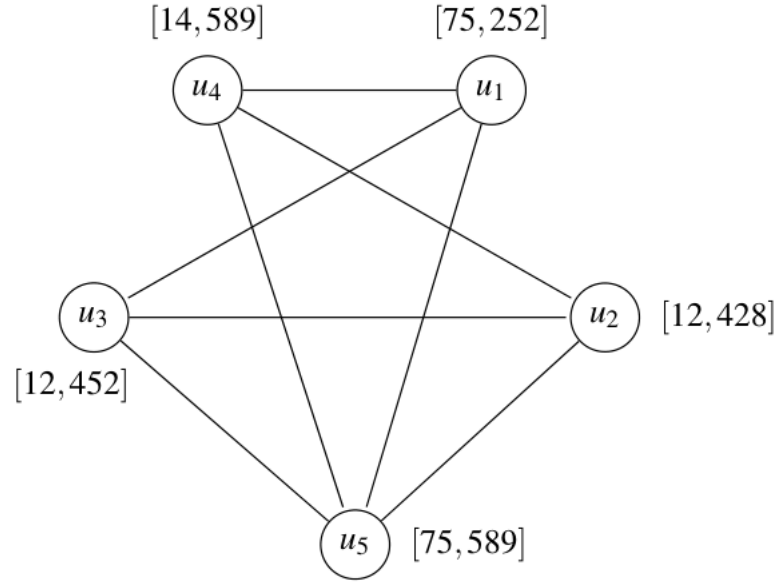


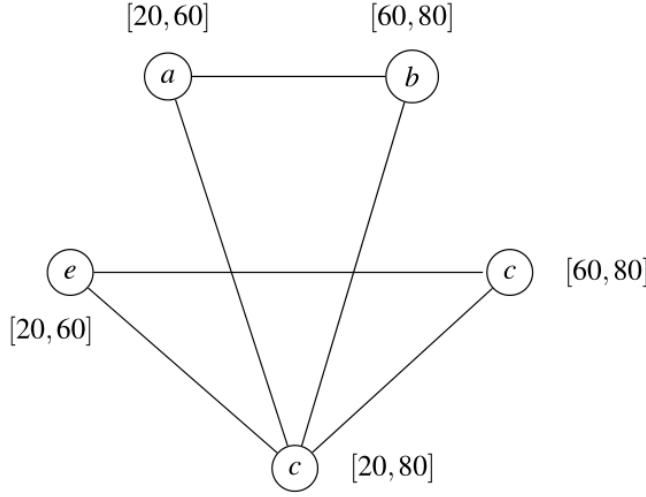
Figure 5.3: S-IDGM of an IDGM, given in Fig. 5.1

same power intervals as shown in Fig. 5.4(a), relabelling of those nodes can take any consecutive order of the running subscript since all the resultant S-IDGMs come out to be isomorphic, see Fig. 5.4(b) and 5.4(c) representing the isomorphic models of Fig. 5.4(a).

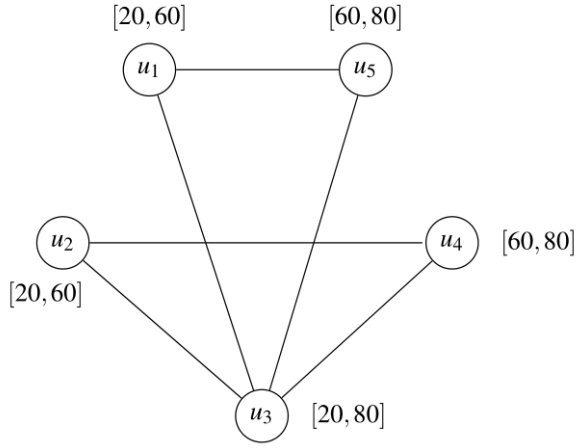
The second requirement of our methodology is to rename minimally assigned power node with leftmost vertex (*LMV*) and maximally assigned power node with rightmost vertex (*RMV*) of the obtained S-IDGM based on the definition introduced in chapter 2 (refer section 2.3.1, Eq. (2.1), (2.2)). Therefore it has been said that a vertex whose subscript is the minimum number is called leftmost vertex (*LMV*) and a vertex whose subscript is the maximum number among all vertices' subscript of a S-IDGM is called rightmost vertex (*RMV*). Hence *LMV* is the minimally assigned power node, having *min PI* and *RMV* is the maximally assigned power node, having *max PI*.

5.3 Link-level interference model

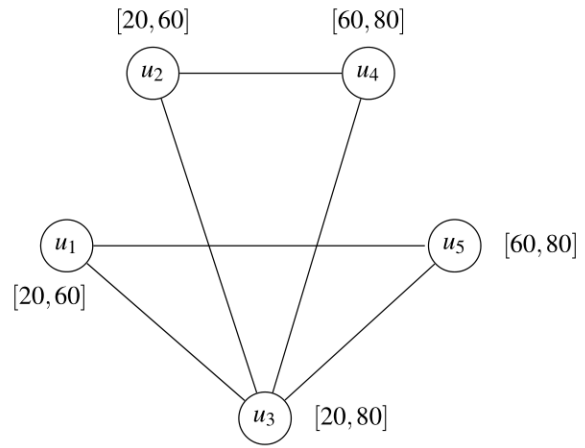
Now introducing Link-level interference model which has been proposed by Sun et al. in 2015 [38] to compute link interference in bidirectional WSNs. For a given graph $G(V, E)$, interference on any bidirectional link uv is denoted by I_{uv} and is equal to the number of distinct interfering links if any of their end node resides inside the disc of node u or v , is called Link-level interference model and is given by Eq. (5.2).



(a) An IDGM where nodes are having same power intervals



(b) First S-IDGM



(c) Second S-IDGM

Figure 5.4: Isomorphic S-IDGMs of an IDGM

$$I_{uv} = |\{xy \mid xy \in E(G), x \text{ or } y \text{ or both} \in D(u, d(u, v)) \text{ or } D(v, d(v, u))\}| \quad (5.2)$$

where xy is another bidirectional link, $d(u, v)$ is an Euclidean distance between u and v , $D(u, d(u, v))$ is a circular disc whose centre is u and radius is $d(u, v)$. Similarly $D(v, d(v, u))$ is a circular disc whose centre is v and radius is $d(v, u)$, where $d(v, u) = d(u, v)$ [12].

An illustration of computing link-level interference model using Eq. (5.2) are shown on network, given in Fig. 5.5, taken from [38]. The link-level interference of link uv using Eq. (5.2) come out to be 7. This link-level interference model outperforms the link interference model described in [3] which counts the number of nodes existing

inside the transmission disc of u or v . According to the model described in [3], the link interference of link uv given in Fig. 5.5, come out 3 based on nodes count. Hence link-level interference model, proposed by Sun et al. [38] provides more significant results.

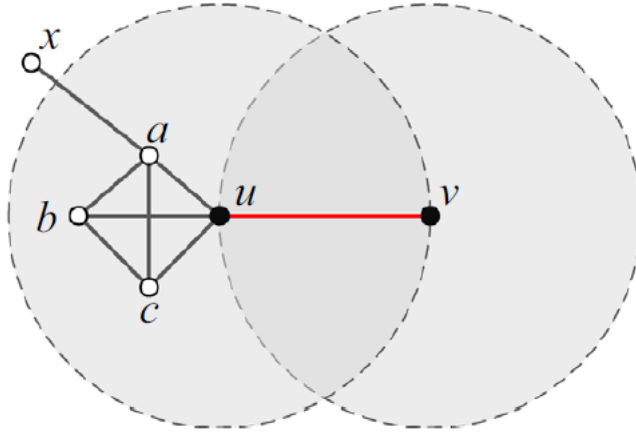


Figure 5.5: Disc topology of link uv

5.4 Dependency analysis of link distance and link-level interference model towards optimality

It has been claimed in many research papers that link distance and its interference are so much depending on each other such as more the distance between the nodes more will be the interference between them [12, 18, 135]. Sun et al. in his paper [38] says that link interference increases with link distance and shows the effect of link distance on link-level interference model with positive correlation.

This motivates us to analyze the effect on interference when the approach is related to obtain distance minimum spanning tree as well as the effect on distance when the approach is related to obtain interference minimum spanning tree. For this, first we put the weight on each link is its distance, compute the MST using Prim's algorithm and observe the reduction in total distance as well as total interference. Then, we put weight on each link is its interference using Eq. (5.2), compute the MST using Prim's algorithm and observe the reduction in total interference as well as total distance. This process has been done on 20 distinct networks and resultant data values have been tabulated in Table 5.1.

Sr. no.	WSNs		Euclidean distance-based MST		Link-level interference model-based MST	
	Nodes	Edges	Total Distance	Total Interference	Total Distance	Total Interference
1	9	27	4.044	38	4.154	34
2	11	30	3.54	44	3.63	43
3	13	38	3.68	60	3.88	55
4	15	49	3.73	97	4.17	88
5	18	100	5.394	96	5.65	87
6	20	120	3.68	124	3.84	117
7	23	129	4.152	144	4.36	137
8	25	141	4.106	180	4.425	172
9	28	202	6.177	165	6.459	161
10	30	268	5.731	174	5.754	173
11	32	291	5.29	240	5.485	218
12	33	265	4.91	245	5.17	243
13	35	307	5.507	254	5.753	239
14	37	304	5.284	310	5.468	283
15	39	433	7.88	251	8.25	230
16	41	461	6.49	248	6.65	239
17	43	436	6.034	353	6.28	300
18	45	479	7.53	341	7.75	334
19	48	608	7.27	363	7.56	326
20	50	608	6.92	394	7.35	377

Table 5.1: Total link distance and total link interference calculation on 20 distinct networks

Some concluding points have been made after observing the tabulated data:

- Resultant data in Table 5.1 reveals that there is always a difference between value of total interference obtained in interference-based MST and distance-based MST i.e. optimal interference reduction is achieved in interference-based MST only, not in distance-based MST, same things happen with link distance. This implies that one cannot optimally reduce both the distance and interference factor by considering the single factor; interference is optimally reduced in interference-based MST and distance is optimally reduced in distance-based MST. Therefore to minimize both the factors simultaneously, a weighted function of link distance and link interference needs to formulate. (Discussion of weighted function is carried forward to next section.)
- Link-level interference is more depending on network density than link distance. More dense the network is, more will be data collision and so more retransmission.
- In dense network, minimization of interference factor when the hop length are not much longer, reduces total distance to some optimal extend as well but this

doesn't happen in sparse network. This implies that in dense network, interference factor has more dominance over distance factor.

5.5 Delay estimation (DE) function

After analysing the results obtained in previous section, a conclusion is made to construct a new function which optimally balances the minimization of link interference with hop length (link distance) when any algorithm is applied after it. Since both the hop length and link-level interference are used metrics to estimate delay between every communicating nodes, therefore this new function is known as 'Delay estimation (DE) function' and is given by Eq. (5.3).

This DE function assigns weight on each edge which is a combination of link-level interference and link distance, using Eq. (5.3). Since $\alpha = 1$ of DE function results interference-based MST providing optimal total interference value and $\alpha = 0$ results distance-based MST providing optimal total link distance value. Exclusion of both these alpha values from Eq. (5.3), the DE function assigns weight to each link with the combination of link distance and link interference which results a weighted MST whose total interference and total hop length values lie close to their respective optimal values (better than when $\alpha = 0$ and $\alpha = 1$).

Moreover, for maintaining the generality, $\alpha = 0.5$ is taken to assign the weight on each link in this chapter. Therefore Eq. (5.3) reduces to Eq. (5.4).

$$DE(uv) = \alpha * I_{uv} + (1 - \alpha) * d(u, v) \quad (5.3)$$

where $\alpha \in (0, 1)$ and uv is any bidirectional link.

$$DE(uv) = I_{uv} + d(u, v) \quad (5.4)$$

Role of DE function:

- DE function assigns weight to each edge using Eq. (5.4) which is nothing but the required delay estimation of each link in the network.
- DE function is used to obtain optimal delay constraint Δ for the complete execution of the proposed algorithm.
- DE function estimates total and maximum topology delay (TD) in the resultant

topology using Eq. (5.4).

5.6 Problem Definition and Proposed Algorithm

To find an energy and delay optimized connected routing topology on S-IDGM $G(V, E)$ of WSN, following is the formulated linear programming problem (LPP) demonstrating the required optimization problem.

$$\min \max_{uv \in E(G)} DE(uv) = \min \max_{uv \in E(G)} \{I_{uv} + d(u, v)\} \quad (5.5)$$

$$\min \max_{u \in V(G)} PI(u) = \min \max_{u \in V(G)} [r_{min}(u), r_{max}(u)] \quad (5.6)$$

subject to

$$\sum_{uv \in E(G)} uv = |V(G)| - 1 \quad (5.7)$$

$$DE(uv) \leq \Delta \quad (5.8)$$

$$r_{min}(u) \leq PowerConsumption(u) \leq r_{max}(u) \quad (5.9)$$

$$\max\{r_{min}(u), r_{min}(v)\} \leq c(uv) \leq \min\{r_{max}(u), r_{max}(v)\} \quad (5.10)$$

$$deg(u) > 0 \quad \forall u \in V(G) \quad (5.11)$$

where uv 's are bidirectional transmission links, Δ is an optimal delay constraint i.e. an optimal max delay bound obtained using delay minimum spanning tree by the proposed algorithm during its execution and $deg(u)$ is a degree of u . Eq. (5.5) and (5.6) depict the desired optimization goals that are minimization of maximum topology delay and minimization of maximum node's power interval respectively. Eq. (5.7)-(5.11) depict the required constraints attached with the given optimization problem. Eq. (5.7) gives a bound on number of links i.e. total number of links in the resultant topology should be equal to one less than the number of nodes, Eq. (5.8) directs to keep only those link whose delay estimation (DE) must be less than or equal to the obtained delay constraint, Eq. (5.9) allows each node to spend its energy within its assigned power interval, Eq. (5.10) allows variation of transmission cost of each link within the given bounds (which is actually an available power interval on each link) as transmission cost is directly depending on the power used by a link during transmission and Eq. (5.11) requires each node to present in the resultant topology so each node must have non-zero degree in the graph.

Energy and Delay Optimization (EDO) algorithm: To the defined optimization problem, a polynomial time algorithm has been proposed, named as 'Energy and Delay Optimization (EDO)' algorithm. The proposed EDO algorithm undergoes three main phases. The very first phase (step 1 and 2) provides an optimal delay constraint i.e. Δ based on DE function and DE MST, second phase (from step 3 to 17) computes the delay constraint energy efficient path cover on S-IDGM of WSN and third phase (step 18 onwards) constructs energy and delay optimized connected routing topology of S-IDGM if the obtained path cover contains more than one path component. Methodology of the proposed EDO algorithm is the motivation of the algorithms given in [2] and in chapter 2.

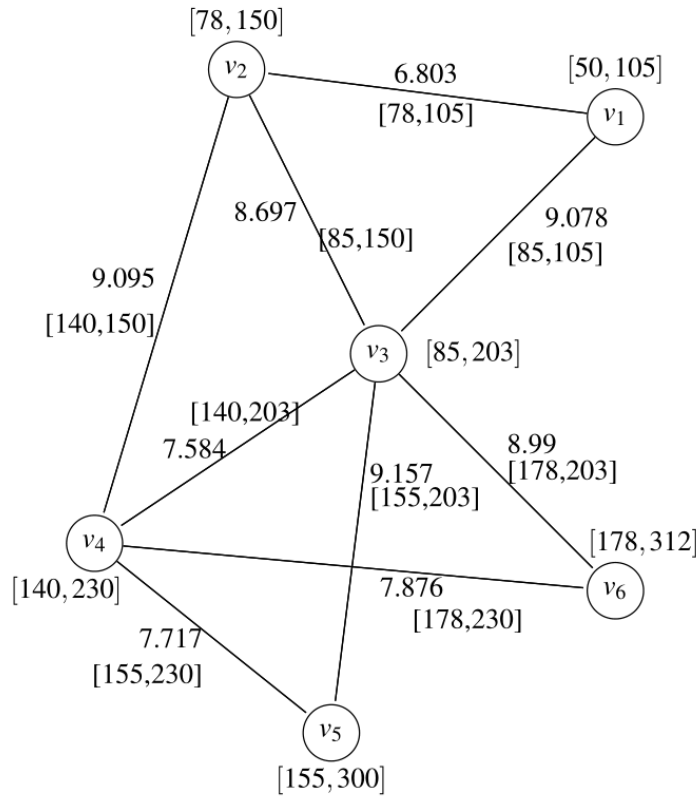


Figure 5.6: A S-IDGM of an arbitrary WSN where weights on each edge is the estimated delay of each link and available power interval on each link.

Working illustration with step-by-step explanation of EDO algorithm: consider a S-IDGM of an arbitrary WSN given in Fig. 5.6 where each node has been weighted with a power interval which results to power interval on each edge as $PI(v_1) = [50, 105]$ and $PI(v_2) = [78, 150]$, then the edge between v_1 and v_2 has $PI(v_1v_2) = PI(v_1) \cap PI(v_2) = [78, 105]$. Likewise each edge has been resulted with a power interval due to its end nodes and so these power intervals are called available

Algorithm 5.1 Energy and Delay Optimization (EDO) algorithm

Input : A S-IDGM $G(V, E)$ where $|V(G)| = n$ and $|E(G)| = m$ of a bidirectional WSN.

Output : A delay-constraint and energy-efficient connected routing topology which results Eq. (5.5) and (5.6) of the given LPP following all the attached constraints.

- 1: Compute $DE(uv)$ of each link uv of $G(V, E)$ using Eq. (5.4).
 - 2: Obtain MST of the delay weighted graph $G(V, E)$ using Prim's algorithm and optimal delay constraint $\Delta = \max_{uv \in MST} \{DE(uv)\}$.
 - 3: Define path cover, $PC = \phi$ initially
 - 4: Let $UncoverV = \{v_1, v_2, \dots, v_n\}$
 - 5: Start $num = 1$ and $Cnum = v_n$ $\triangleright$ (num shows the number of path components; $Cnum$ shows the running path component)
 - 6: $UncoverV = UncoverV / v_n$
 - 7: **while** $UncoverV \neq \phi$ **do**
 - 8: Compute $S = \{v_k \mid v_k \in UncoverV, DE(v_k v_n) \leq \Delta \text{ and } k > n\}$ and $T = \{v_k \mid v_k \in UncoverV, DE(v_k v_n) \leq \Delta \text{ and } k < n\}$
 - 9: **if** S and T of v_n is empty **then**
 - 10: $[v = RMV(UncoverV); num = num + 1; Cnum = v]$
 - 11: **if** $S \neq \phi$ **then**
 - 12: $[v = RMV(S); Cnum = Cnum \cup v]$ $\triangleright$ ($\cup$ shows the joining of vertices)
 - 13: **else**
 - 14: $[v = LMV(T); Cnum = Cnum \cup v]$
 - 15: $v_n = v$
 - 16: $uncoverV = uncoverV / \{v_n\}$
 - 17: **return** $PC = \{C_1, C_2, \dots, Cnum\}$
 - 18: **if** $|PC| = 1$ **then**
 - 19: C_1 is the required solution
 - 20: **else**
 - 21: Choose a component C_i from PC and $PC \leftarrow PC / \{C_i\}$
 - 22: **while** $PC \neq \phi$ **do**
 - 23: Pick another component $C_j \in PC$ such that C_j and C_i are intersecting
 - 24: Compute $nghC_j = \{v_i \mid v_i \in C_i, \text{ providing the direct connectivity to } C_j \text{ under } \Delta\}$ and $nghC_i = \{v_j \mid v_j \in C_j, \text{ providing the direct connectivity to } C_i \text{ under } \Delta\}$
 - 25: **if** $LMV(nghC_i) < LMV(nghC_j)$ **then**
 - 26: $C_i = C_i \cup C_j$ will be the new path with the joining of $LMV(nghC_i)$ to the leftmost adjacent vertex from $nghC_j$.
 - 27: **else**
 - 28: $C_i = C_i \cup C_j$ will be the new path with the joining of $LMV(nghC_j)$ to the leftmost adjacent vertex from $nghC_i$
 - 29: $PC = PC / \{C_j\}$
 - 30: C_i is the required delay constraint energy efficient connected routing topology.
 - 31: For every link $uv \in C_i$, update $DE(uv)$ using Eq. (5.4) and compute $\min \max_{uv \in C_i} DE(uv)$, $\min \sum_{uv \in C_i} DE(uv)$.
 - 32: For each node $v \in C_i$, update $PI(v) = [\min\{PI_{vu}, \forall vu \in C_i\}, \max\{PI_{vu}, \forall vu \in C_i\}]$ and compute $\min \max_{v \in C_i} PI(v)$.
-

power intervals, shown in Fig. 5.6. Weight other than the available power interval on each edge in Fig. 5.6 is a delay estimation.

Step 1 Computes delay estimation on each edge of graph, given in Fig. 5.6, using Eq. (5.4) and so $DE(v_1v_2) = 6.803$, $DE(v_1v_3) = 9.078$, $DE(v_2v_3) = 8.697$, $DE(v_3v_4) = 7.584$, $DE(v_3v_5) = 9.157$, $DE(v_3v_6) = 8.99$, $DE(v_4v_5) = 7.717$, $DE(v_4v_6) = 7.876$.

Step 2 DE-MST is obtained using Prim's algorithm on Fig. 5.6 with edges $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_4v_6\}$ and results $\Delta = \max\{6.803, 8.697, 7.584, 7.717, 7.876\} = 8.697$.

Step 3 Path cover is defined as $PC = \phi$ initially depicts that no vertex is covered by any path component of PC .

Step 4 $UncoverV$ is the set of unvisited vertices which are not covered by PC . So in this illustration $UncoverV = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ since no vertex is covered by PC .

Step 5 $num = 1$ shows that PC starts to obtain its first path component C_1 as PC is like $\{C_1, C_2, \dots, C_{num}\}$. C_1 visits maximally assigned vertex first i.e. $RMV(UncoverV)$, therefore $C_1 = v_6$.

Step 6 $UncoverV$ updates to $UncoverV = \{v_1, v_2, v_3, v_4, v_5\}$ as v_6 is covered by C_1 .

Step 7 Initialization of while loop brings execution of steps from 8 to 16 until $UncoverV$ becomes empty.

Step 8 S and T has been computed for each node present in the original graph, given in Fig. 5.6. For $v_n = v_6$, $S(v_6) = \phi$ since S comprises only those unvisited neighbour nodes of v_6 whose subscript is greater than 6 with link delay ≤ 8.697 . Unlikely $T(v_6) = \{v_4\}$ comprises of those unvisited neighbour nodes of v_6 whose subscript is less than 6 with link delay ≤ 8.923 .

Step 9 Checks the emptiness of S and T .

Step 10 If incase both S and T are empty then C_1 gets terminated with the initialization of C_2 which again starts with RMV of $UncoverV$ and jumps to step 15.

Step 11 Checks the non-emptiness of set S if the condition of step 9 is not satisfied. If S comes out non-empty then

Step 12 C_1 extends itself by adjoining RMV from S .

Step 13 Checking of T 's emptiness if the condition of step 11 is not approved like in the given illustration where S is empty and T is non-empty then

Step 14 C_1 extends itself by adjoining LMV from T so C_1 becomes $C_1 = v_6v_4$.

Step 15 Now v_4 becomes v_n i.e $v_n = v_4$.

Step 16 Therefore $UncoverV = \{v_1, v_2, v_3, v_5\}$.

(Since $UncoverV \neq \phi$ execution of while loop causes computation of S and T for v_4 , results $S = \{v_5\}$ and $T = \{v_3\}$ and $C_1 = v_6v_4v_5$. Now $v_n = v_5$, $UncoverV = \{v_1, v_2, v_3\}$,

$S(v_5) = \emptyset$ and $T(v_5) = \emptyset$ therefore step 9 and 10 terminate C_1 with the initiation of $C_2 = RMV(UncoverV)$ i.e. $C_2 = v_3$. Now $v_n = v_3$, $UncoverV = \{v_1, v_2\}$, $S(v_3) = \emptyset$ and $T(v_3) = \{v_2\}$ so C_2 extends to v_3v_2 and likewise $v_n = v_2$ results $C_2 = \{v_3v_2v_1\}$. Finally $UncoverV$ becomes empty and while loop gets terminated.)

Step 17 Return $PC = \{C_1, C_2\}$ i.e. $PC = \{v_6v_4v_5, v_3v_2v_1\}$.

Step 18 Checks the cardinality of PC , if $|PC| = 1$ then

Step 19 C_1 is the required solution.

Step 20 In the given illustration we follow the else part as $|PC| = 2$.

Step 21 Let $C_i = v_6v_4v_5$ and update $PC = \{v_3v_2v_1\}$.

Step 22 Initialization of while loop since $PC \neq \emptyset$.

Step 23 Let $C_j = v_3v_2v_1$, also C_i and C_j are intersecting in the original graph, given in Fig. 5.6.

Step 24 $nghC_i$ and $nghC_j$ shows the direct neighbourhood of path component C_i and C_j respectively under the delay constraint. Therefore $nghC_j = \{v_4\}$ and $nghC_i = \{v_3\}$ for the given illustration.

Step 25 $LMV(nghC_i)$ is v_3 , $LMV(nghC_j)$ is v_4 and $v_3 < v_4$, this satisfies the condition that $LMV(nghC_i) < LMV(nghC_j)$.

Step 26 Therefore C_i is joined to C_j by opting the connection from v_3 to v_4 , shown in Fig. 5.7.

Step 27 If incase condition of step 25 doesn't satisfy then

Step 28 C_i is joined to C_j by making connection of $LMV(nghC_j)$ to its leftmost adjacent vertex from $nghC_i$.

Step 29 PC becomes empty for the given illustration, therefore end of while loop.

Step 30 Resultant C_i , given in Fig. 5.7 is the optimized connected routing topology of Fig. 5.6.

*(Fig. 5.7 shows the updated weights on nodes as well as on edges.)

Step 31 Computation of link delay of the optimized topology using Eq. (5.4). Delay weights are updated in Fig. 5.7 which results $\min \max_{uv \in C_i} DE(uv) = 4.697$ and $\min \sum_{uv \in C_i} DE(uv) = 18.677$.

Step 32 Updation of power assignment of each node of the optimized topology of Fig. 5.7 using Eq. (2.5): $r'(v_1) = [78, 105]$, $r'(v_2) = [\min\{78, 85\}, \max\{105, 150\}] = [78, 150]$, $r'(v_3) = [\min\{85, 140\}, \max\{150, 203\}] = [85, 203]$, $r'(v_4) = [\min\{140, 155, 178\}, \max\{203, 230\}] = [140, 230]$, $r'(v_5) = [155, 230]$ and $r'(v_6) = [178, 230]$. Therefore $\min \max PI = [140, 230]$ at node v_4 .

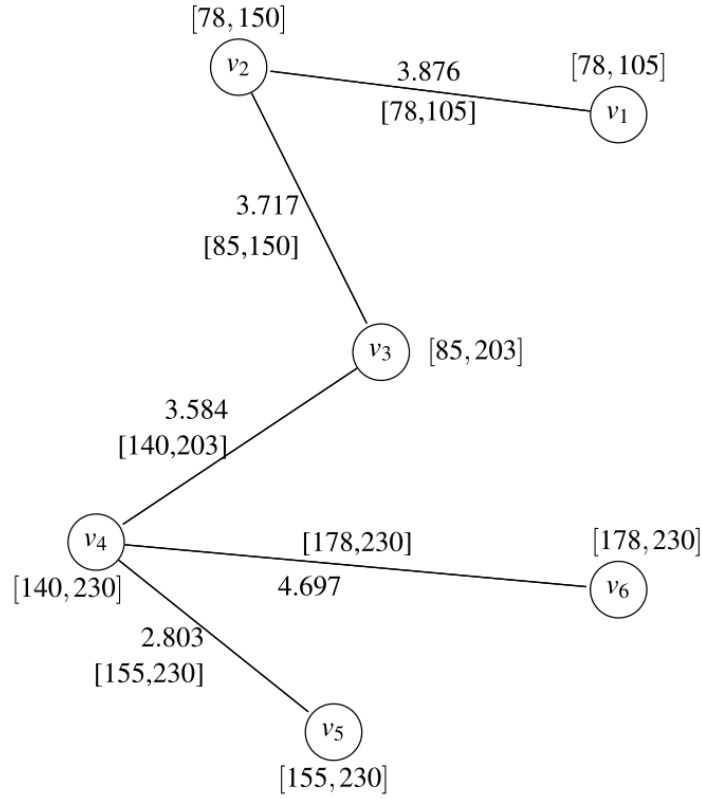


Figure 5.7: Optimized Connected topology by EDO algorithm on S-IDGM of Fig. 5.6

Remark 5.6.1. Max TD on S-IDGM, given in Fig. 5.6 is 9.157 and max node PI is $[178, 312]$. EDO algorithm reduces max TD from 9.157 to 4.697 and max node PI from $[178, 312]$ to $[140, 230]$ (refer Fig. 5.7). Also total TD on the S-IDGM has been reduced from 74.997 to 18.677 on the optimized topology, shown in Fig. 5.7.

Remark 5.6.2. Minimization of max node PI leads to the minimization of max power consumption as 320 is the maximum power consumption of atleast one link in the original graph and hence it reduces to 230 in the optimized topology (refer Fig. 5.6 and 5.7). This leads to energy saving of the network. Therefore % of energy saving by EDO algorithm when max node PI reduces from $[178, 312]$ to $[140, 230]$ is obtained by calculating $\frac{(312-230)}{312} \times 100 = 26\%$.

Theorem 5.6.1. Δ is an optimal delay constraint obtained by EDO algorithm.

Proof. Δ is the maximum edge weight on the obtained MST where the edge weight is the delay estimation of each link (computed using Eq. (5.4)). Assume that Δ is not an optimal constraint this means that there exists another spanning tree say Tr whose maximum edge weight must be less than Δ . But this is not possible as Tr cannot be another MST giving different solution of the same graph. Hence Δ is an optimal delay constraint. $\square$

Theorem 5.6.2. EDO algorithm results energy and delay optimized connected routing topology which minimizes maximum delay as well as node's power interval simultaneously on S-IDGM of a WSN.

Proof. We proceed the proof by considering two possible cases.

Case(i) Maximum power consumption and maximum delay exist on distinct links:

Let v_i be any node having *max PI* on the obtained topology T_{edo} by EDO algorithm. This implies that there exists atleast one link of max power consumption incident on v_i , let l be that link. Let $l' \in T_{edo}$ be another link, distinct from l , having maximum delay. Let us assume that T_{edo} is not an energy and delay optimal topology obtained by EDO algorithm and T' is an optimal one where the cardinality of path cover obtained for T' must not be greater than the cardinality of path cover obtained for T_{edo} construction. Hence links l and l' which are having max power consumption and max delay respectively must not belong to T' (otherwise T_{edo} would have been optimal). This means that a connected routing topology can be constructed using links whose power consumption and delay estimation are less than l and l' respectively. In that case EDO algorithm would have terminated before processing l and l' and this leads to contradiction. Hence T_{edo} is the optimal topology.

Case(ii) Maximum power consumption and maximum delay lie on a same link:

Let l be the link of max power consumption and max delay on the topology T_{edo} obtained by EDO algorithm, incident on node v_i having *max PI*. The proof carries forward by assuming the contradicting situation in the same way as we did in case(i) which finally results that T_{edo} is the energy and delay optimal topology, obtained by EDO algorithm. $\square$

Theorem 5.6.3. Time complexity of EDO algorithm is $\mathcal{O}(n^2)$ where n is number of nodes.

Proof. Step 1 takes $\mathcal{O}(n^2)$ to compute weighted graph. Step 2 also takes $\mathcal{O}(n^2)$ to obtain MST using Prim's algorithm. Step 4 takes $\mathcal{O}(n)$ and steps 3,5,6 are done in constant time. Step 7 initiates the first while loop and executes the steps from 8 to 16 atmost $\mathcal{O}(n)$ time. Step 8 of finding S and T takes $\mathcal{O}(\text{degree}(v_k))$ and maximum complexity from steps 9 to 14 can be $\mathcal{O}(n)$ since finding LMV and RMV of any component can take $\mathcal{O}(n)$. Steps 15 and 16 take constant time and here's the end of first while loop. Now step 18 is done in $\mathcal{O}(n)$ time and steps 19 to 21 take constant time. Step 22 initiates the second while loop and executes the steps from 23 to 29 atmost $\mathcal{O}(n)$. Steps 23 and 24 can take $\mathcal{O}(n^2)$ to find intersection and adjacency of two distinct components. Step 25 takes $\mathcal{O}(|V(C_i)|) + \mathcal{O}(|V(C_j)|)$ where $|V(C_i)|$ and $|V(C_j)|$ depict the number of nodes in components C_i and C_j respectively. Step 26 takes $\mathcal{O}(|V(C_j)|)$ and step 28 takes $\mathcal{O}(|V(C_i)|)$. Final steps 31 and 32 can be done in $\mathcal{O}(n^2)$. Therefore the maximum

time complexity of EDO algorithm is $\mathcal{O}(n^2)$. □

5.7 Performance evaluation

Let us now evaluate the performance of EDO algorithm in term of topology delay minimization and energy efficiency by computing max node's power interval and percentage of energy saving and show the comparison with predefined algorithms, given in [2] and chapter 2 on interval data based graph models of WSNs. Algorithm, given in [2], constructs a spanning tree of minimum number of pendant vertices by joining disjoint path component of path cover, obtained by Arikati and Rangan in [1] on an interval graph. The path cover obtained in [1] is called optimal in term of minimum cardinality (minimum number of path components) of the obtained path cover.

The Algorithm 2&3, introduced in chapter 2 provides energy optimal path cover and energy efficient connected routing topology which minimizes max node's power interval and max link's transmission cost of worst case of power consumption. This routing topology is more energy efficient than the spanning tree obtained in [2] on interval graphs.

Here we will show that EDO algorithm outperforms both the algorithms on S-IDGMs of WSNs in simultaneous minimization of topology delay and nodes' power consumption interval using simulation results, graphical plotting and statistical t -test analysis. Using t -test, we show that topology formed by EDO algorithm has no significant difference in optimizing power consumption to save network energy, comparing with the energy efficient topology obtained by energy optimal Algorithm 2.2 and 2.3.

In addition, comparison of EDO algorithm with delay minimum spanning tree where the weight on each edge is computed using Eq. (5.4) and MST has been obtained using Prim's algorithm as for conveniency we call it DE MST, has also been discussed. No doubt, this DE MST is the most optimal in minimizing max topology delay and total topology delay still our evaluation shows that DE MST is not as energy efficient as the topology formed by EDO algorithm.

5.7.1 Simulation environment

We run the simulation on large number of WSNs whose underlying graphs are interval graphs in Spyder (Python 3.7) to show the required comparison. Input of arbitrary interval graphs are randomly generated in term of number of nodes, number

of edges, list of edges and list of all intervals in increasing order of their upper bounds. The data of each interval graph has been given to the python code of EDO algorithm to evaluate a connected routing topology and compared it with the topology formed by using algorithm given in [2], Algorithm 2.2 and 2.3 and DE MST.

Simulation parameter α of EDO algorithm: The variable α when approaching to 1 gives more weightage to link level interference and α approaching to 0 gives more weightage to link distance by Eq. (5.3). After observing the relationship between link distance and link interference on distinct WSNs towards optimality in section 5.4, α is fixed to a constant 0.5 in simulation work of EDO algorithm in order to consider both the metrics of delay estimation at equal weightage.

5.7.2 Metric 1: Topology delay minimization

The obtained data after simulation on maximum topology delay and total topology delay of the routing topology by each algorithm has been noted down in Table 5.2. It has been seen from the data given in Table 5.2 and their corresponding plotting in Fig. 5.8 that EDO algorithm performs better in minimizing maximum topology delay than Algorithm 2.2 and 2.3 and algorithm given in [2]. Moreover, data plotting in Fig. 5.9 depicts that EDO algorithm performs better in minimizing total topology delay than Algorithm 2.2 and 2.3 in each case. Comparing with Algorithm [2], EDO algorithm minimizes more total topology delay in dense networks. Further, we use two-tailed t -test to show that EDO algorithm optimizes topology delay at 2% level of significance, comparing with DE MST where DE MST has been optimal in minimizing topology delay.

t -test on the data given in Table 5.2: t -test compares mean of data obtained by two different methodologies where results which are obtained on samples, are believed to exist on large number of networks (or we call it population of S-IDGMs).

To examine the performance of EDO algorithm with respect to DE MST through 'two tailed t -test', two types of hypothesis have to make. The first hypothesis is called null hypothesis comprises of a statement which we want to claim/proof and the other one is called alternate hypothesis whose statement is against the claim. Presently our claim for a null hypothesis is that the topology delay minimization by EDO algorithm is as optimal as DE MST. For this to test, null hypothesis becomes 'mean of data by

Number of nodes	Number of edges	Maximum Topology Delay				Total Topology Delay			
		DE MST	EDO algorithm	Algorithm 2.2 and 2.3	Algorithm in [2]	DE MST	EDO algorithm	Algorithm 2.2 and 2.3	Algorithm in [2]
6	9	4.923	4.923	4.997	4.923	15.345	15.606	18.081	15.606
9	27	6.931	6.931	7.970	7.89	38.154	42.898	56.615	48.6
11	30	7.445	8.445	9.722	9.577	45.595	52.083	81.405	55.646
13	38	9.566	11.566	11.793	11.614	58.875	87.303	95.438	92.79
15	49	11.336	11.512	13.6	13.6	95.002	116.324	145.06	136.54
18	100	11.7	11.7	16.923	16.915	91.626	126.336	253.425	176.82
20	120	17.479	18.479	18.602	18.586	121.793	246.42	294.52	214.1
23	129	17.557	17.557	21.59	20.653	142.275	265.92	340.22	228.676
25	141	17.614	17.614	23.424	19.542	178.243	281.27	378.288	303.522
28	202	14.3	16.468	27.046	27	170.4	302.278	503.12	419.67
30	268	11.373	20.44	28.84	28.614	177.744	232.017	600.5	456.34
32	291	13.282	23.35	30.856	29.714	226.422	305.87	761.024	674.07
35	307	17.685	18.31	33.712	33.67	244.72	364.86	740	634.65
37	304	26.786	28.786	35.667	33.582	288.433	672.15	856.106	698.47
39	433	12.363	23.853	38.1	38	237.133	406.834	1041.672	668.4
41	461	10.305	16.367	40	38.654	246.535	355.252	1065.673	774.827
43	436	20.513	20.513	41.819	37.58	306.282	461.506	1192.872	796.134
45	479	14.3	20.368	42.758	42.854	340.684	412.144	1279.416	943.376
48	608	15.178	22.446	46.847	45.817	333.56	666.5	1483	1100
50	608	16.27	18.22	49	48.757	384.235	486	1679.355	1324.142

Table 5.2: Data obtained on max topology delay and total topology delay after simulation

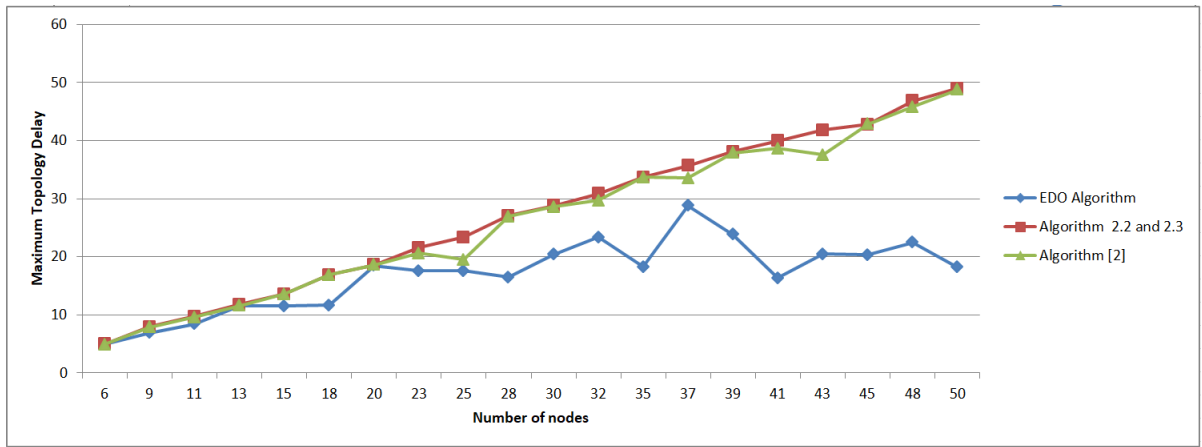


Figure 5.8: Max Topology Delay with network density

EDO algorithm is equal to the mean of data by DE MST' and alternate hypothesis contradicts the null hypothesis and becomes 'mean of data by EDO algorithm is not equal to the mean of data by DE MST'.

Mean of max TD by DE MST is 13.844 and mean of total TD by DE MST is 187.175 on the networks, given in Table 5.2.

Let μ_0 be the mean of max TD and μ_1 be the mean of total TD by EDO algorithm on population of S-IDGMs.

Null hypothesis H_0 : $\mu_0 = 13.844$ and $\mu_1 = 187.175$.

Alternate hypothesis H_a : $\mu_0 \neq 13.844$ and $\mu_1 \neq 187.175$.

Now computing t -values for both the max TD and total TD using formula $t_0 = \frac{\bar{x}_0 - \mu_0}{SD_0 / \sqrt{n}}$ and $t_1 = \frac{\bar{x}_1 - \mu_1}{SD_1 / \sqrt{n}}$ respectively where n is number of sample networks, $\bar{x}_0$ is mean of max TD , SD_0 is standard deviation of max TD by EDO algorithm on the networks, given in Table 5.2. Likewise $\bar{x}_1$ is mean of total TD and SD_1 is standard deviation of total TD by EDO algorithm on the networks, given in Table 5.2.

Therefore $n = 20$, $\bar{x}_0 = 16.89$, $\bar{x}_1 = 294.978$, $SD_0 = 6.978$ and $SD_1 = 190.688$. Substituting

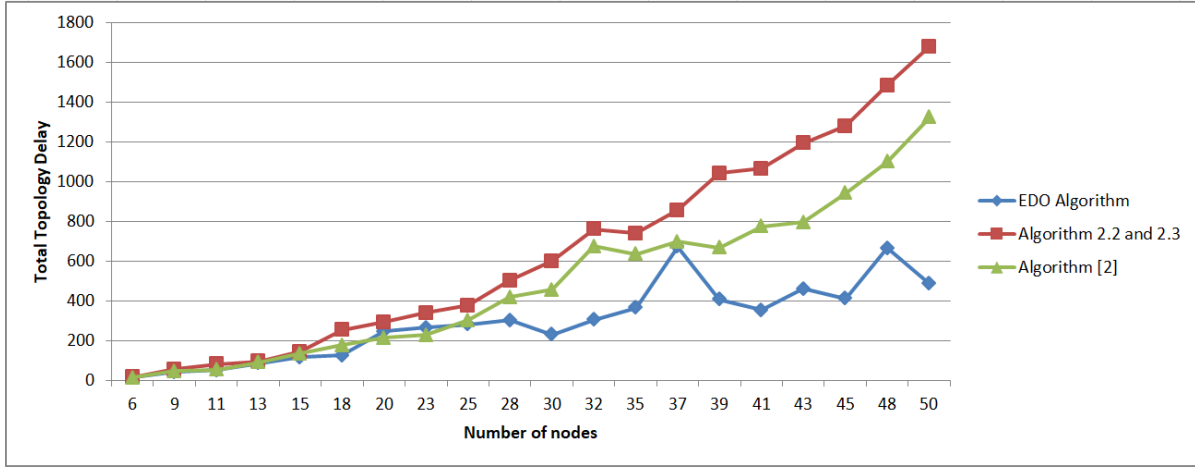


Figure 5.9: Total Topology Delay with network density

these values in the formulas (given above) result $t_0 = 2.23395$ and $t_1 = 2.52827$. Hence the null hypothesis is accepted for both μ_0 and μ_1 with 2% level of significance. It can be said that topology delay minimization by EDO algorithm is as optimal as DE MST at 2% level of significance.

Remark 5.7.1. t -values of Algorithm 2.2 and 2.3 and Algorithm [2] for max TD come-out to be 4.36 and 4.20 respectively, are significantly different from optimal value at 2% level of significance. Also t -values of Algorithm 2.2 and 2.3 and Algorithm [2] for total TD come out to be 4.0 and 3.536, are again significantly different from optimal value at 2% level of significance. Therefore overall performance of EDO algorithm in optimizing TD is much better than Algorithm 2.2 and 2.3 and Algorithm [2] on interval data based graph models of WSNs.

5.7.3 Metric 2: Energy efficient topology

The obtained data after simulation on max node power interval and percentage of energy saving on the topology by each algorithm has been noted down in Table 5.3. It has been remarked already that percentage of energy saving is directly depending on max node power interval; more is the upper bound of PI lesser is the energy saving. Fig. 5.10 which shows the percentage of energy saving by each respective algorithm, also depicts that more energy saving by any algorithm implies lesser maximum power consumption and this can be seen in Table 5.3 as well.

Data in Table 5.3 and plotting in Fig. 5.10 are explicitly showing that EDO algorithm performs better in minimizing max node power consumption than DE MST and algorithm, given in [2] in each case and so EDO algorithm conserves more energy than DE MST and algorithm, given in [2] on interval data based graph models of WSNs.

However, it has been observed that in some networks EDO algorithm conserves more energy than Algorithm 2.2 and 2.3 while in other networks EDO algorithm conserves equal and lesser energy than Algorithm 2.2 and 2.3. This is because Algorithm 2.2 and 2.3 has been an energy optimal in providing connected routing topology on interval graphs without considering delay factors and EDO algorithm which works to obtain an energy and delay minimized connected routing topology, considers DE function depending on link interference and distance factor which helps in minimizing topology delay and power consumption simultaneously. Since tradeoff between energy consumption and delay minimization brings fluctuation in energy saving while minimizing topology delay. Therefore to evaluate the performance of EDO algorithm in compare to the Algorithm 2.2 and 2.3, two-tailed $t - test$ has been used on the obtained data, given in Table 5.3 to show that topologies formed by EDO algorithm and Algorithm 2.2 and 2.3 have no significant difference in saving nodes' energy.

Number of nodes	Number of edges	Max node power interval				Percentage of energy saving			
		DE MST	EDO algorithm	Algorithm 2.2 and 2.3	Algorithm in [2]	DE MST	EDO algorithm	Algorithm 2.2 and 2.3	Algorithm in [2]
6	9	[155,312]	[155,230]	[155,230]	[161,312]	0	26	26	0
9	27	[16,133]	[42,116]	[16,116]	[22,116]	4	16	16	16
11	30	[81,169]	[103,169]	[55,131]	[103,193]	20.3	20.3	38	9
13	38	[123,176]	[72,162]	[72,162]	[121,236]	31	36	36	7
15	49	[68,227]	[9,189]	[9,246]	[93,277]	23	35.5	16	5.5
18	100	[87,226]	[87,207]	[47,190]	[84, 263]	21	28	34	8
20	120	[197,369]	[10,366]	[10,366]	[319,394]	8	8.5	8.5	2
23	129	[86,371]	[99,349]	[99,349]	[272,349]	7	12.5	12.5	12.5
25	141	[226,478]	[273,457]	[273,478]	[226,481]	1	5	1	0
28	202	[165,408]	[165,375]	[24,404]	[165,446]	9.5	16.9	10.4	1
30	268	[332,492]	[12,477]	[15,485]	[93,560]	18	20.5	19.2	6.7
32	291	[308,569]	[350,543]	[41,534]	[308,581]	6.9	11.13	12.6	5
35	307	[516,663]	[479,663]	[455,663]	[516,679]	5	5	5	2.6
37	304	[44,646]	[328,612]	[544,619]	[359,694]	9.3	14	13	2.5
39	433	[486,725]	[412,671]	[412,671]	[156,771]	6.2	13.2	13.2	0.3
41	461	[287,746]	[282,696]	[287,696]	[270,806]	8.13	14.3	14.3	0.71
43	436	[546,825]	[548,807]	[396,803]	[548,825]	4	6.1	6.5	4
45	479	[205,875]	[205,875]	[351,823]	[367,894]	2.2	2.2	8	0.1
48	608	[641,927]	[16,803]	[568,919]	[641,940]	3	16	4	1.7
50	608	[513,942]	[443,883]	[163,870]	[604,959]	5.5	11.4	12.7	4

Table 5.3: Data obtained on max node power interval and % of energy saving after simulation

t -test on the data given in Table 5.3: Routing topology formed by Algorithm 2.2 and 2.3 has been energy optimal and we claim that topology formed by EDO algorithm is as energy optimal as by Algorithm 2.2 and 2.3 while optimizing topology delay. So null hypothesis becomes 'Mean of percentage of energy saving by EDO algorithm is equal to the mean of percentage of energy saving by Algorithm 2.2 and 2.3' and alternate hypothesis contradicts the null hypothesis and becomes 'Mean of percentage of energy saving by EDO algorithm is not equal to the mean of percentage of energy saving by Algorithm 2.2 and 2.3'.

Mean of percentage of energy saving by Algorithm 2.2 and 2.3 is 15.36 on the networks, given in Table 5.3 and let μ_0 is the mean of percentage of energy saving by EDO algorithm on population of S-IDGM.

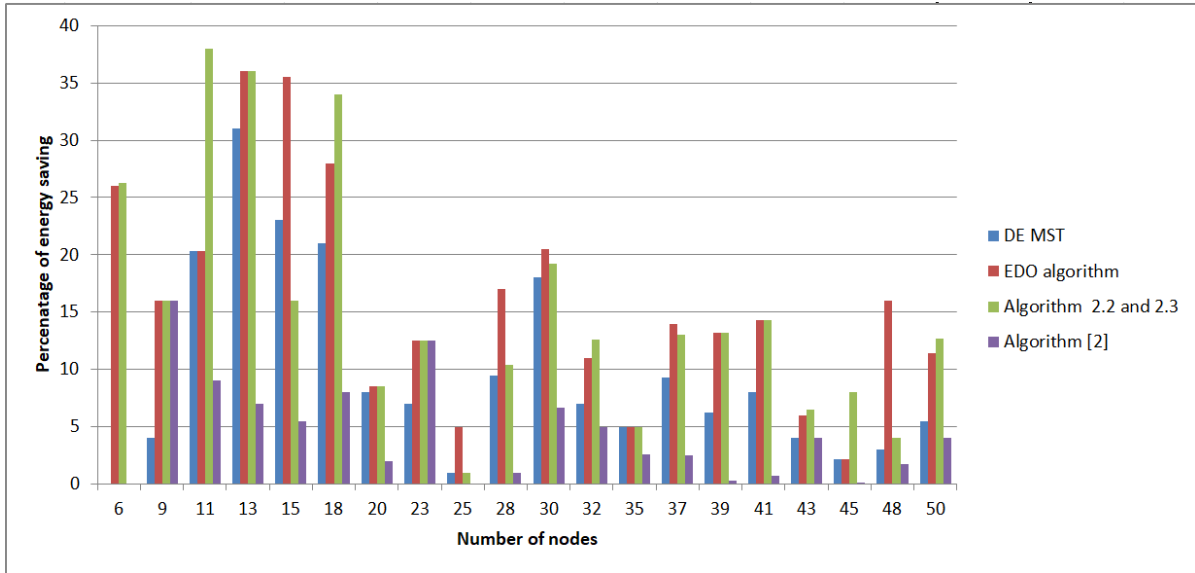


Figure 5.10: Percentage of energy saving

Let the null hypothesis $H_0: \mu_0 = 15.36$ and alternate hypothesis $H_a: \mu_0 \neq 15.36$.

Mean of percentage of energy saving by EDO algorithm on the networks, given in Table 5.3 is 15.92 i.e. $\bar{x} = 15.92$ and standard deviation is $SD = 9.53026$. Computing t -value using formula $t = \frac{\bar{x} - \mu_0}{SD/\sqrt{n}}$ where n is the number of sample networks, in this case $n = 20$. Substituting the values in the given formula results $t = 0.2628$. Hence the null hypothesis is accepted with 5% level of significance. This shows that there is no significant difference in minimizing maximum nodes' power interval and percentage of energy saving between EDO algorithm and Algorithm 2.2 and 2.3. It can be said that EDO algorithm results energy optimal topology while minimizing topology delay on S-IDGMs of WSNs.

Remark 5.7.2. t -values of DE MST and Algorithm [2] for average percentage of energy saving come out to be -3.0381 and -11.12 respectively, are significantly different from optimal value at 5% level of significance. Therefore overall performance of EDO algorithm in optimizing max power consumption is much better than DE MST and Algorithm [2] on interval data based graph models of WSNs.

5.8 Conclusion

In this work, we have proposed an energy and delay optimizing polynomial time algorithm, named as EDO algorithm, to construct an energy and delay optimized connected routing topology on heterogenous bidirectional connected WSNs where nodes are allowed to vary their transmission range within their assigned power interval. A

sorted interval data based graph model (S-IDGM) has been introduced to show nodes power interval explicitly and proposed EDO algorithm runs on S-IDGM to result an optimized connected topology which minimizes max topology delay, total topology delay and max nodes' power consumption interval. Minimization of max nodes' power interval consequently results efficient energy saving on S-IDGMs. Additionally, dependency analysis between link distance and link interference has been shown which helps to formulate a new weighted 'Delay estimation (DE)' function towards the optimality of EDO algorithm.

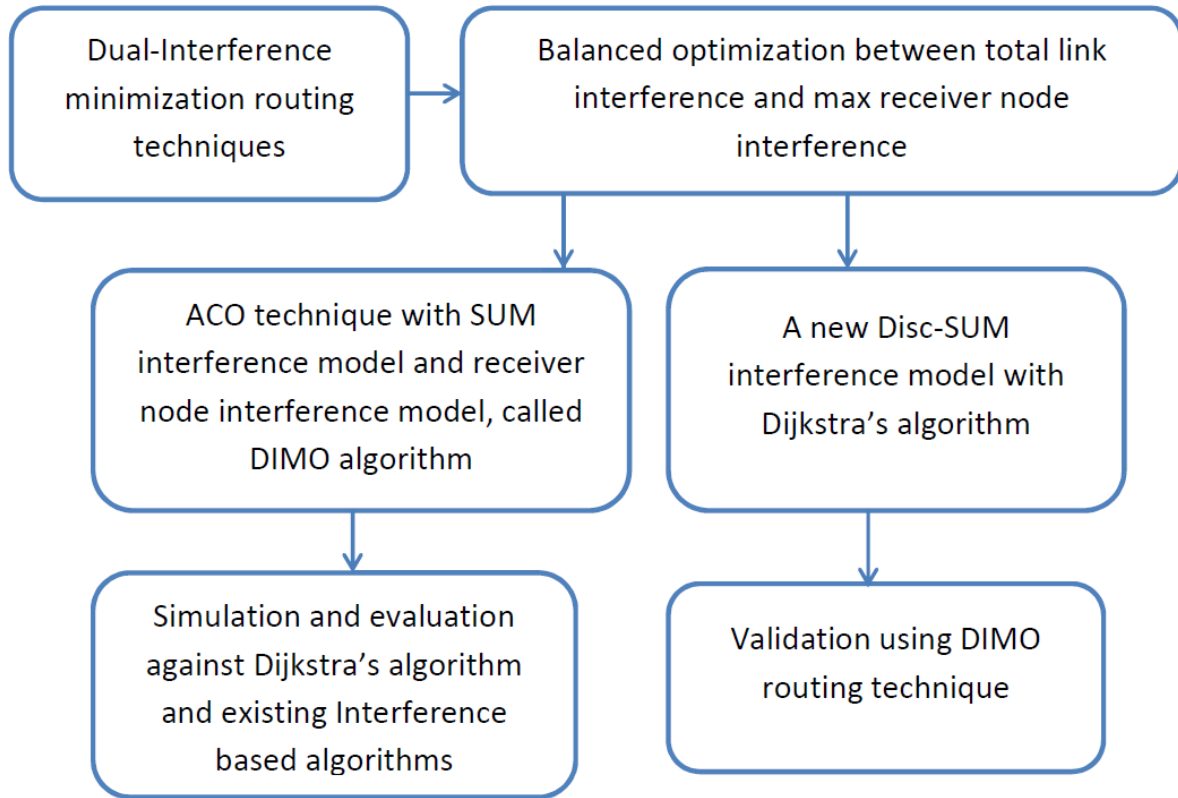
Chapter 6

Dual-interference minimization routing techniques in WSNs

In this chapter¹, we have introduced two new methodologies to minimize dual-interference as interference minimization whether on topology or routing level has been a prominent concern in WSNs. This chapter focuses on establishing an efficient routing path from the source to the destination node of minimum total link interference and maximum receiver node interference by optimally balancing between both the interference levels on any bidirectional connected graph model of WSN. We have proposed a dual-interference model based optimization (DIMO) routing algorithm by engaging ant colony optimization technique with two distinct interference models simultaneously that are link SUM interference (I_{SUM}) model and receiver interference (I_R) model. Moreover, a new interference model named as Disc-SUM model has also been developed whose implementation with Dijkstra's algorithm results in a dual-interference minimized routing path in quadratic time complexity. Simulation work has been carried out to validate the DIMO algorithm and the Disc-SUM model for implementing a routing path between the source to the destination node with optimally reduced total link interference, maximum receiver node interference, total transmission cost, and hop counts.

¹The content of this chapter is based on a research paper “Dual-interference minimization routing techniques in wireless sensor networks”, *Wireless Networks* (2023). <https://doi.org/10.1007/s11276-023-03602-3> (SCIE, Impact Factor: 3)

Graphical abstract of the chapter:



6.1 Introduction

The trend of WSN has been growing very rapidly with time as it has got a wide range of applications in various fields like power monitoring, inventory location monitoring, military surveillance, hospital services, tracking objects, animals, humans, and vehicles, etc. These applications are functioned by numerous battery-powered, light weight and omnidirectional wireless sensors when they are deployed in any target area [9, 14]. These sensor nodes sense information from any target area and pass on the sensed data to other nodes till the collective information in the form of signals reached to a destination node [137]. This leads to frequent data transmission from the source node to the destination node via relay nodes and so, multiple signal receptions to receiver nodes cause lots of data collision and retransmission in the network. The collision of data packets leads to transmission hindrance, and retransmission which is called interference in WSNs. Interference contributes to more power consumption of each sensor node in WSNs where sensors are power constraint and non-rechargeable [18, 40]. In history, many researchers have attempted their best to get rid of network interference by developing many distinct interference models to

measure hindrances/obstacles existing on links and nodes during data transmission in WSNs. These interference models have been further engaged to construct the interference-reduced topology of sensors through some interference minimization techniques, and algorithms [3, 12, 19, 38, 126, 138].

Existing interference minimization algorithms primarily focus on developing a topology of minimum interference by connecting all sensor nodes yet this does not bring lesser interference at both the link and node level when transmission happens from source to destination node on the connected topology. Also existing interference minimization algorithms for route determination between source to destination node work on single interference model at a time and achieve optimality either at link or node level. This motivates us to bring new routing techniques for determining an efficient path between source to destination node under distinct interference models to achieve simultaneous reduction of max receiver node interference and total path interference during transmission on bidirectional connected graph models of WSNs. So we have proposed a dual-interference minimization (DIM) approach by introducing ant colony optimization technique (ACO) with SUM interference model [3] and receiver interference model [19]. The proposed routing technique is named as DIM-ACO algorithm finds an efficient routing path between source to destination node with optimal reduction of both the max receiver node interference and total path interference on bidirectional graph models of WSNs. This is very first time that ACO technique has been engaged to find interference minimum routing path under two distinct interference models simultaneously. A simulation work has been done to validate DIM-ACO routing algorithm on the proposed ground by comparing it with interference minimum path by Dijkstra's algorithm and interference minimum topology by algorithm, introduced in [3].

Existing interference minimization algorithms primarily focus on developing a topology of minimum interference by connecting all sensor nodes yet this does not bring lesser interference at both the link and node level when transmission happens from the source to the destination node on the connected topology. Also existing interference minimization algorithms for route determination between the source to the destination node work on a single interference model at a time and achieve optimality either at link or node level. This motivates us to bring new routing techniques for determining an efficient path between the source to the destination node to achieve a simultaneous reduction of maximum receiver node interference and total link interference during transmission on bidirectional connected graph models of WSNs.

Briefing the chapter contribution:

1. For the very first time, ACO technique has been introduced with two distinct interference models to result in dual-interference reduced routing path from the source to the destination node. The technique is named as DIMO algorithm.
2. DIMO algorithm simultaneously minimizes max receiver node interference and total link interference on bidirectional routing paths from the source to the destination node.
3. A new link interference model named as Disc-SUM model has been introduced to compute interference on each bidirectional link based on transmission discs interruption for the very first time.
4. Another routing algorithm has been proposed with an implementation of the Disc-SUM model with the Dijkstra's algorithm, named as DSD algorithm which not only minimizes total link interference but also minimizes max receiver node interference in quadratic time complexity.
5. Proposed work validation has been shown via simulation, graphical, and statistical t -test analysis on the ground of total link interference, max receiver node interference, transmission cost, and hop counts minimization.

The organization of the chapter is as follows, section 6.2 defines the problem behind the proposed work. Then section 6.3 explains the methodology of the proposed DIM-ACO routing technique. Description of Disc-SUM model and DSD algorithm have been given in section 6.4. Simulation and evaluation is given in section 6.5 and section 6.6 concludes the chapter.

6.2 Problem Definition

On any wireless sensor network whose graphical layout is a bidirectional connected graph, a transmission route from the source to the destination node of minimum interference at both the link and node level is required to find. So, the problem is stated as follows.

Dual-interference minimization routing problem (DIMR problem): Find an

efficient routing path from the source to the destination node such that the resultant path has a minimum of maximum receiver node interference with a simultaneous reduction in total link interference on bidirectional connected graph models of WSNs.

To deal with DIMR problem, two new approaches and a new interference model have been proposed and hence we have accomplished this via two distinct ways:

1. Introducing ACO-technique with link and node interference models simultaneously.
2. Proposal of a new interference model and engaging it with Dijkstra's algorithm.

Basic concept of ACO-technique has been explained in definition 1.16 and Dijkstra's algorithm has been defined as follows.

Definition 6.1. Dijkstra's algorithm: It is a polynomial time algorithm used to find a shortest path between the source to the destination node on any directional/bidirectional graph where each edge of the graph has been weighted with the Euclidean distance (or any positive weight).

In this chapter, well known Dijkstra's and ACO routing techniques have been engaged on bidirectional graph models of WSNs where each edge/link has been weighted with interference instead of distance.

6.3 Dual-interference model based optimization (DIMO) technique

The first proposed technique is named as DIMO routing algorithm, given by Algorithm 6.1, introduces ACO with a new probability and pheromone update function under two distinct interference models that are I_R (explained in definition 1.15) and I_{SUM} (explained in definition 1.12) models. The probability function is given by Eq. (6.1).

DIMO routing algorithm has been proposed in which ACO technique is introduced with a new probability function based on two distinct interference models that are receiver node interference I_R (explained in definition 1.15) and SUM interference I_{SUM} (explained in definition 1.12) models. The new probability function is given by Eq. (6.1).

$$P_{ij}^k(t) = \frac{\tau_{ij}^\alpha(t) \times (\frac{1}{I_R(j)})^\beta}{\sum_{il \in E(G)} \tau_{il}^\alpha(t) \times (\frac{1}{I_R(l)})^\beta} \quad (6.1)$$

where $P_{ij}^k(t)$ calculates the probability for k^{th} ant to pick its next move from i^{th} to j^{th} node for data transmission between the source to the destination node at iteration t .

Algorithm 6.1 DIMO algorithm

Input: Bidirectional connected graph $G(V, E)$ with $|V| = n$, source node $v_s = v_1$ and destination node $v_d = v_n$

Output: DIM-routing path from v_s to v_d i.e. $DIMP(v_s \rightarrow v_d)$

- 1: Define a set of unvisited nodes as $UncoverV = V \setminus \{v_1\}$
- 2: Initialise ACO by setting number of ants = m , iteration = IN and $\tau_e(t) = 1 \forall e \in E$
- 3: **for** $t \in [1, IN]$ **do**
- 4: Set α , β and ρ in $[0, 1]$
- 5: **for** $k \in [1, m]$ **do**
- 6: Evaluate $P_{v_s v_j}^k(t)$ for each $v_s v_j \in E$ and $v_j \in UncoverV$ using Eq. (6.1)
- 7: Select v_j as next node whose $P_{v_s v_j}^k(t)$ is maximum
- 8: **if** $v_j \neq v_d$ **then**
- 9: $v_s \leftarrow v_j$ and go to step 6
- 10: Return $Path(v_s \rightarrow v_d)$
- 11: Compute $\max\{I_R(v)\} \forall v \in Path(v_s \rightarrow v_d)$ using Eq. (1.3) and $\sum\{I_{SUM}(e)\} \forall e \in Path(v_s \rightarrow v_d)$ using Eq. (1.2)
- 12: **if** $\max\{I_R(v)\}$ and $\sum\{I_{SUM}(e)\}$ do not satisfy the required criteria of the DIMR problem **then**
- 13: Back to step 4
- 14: **if** $\max\{I_R(v)\}$ and $\sum\{I_{SUM}(e)\}$ satisfy the required criteria of the DIMR problem **then**
- 15: $DIMP(t) = Path(v_s \rightarrow v_d)$
- 16: Evaluate pheromone update on $DIMP(t)$ for next iteration using Eq. (6.2) and (6.3)

τ_{ij} represents the pheromone quantity on edge from i^{th} to j^{th} node based on link interference. $I_R(j)$ computes the receiver interference at each unvisited j^{th} node using Eq. (1.3). Parameters α and β are engaged to regulate decisions based on link interference and receiver node interference respectively by varying their value between 0 and 1.

Each ant selects a next node of higher probability using Eq. (6.1) to transmit its data on a bidirectional connected graph model $G(V, E)$ as a node with higher probability implies lesser interference on transmission between the sensors.

The significance of using t lies with pheromone update on paths followed by m ants in each iteration where a total number of iterations is denoted by IN in the algorithm.

The new definition of pheromone update function based on link interference model I_{SUM} is given by Eq. (6.2) and (6.3).

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k \quad (6.2)$$

$$\Delta\tau_{ij}^k = \frac{1}{I_{SUM}(ij)} \quad (6.3)$$

where $\rho \in [0, 1]$ is engaged to regulate the evaporation rate of previously deposited

pheromone in each iteration. $I_{SUM}(ij)$ computes link interference on each bidirectional link ij using Eq. (1.2). $\sum_{k=1}^m \Delta\tau_{ij}^k$ is the total value of pheromone deposit on an edge ij traveled by m number of ants in the current iteration. If some edge is traveled by 5 ants in a single iteration then the pheromone deposit on the edge is summed up to 5 times based on the Eq. (6.2) and (6.3). Also Eq. (6.2) and (6.3) reveal that the update of the pheromone in the next iteration depends on the pheromone quantity deposited till previous iterations. So initially when $t = 1$, the pheromone on each edge is assumed to be the same constant or 1.

Step-by-step explanation of the DIMO algorithm: Step 1 defines a set of unvisited nodes as $UncoverV$ which are not yet traveled or visited and initially all vertices except source node are unvisited. Step 2 initialises ACO technique with total number of desirable ants m , iterations IN and pheromone value 1 on all edges e . Step 3 initiates a for loop which iterates IN times and step 4 can choose any value of α, β, ρ in interval $[0, 1]$ during the execution of the algorithm. Step 5 starts the travel for m number of ants one-by-one from source v_s to destination v_d . Steps 6-15 result a routing path optimal to the DIMR problem when m ants complete their rounds from source-destination-source using Eq. (6.1), (6.2) and (6.3) at some α, β, ρ after comparing the various solutions to the DIMR problem at distinct values of α, β, ρ in each iteration $t \in [1, IN]$. Step 16 updates the pheromone quantity in each iteration for better and fast convergence of ants to a single solution. Hence, a resultant routing path with balanced minimization of max receiver interference and total link interference at IN^{th} iteration is considered as optimal.

Theorem 6.3.1. The time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$ where IN is number of iterations, m is number of ants and n is total number of nodes.

Proof. Step 1 and 2 can be done in constant time. Step 3 repeats at the most IN times. Step 4 takes constant time and step 5 runs atleast m times. The total time complexity from steps 6 to 10 is $\mathcal{O}(n^2)$. Steps 11 can be done in $\mathcal{O}(n)$ time. Steps 12-15 take constant time and step 16 takes $\mathcal{O}(n)$. Hence total time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$. $\square$

6.4 Proposed Disc-SUM interference model

DIMO algorithm may reach to higher time complexity due to presence of the IN and m variables. Hence to deal with the DIMR problem with lesser time complexity,

a new interference model named as Disc-SUM model has been proposed. The proposed model computes link interference based on a number of transmission discs intersecting or covering the link on a routing topology of WSN. Mathematically Disc-SUM model is given by Eq. (6.4) where $D(w, r_w)$ is a transmission disc of node w with transmission range r_w .

$$Disc-SUM(I_{uv}) = \{w \in V \setminus \{u, v\} \mid uv \in D(w, r_w) \text{ or } u/v \in D(w, r_w)\} \quad (6.4)$$

A topological illustration of the Disc-SUM model on bidirectional link uv is shown in Fig. 6.1 where the transmission disc of node x is covering the link uv and the transmission disc of nodes y, w, z are intersecting the link uv . All the interrupting transmission discs are shown using blue color.

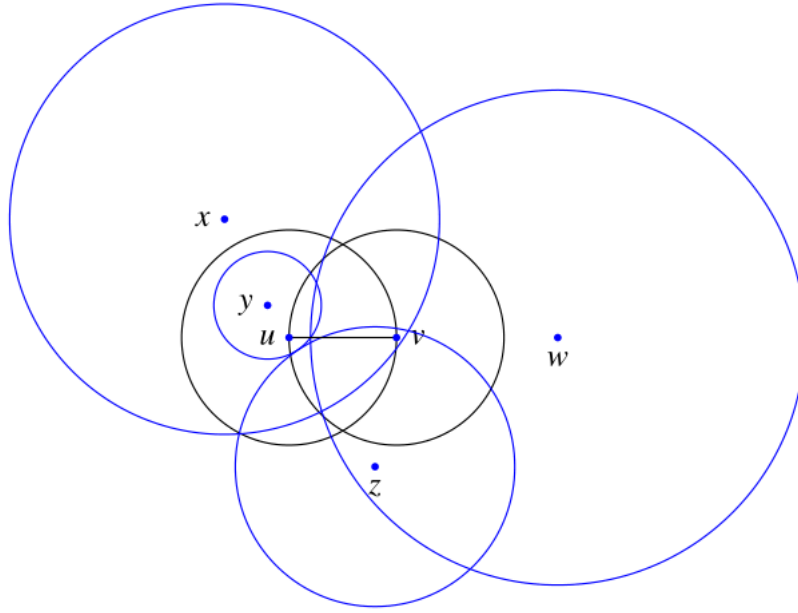


Figure 6.1: Topology of Disc-SUM interference model on bidirectional link uv

Remark 6.4.1. Interference computation by the Disc-SUM model on the bidirectional link uv given in Fig. 6.1 is found to be 4 whereas the SUM model introduced by Panda and Shetty in [3] computes interference quantity on uv is 1 only. It is because the model, given in [3] measures link interference by counting the number of nodes inside the transmission disc of u and v or both, and in the proposed model nodes outside the transmission disc of uv are also taken into account based on transmission discs interruption of the nodes to the link.

Remark 6.4.2. To the best of our knowledge, this is the first time that transmission disc interruption based link interference has been taken into account whose result will be shown on simultaneous minimization of maximum receiver node interference and total path interference, reduction of intermediate hop counts and total transmission cost in the subsequent sections.

6.4.1 Disc-SUM Dijkstra's (DSD) algorithm

Introducing Dijkstra's algorithm on bidirectional connected graph models of WSNs where each edge is weighted with link interference measured using the proposed Disc-SUM model. The approach of finding a dual-interference minimum routing path from the source to the destination node using the Dijkstra's algorithm with the Disc-SUM model is called Disc-SUM Dijkstra's (DSD) algorithm.

Algorithm 6.2 Disc-SUM Dijkstra's (DSD) algorithm

Input: Bidirectional connected graph $G(V, E)$ with $|V| = n$, source node $v_s = v_1$ and destination node $v_d = v_n$.

Output: DIM-routing path from v_s to v_d .

- 1: Compute $Disc-SUM(I_e) \forall e \in E(G)$ using Eq. (6.4).
 - 2: Initiate Dijkstra's algorithm.
 - 3: Return $Path(v_s \rightarrow v_d)$
 - 4: Compute $\max\{I_R(v)\} \forall v \in Path(v_s \rightarrow v_d)$ and $\sum\{Disc-SUM(I_e)\} \forall e \in Path(v_s \rightarrow v_d)$.
-

Proposition 6.4.1. DSD algorithm has $\mathcal{O}(n^2)$ time complexity where n is number of nodes.

Theorem 6.4.1. DSD algorithm minimizes total link interference of a routing path from the source to the destination node.

Proof. Dijkstra's algorithm has been optimal in finding a path from the source to the destination node of minimum total weight and when weight is link interference by the Disc-SUM model then the DSD algorithm results in an optimal path of minimum total link interference. □

Remark 6.4.3. DSD algorithm minimizes max receiver node interference of a routing path from the source to the destination node which is shown in the subsequent section through simulation.

6.5 Simulation and evaluation

To evaluate the performance of the proposed DIMO and DSD algorithms in terms of simultaneous minimization of max receiver node interference and total link interference on a routing path from the source to the destination node, simulation work on hypothetical bidirectional connected graph models of WSNs has been carried out. On each bidirectional connected graph model $G(V, E)$ of n number of nodes, it is assumed that $v_1 \in V$ is a source node and $v_n \in V$ is a destination node under each simulation. We have opted Python language for simulations and analyzed the resultant data using graphical plotting and statistical t -test analysis in IBM SPSS.

6.5.1 Performance evaluation of the DIMO algorithm

To evaluate the performance of the DIMO algorithm, we have compared the data obtained by DIMO algorithm with the Dijkstra's algorithm, MISBT coverage algorithm, given in [3] and MRI-TPL algorithm, given in [4]. Benchmarks of performance evaluation are max receiver node interference i.e. $\max I_R$ and total link interference i.e. $\sum I_{SUM}$ of a routing path traced from the source to the destination node on bidirectional connected graph models of WSNs.

The significance of comparing our simulation results with Dijkstra's algorithm is lying in the fact that Dijkstra's algorithm always provides an optimal solution by minimizing total weights of the path traced. Further, comparison of DIMO algorithm with MISBT coverage algorithm [3] and MRI-TPL algorithm [4] is lying in the fact that both the algorithms have been developed to result an interference optimal topology under I_{SUM} and I_R model respectively. According to our benchmark, we have focused on a transmission route from the source to the destination node on the topologies obtained by the MISBT and MRI-TPL algorithms for comparison.

Table 6.1 shows the simulation results on the DIMR problem by the DIMO algorithm, Dijkstra's algorithm, MISBT coverage algorithm, and MRI-TPL algorithm on various graph models of WSNs. It can be observed from the data given in Table 6.1 that the DIMO algorithm results in better dual-minimization by better balancing between max receiver node interference and total link interference in comparison to Dijkstra's, MRI-TPL [4] and MISBT coverage algorithm [3]. Dijkstra's algorithm works on a single objective i.e. optimal minimization of total link interference, does not control the max receiver node interference on the transmission route. Also, MISBT and MRI-TPL

WSNs (nodes, links)	DIMO algorithm		Dijkstra's algorithm		MISBT algorithm, given in[3]		MRI-TPL algorithm, given in [4]	
	α, β, ρ	$\max I_R$ ΣI_{SUM}	$\max I_R$ ΣI_{SUM}	$\max I_R$ ΣI_{SUM}	$\max I_R$ ΣI_{SUM}	$\max I_R$ ΣI_{SUM}		
(7,10)	.5,.6,,3	3 4	6 3	6 3	6 3	6 3		
(8,23)	.9,.4,.2	19 3	19 3	36 7	36 8			
(9,22)	.5,.6,.2	16 7	25 4	25 10	24 8			
(10,28)	.7,.5,.2	28 6	35 3	35 7	28 6			
(11,31)	.6,.4,.2	15 10	18 9	15 10	18 10			
(12,40)	.6,.4,.2	27 8	33 5	39 12	33 10			
(13,51)	.5,.6,.4	21 9	31 6	31 6	38 9			
(14,36)	.5,.3,.3	17 12	24 9	18 12	24 11			
(15,58)	.5,.4,.3	57 8	73 5	73 6	73 12			
(16,77)	.5,.4,.3	48 11	52 7	52 13	54 15			
(17,64)	.6,.4,.3	42 17	42 15	57 28	42 22			
(17,94)	.6,.3,.2	48 6	57 5	68 24	68 25			
(18,100)	.6,.3,.2	58 17	75 9	74 10	74 10			
(19,100)	.7,.4,.2	50 10	83 9	92 12	83 10			
(20,124)	.6,.5,.2	72 14	89 12	103 16	89 11			
(21,114)	.6,.4,.2	68 6	87 5	68 6	86 8			
(22,114)	.5,.2,.2	71 25	76 19	88 26	97 24			
(23,149)	.8,.4,.2	81 18	114 17	121 31	97 26			
(24,167)	.6,.5,.2	97 8	112 5	112 7	112 7			
(25,158)	.8,.4,.4	103 26	144 22	121 42	103 37			
(26,150)	.6,.5,.3	80 29	99 21	143 34	166 29			
(27,186)	.7,.4,.1	85 23	115 18	160 28	171 44			
(28,171)	.8,.3,.1	81 22	117 19	125 23	81 24			
(29,207)	.8,.4,.3	133 20	200 17	160 18	225 36			
(30,238)	.7,.6,.3	165 24	216 18	230 38	190 41			

Table 6.1: Data obtained on max receiver node interference under I_R model and total link interference under I_{SUM} model via simulation

algorithms perform well in minimizing total link interference, not max receiver node interference.

Fig. 6.2 clearly shows that the DIMO algorithm is far better in minimizing max receiver node interference on routing path from the source to the destination node. Though minimization of total link interference by the DIMO algorithm is not as much effective as by the Dijkstra's algorithm yet it is better fulfilling our goal of the DIMR problem as seen from Fig. 6.3. Plots of Fig. 6.3 have been derived by subtracting the resultant data, given in Table 6.1, of the DIMO algorithm from the Dijkstra's algorithm. The difference on positive axis depicts that the DIMO algorithm outperforms the Dijkstra's by how much and difference on negative axis depicts that the DIMO is lacking behind by how much in compare to the Dijkstra's. Therefore it is clearly understood from Fig. 6.3 that difference of minimized max receiver node interference by the DIMO from the Dijkstra's is much larger than the difference of minimized total link interference by the

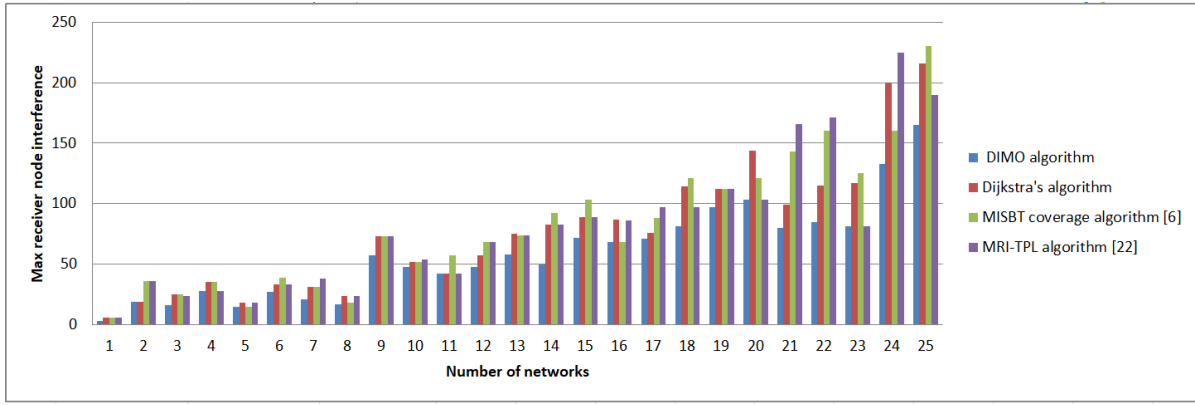


Figure 6.2: Comparison of max receiver node interference corresponding to the data given in Table 6.1

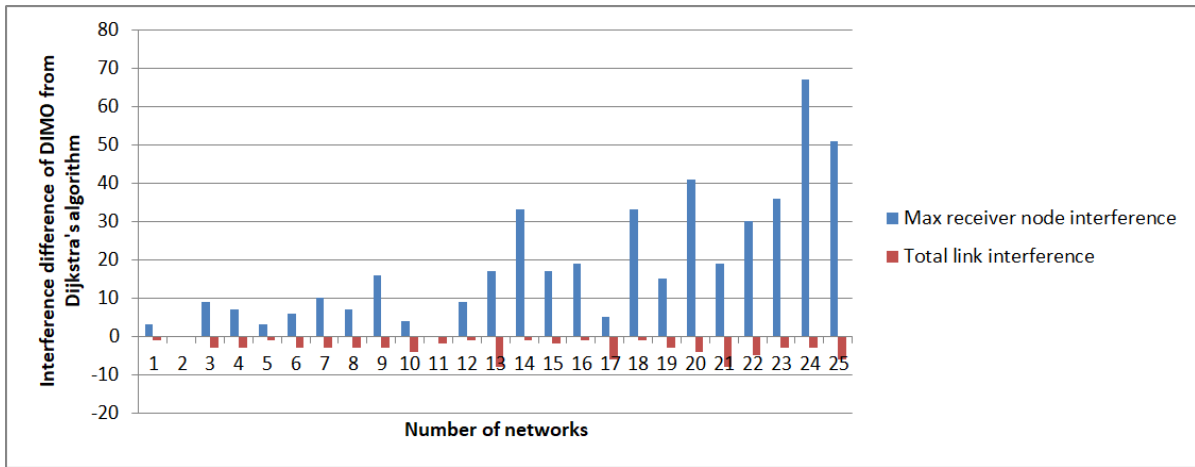


Figure 6.3: Difference of reduced max receiver node interference and total link interference of DIMO from Dijkstra's algorithm

Dijkstra's algorithm from the DIMO. It can be said that the routing path by the Dijkstra algorithm has a poor balance between the minimization of total link interference and max receiver node interference. Only focus on minimizing total link interference results in higher receiver node interference on the routing path as seen from Table 6.1, Fig. 6.2, and Fig. 6.3. Hence DIMO algorithm is more optimal in dealing with the DIMR problem than the Dijkstra's algorithm.

Likewise, Fig. 6.4 and Fig. 6.5 depict the difference of resultant data (given in Table 6.1) of the DIMO algorithm from the MISBT coverage algorithm [3] and the MRI-TPL algorithm [4] respectively. Plots of the figures clearly depict that the DIMO algorithm is not only better in minimizing max receiver node interference but also the total link interference of a routing path from the source to the destination node on various networks in comparison of the MISBT, and MRI-TPL algorithms.

Remark 6.5.1. The proposed DIMO algorithm outperforms Dijkstra's algorithm, MISBT coverage algorithm [3], and MRI-TPL algorithm [4] in dealing with the DIMR problem.

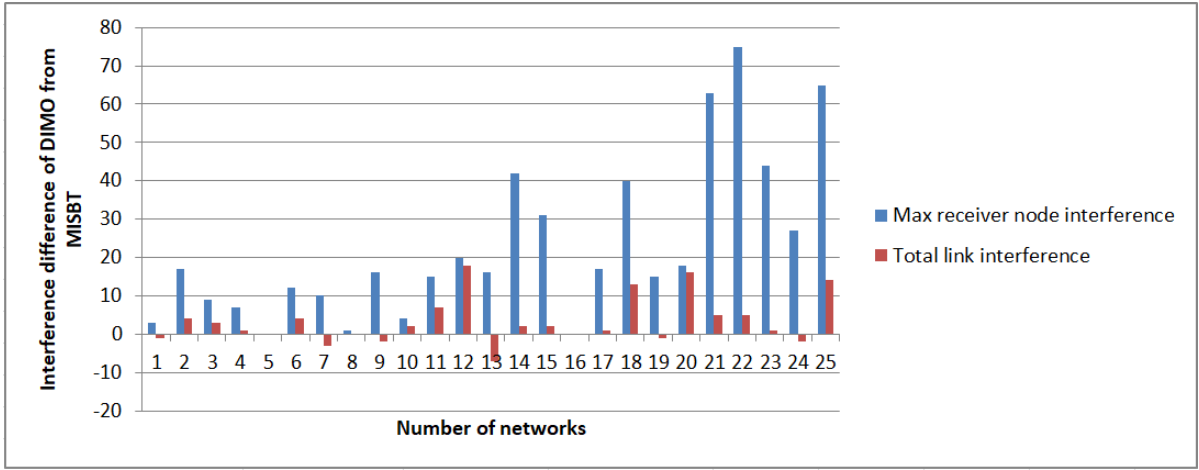


Figure 6.4: Difference of reduced max receiver node interference and total link interference of DIMO from MISBT coverage algorithm [3]

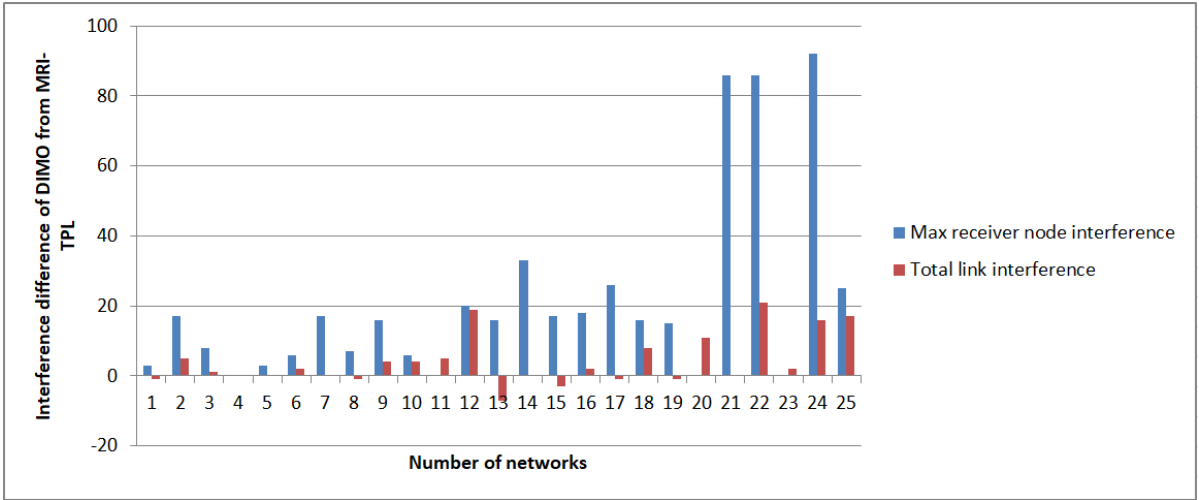


Figure 6.5: Difference of reduced max receiver node interference and total link interference of DIMO from MRI-TPL algorithm [4]

6.5.2 Performance evaluation of the DSD algorithm

Proposed DSD algorithm comprises of Disc-SUM model with Dijkstra's algorithm. As Dijkstra's algorithm results optimal path of total edge weight so it can be claimed that total link interference on routing path from source to destination node obtained using DSD algorithm would be optimal as well. Furthermore minimization of total link interference on routing paths is shown by evaluating total transmission cost as link interference directly effects the link's transmission cost [18]. For any bidirectional link uv of Euclidean distance ' d ', link transmission cost has been evaluated as $\|d^2\|$ [3]. Total transmission cost of each routing path by DSD algorithm has been evaluated and compared with DIM-ACO algorithm. Likewise optimality of max receiver node interference on each routing path by DSD algorithm is compared with DIM-ACO as

DIM-ACO has been validated as optimal in dealing with DIMR problem in previous section.

The proposed DSD algorithm comprises of the Disc-SUM model with the Dijkstra's

WSNs (nodes, links)	Total transmission cost		max receiver interference	
	DSD algorithm	DIMO algorithm	DSD algorithm	DIMO algorithm
(7,10)	165	180	3	3
(8,23)	6	6	19	19
(9,22)	14	28.53	16	16
(10,28)	34	30.42	26	28
(11,31)	211.8	211.8	15	15
(12,40)	87.5	87.5	27	27
(13,51)	100	100	21	21
(14,36)	128.33	102	17	17
(15,58)	22.45	22.45	57	57
(16,77)	102	102	48	48
(17,64)	226	94.8	42	42
(17,94)	79.6	79.6	48	48
(18,100)	128.3	128.3	58	58
(19,100)	51.7	51.7	50	50
(20,124)	101	98	31	72
(21,114)	15.5	15.5	68	68
(22,114)	171	171	71	71
(23,149)	121.4	121.4	81	81
(24,167)	19.73	19.73	97	97
(25,158)	141.73	141.73	103	103
(26,150)	100.33	144	105	80
(27,186)	86.53	86.53	85	85
(28,171)	94.77	94.77	81	81
(29,207)	76.53	76.53	133	133
(30,238)	103.64	79.3	165	165

Table 6.2: Resultant data on total transmission cost and max receiver node interference on routing paths via simulation

algorithm. As Dijkstra's algorithm results in an optimal path of minimum total edge weight so it can be claimed that the total link interference on a routing path from the source to the destination node obtained using the DSD algorithm is optimal as well. Furthermore, minimization of the total link interference on the routing paths are shown by evaluating total transmission cost as link interference reduction directly affects the link's transmission cost [18]. For any bidirectional link uv of Euclidean distance ' d ', link transmission cost has been evaluated as $\|d^2\|$ [3]. The total transmission cost on each routing path by the DSD algorithm has been evaluated and compared with the DIMO algorithm. Likewise, optimality of the max receiver node interference on each

routing path by the DSD algorithm has been compared with the DIMO algorithm as DIMO algorithm has been declared as optimal in dealing with the DIMR problem in the previous section.

Table 6.2 shows the evaluated total transmission cost as well as max receiver node interference on the resultant routing path using DSD and DIMO algorithms. It is observed from Table 6.2 that in some cases DSD algorithm performs better than DIMO algorithm and vice versa. Therefore, for better analysis of resultant data by the DSD algorithm and the DIMO algorithm on total transmission cost and max receiver node interference, paired sample '*t*-test' statistical tool has been used in IBM-SPSS software.

To examine the validation of the DSD algorithm through the '*t*-test', two types of hypothesis have to make. The first hypothesis is called the null hypothesis comprises a statement that we want to claim and the other one is called the alternate hypothesis whose statement is against the claim. Presently our claim for a null hypothesis is that the dual-interference minimization by the DSD algorithm is as optimal as the DIMO algorithm. For this to test, the null hypothesis becomes 'mean of data by the DSD algorithm is equal to the mean of data by the DIMO algorithm' and the alternate hypothesis becomes opposite to the null hypothesis i.e. 'mean of data by the DSD algorithm is not equal to the mean of data by the DIMO algorithm'. Fig. 6.6 and 6.7 show the paired sample *t*-test results on total transmission cost and max receiver node interference respectively, obtained in the IBM SPSS. In Fig. 6.6, resultant *p*-value in column Sig. (2-tailed) is .436, shows the significance level greater than .05 and so, the null hypothesis is accepted with a 5% level of significance. Hence total transmission cost by the DSD algorithm is as optimal as the DIMO algorithm. Likewise in Fig. 6.7, resultant *p*-value in column Sig. (2-tailed) is .716, greater than .05 implies that the null hypothesis is accepted with a 5% level of significance. Hence average max receiver node interference by the DSD algorithm is as optimal as the DIMO algorithm. Moreover, the effect of interference minimization by the DSD algorithm has been shown on hop counts in Fig. 6.8. Hop counts are the number of intermediate nodes on a routing path from the source to the destination node, and more the hop counts more is a delay in data delivery. Fig. 6.8 reveals that the routing path by the DSD algorithm has lesser hop counts than the routing path by the DIMO algorithm in many cases. Hence interference minimization by the DSD algorithm favors delay reduction as well.

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pair 1 DSD - DIMO	4.61000	29.10673	5.82135	-7.40467	16.62467	.792	24	.436

Figure 6.6: Paired sample t -test of DSD and DIMO algorithms on total transmission cost

		Paired Differences				t	df	Sig. (2-tailed)	
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower				Upper
Pair 1	DSD - DIMO	-.7200	9.7831	1.9566	-4.7583	3.3183	-.368	24	.716

Figure 6.7: Paired sample t -test of DSD and DIMO algorithms on max receiver node interference

Remark 6.5.2. DSD algorithm is as effective as DIMO algorithm in minimizing max receiver node interference and total link interference of a routing path from the source to the destination node on bidirectional connected graph models of WSNs.

Remark 6.5.3. The significance of using the DSD algorithm for the DIMR problem instead of the DIMO algorithm is lying on the ground of time complexity as the total time complexity of the DSD algorithm is lesser than the DIMO algorithm in dealing with the DIMR problem.

Remark 6.5.4. The total time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$ where IN is number of iterations, m is number of ants and n is number of nodes of a WSN whereas the total time complexity of the DSD algorithm is $\mathcal{O}(n^2)$.

Remark 6.5.5. The effect of interference minimization by the DSD algorithm is seen on reduction in hop counts on a routing path from the source to the destination node. DSD algorithm has lesser hop counts than DIMO algorithm and so, it can be said that DSD algorithm favors delay reduction along with transmission cost on a routing path from the source to the destination node on bidirectional connected graph models of WSNs.

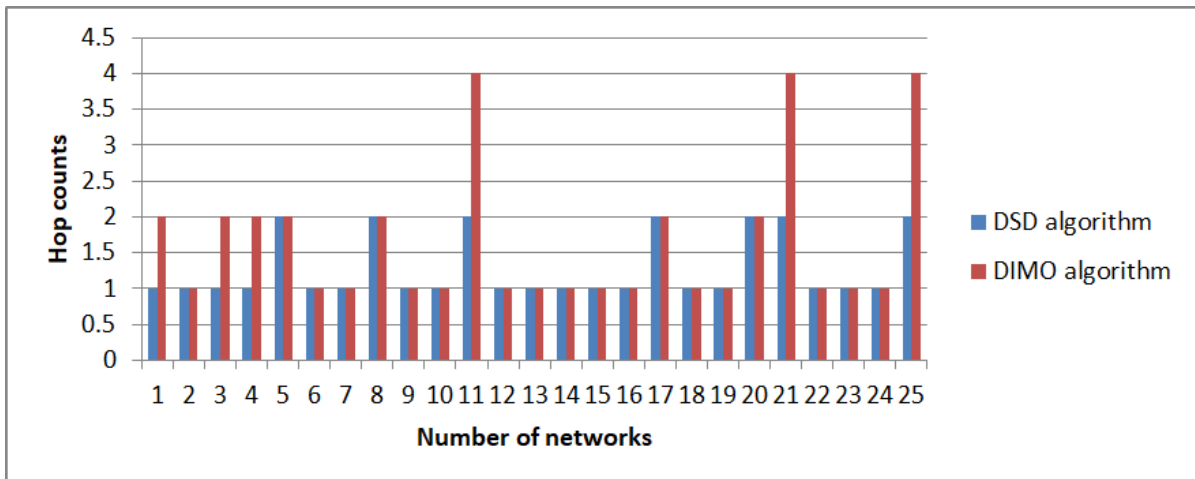


Figure 6.8: Comparison of hop counts from the source to the destination node

6.6 Conclusion

In this chapter, two new dual-interference minimization routing approaches have been proposed which simultaneously minimize total link interference and max node interference during data transmission from the source to the destination node on bidirectional connected graph models of WSNs. The first approach, named as DIMO algorithm engages ACO technique with two distinct interference models via probability and pheromone update function. This is the very first time that ACO has been introduced with interference models to minimize dual-interference in WSNs. The second approach is based on the newly proposed interference model named as DiscSUM model which provides new weights to each edge as link interference and use of Dijkstra's algorithm on the weighted graph results in a dual-interference minimum routing path from the source to the destination node in quadratic time complexity. The validation of the proposed techniques and model has been done by carrying out simulations. Data obtained via simulations has been analysed and compared against the algorithms from similar grounds. Analysis on minimization of max receiver node interference, total link interference, total transmission cost, and hop counts have been accomplished graphically and statistical t -test tool in IBM SPSS.

The proposed techniques are better in minimizing dual-interference on the routing paths by optimally balancing both interference levels during data transmission. In future, dual-interference minimization would be extended to dynamic routing paths under different interference models or by developing some new suitable models for dynamic WSNs.

Summary and future scope of the work

WSNs enable new applications with time and require non-conventional paradigms to design wireless sensors' topology and routing between the sensors in such a way that it can bring out efficient communication by negotiating the use of node's power since each sensor node is power constraint. By efficient communication, we mean that nodes should have that much amount of power so that they can sense, process and transmit the data to the required nodes in such a way that their communication whether at topology or routing level can deal with different problems and accomplish the proposed benchmark. These benchmarks are basically the prominent concerns/requirements of WSNs on which researchers have been working since past many years. Most of the prominent concerns that are energy conservation, nodes' power consumption, network lifetime, load balancing, topology connectivity, communication complexity, interference and delay, become a part of motivation for us to put huge efforts to develop energy and computationally efficient techniques and algorithms.

All the efforts which have been devoted by us to resolve or mitigate these concerns of WSNs, collectively given in this thesis from chapter 2 to chapter 6. We have adopted and designed graph theoretic approaches/algorithms to optimize underlying network of sensor nodes on both the topology and routing level.

In chapter 2, we have introduced a new graph model of WSNs where nodes are allowed to vary their transmission range from minimum to maximum power required by them during transmission. This min to max power requirement becomes a power interval, assigned to each node in such a way that these assigned power intervals satisfy the properties of being an interval graph and so a new 'right-end ordered interval graph' has been modelled in this chapter. Three power optimization algorithms have been proposed which work to result bidirectionally connected topology of sensor nodes by minimizing maximum node's power consumption as well as link's transmission cost of worst case situation within quadratic time complexity. To the best of our

knowledge, the proposed models and algorithms are the very first attempt in topology control of WSNs where nodes can vary their transmission range and optimization has been shown explicitly in the term of power interval.

The graph model and algorithms proposed in chapter 2, have been energy optimal and designed for right-end ordered interval graph models of WSNs where nodes are immobile in nature. Also the energy optimal topology has spanning tree-like structure so if any node gets die in the network, it would cause disconnectivity and transmission loss. Hence in chapter 3, we have considered the fact of nodes death in the network due to drainage of their power and their immobile behavior requires new algorithms, not restricted to work on interval graphs only, to reconnect the disconnected topology after the removal of dead node from the energy optimal topology. We have done sensitivity analysis on disconnectivity situations due to removal of any intermediate node from the energy optimal topology and change on optimal value of max power consumption on the reconnected topologies by newly proposed algorithms. We have explored and discussed 5 distinct disconnectivity situations under which the proposed algorithms result reconnected topologies of minimum effect on optimal value. Furthermore a methodology to connect disjoint components of a separable interval graph without its cut-vertex has been proposed in this chapter.

Then chapter 4 have brought-up the concept of entering/joining of new nodes to the energy optimal topology introduced in chapter 2, as some application of WSNs can require urgent addition of some new sensor nodes to the already existing topology. We have considered two types of nodes addition based on intersection of power interval; first one is intersecting node and second is non-intersecting node. As algorithms introduced in chapter 2 can join new node only when the existing topology along with new node makes right-end ordered interval graph which cause higher time complexity so new algorithms of lesser time complexity under different situation of nodes addition have been proposed. Also sensitivity analysis have been carried out to discuss the % effect on optimal value of max power consumption and it has been seen that proposed algorithms bring no or minimum change on optimal value even after joining multiple new nodes to the energy optimal topology.

The interval graph model proposed in chapter 2, has not been applicable to those WSNs who have not satisfied interval graph properties so a new and generalized graph model has been proposed in chapter 5 named as 'sorted-interval data based graph model (SDGM)'. In this chapter, we have considered topology delay and energy

consumption factors to minimize simultaneously on bidirectionally connected SDGM. For this to achieve, we have formulated a weighted function based on link interference and link length as both the factors contribute to delay and nodes energy consumption in the network. Also we have discussed the dependency analysis between link interference and link length towards optimality. A new energy and delay optimization algorithm has been proposed and explained with an illustration. Simulation work has shown the effectiveness of the proposed work in minimizing the topology delay and power consumption on various SDGMs.

Till chapter 5, we have discussed and investigated on topology control techniques in WSNs therefore in chapter 6 we have introduced two new routing techniques which minimize interference between source to destination node at both the link and node level along with transmission cost on any bidirectional connected graph models of WSNs. To the best of our knowledge, the very first time ACO technique has been engaged with two distinct interference models via probability function and pheromone update function and results an optimal balanced minimization between receiver node interference and total link interference. Also a new link interference model has been proposed which results a routing path of minimum of max receiver node interference and total transmission cost in polynomial time complexity by using Dijkstra's algorithm. This also results reduced hop counts along the route between source to destination node.

In our next work, we would be thinking to optimize dynamic WSNs in term of power, location error, reliability, interference and delay as in dynamic WSNs, nodes can move independently so it becomes difficult to access their correct location during transmission which may lead to transmission error, data delivery delay and so unreliability.

As graph based models of WSNs are less stable and less adaptable to nodes movement so we are thinking to explore other relevant theories which would give us more stable and adaptable models for WSNs to cope-up with dynamic behavior of wireless nodes in lesser time complexity.

Also the proposed methods in this thesis have been centralized in nature so we have been looking forward to introduce distributed methods/algorithms on the defined problems by considering more benchmarks to validate the work.

As delay and energy consumption factors are contradicting to each other when it comes to minimize them simultaneously hence more methods need to explore on both the routing level and topology level such that nodes energy can be saved with

minimum delay in transmission where delay can be taken on any other factors like sleep/wakeup cycle of nodes, hop counts, amount of data transmission etc.

Optimization in WSNs can be more complex and multi-variate with respect to distinct constraints and objectives therefore more robust methods using nature inspired techniques like ACO, particle swarm optimization (PSO) etc would be taken into account in future work.

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List of Publications

1. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *Interval graph based energy efficient routing scheme for a connected topology in wireless sensor networks*, Wireless Networks, Springer, **27**, 5085-5104 (2021) (**SCI, Impact Factor: 3**).
 2. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *Systematic study of topology control methods and routing techniques in wireless sensor networks*, Peer-to-Peer Networking and Applications, Springer, **15 (4)**, 1862-1922 (2022) (**SCIE, Impact Factor: 4.2**).
 3. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *Optimization of energy and delay on interval data based graph model of wireless sensor networks*, Wireless Networks, Springer, **29**, 2293-2311 (2023) (**SCI, Impact Factor: 3**).
 4. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *Dual-interference minimization routing techniques in wireless sensor networks*, Wireless Networks, Springer (2023). <https://doi.org/10.1007/s11276-023-03602-3> (**SCI, Impact Factor: 3**).
 5. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *Sensitivity analysis on interval graph based energy optimal spanning tree topology of wireless sensor networks with respect to node removal*, **Communicated**.
 6. **Radhika Kavra**, Anjana Gupta, Sangita Kansal; *New approaches of extending energy optimal topology on interval graph model of wireless sensor networks with sensitivity analysis*, **Communicated**.
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Papers Presented in International Conferences

- Presented a research paper entitled “**Energy optimized routing topology for a connected graph model of wireless sensor networks**” in *2nd International conference on Applied Mathematics in Science and Engineering (AMSE 2022)* held at the centre for Data Science, ITER, Bhubaneswar, India, during March 24-26, 2022.
 - Presented a research paper entitled “**A new routing approach based on interval graph to optimize power consumption in wireless sensor networks**” in *International conference on Graphs, Networks and Combinatorics (ICGNC 2023)* held at Ramanujan college, Delhi University, during 10-12 January 2023.
 - Presented a research paper entitled “**New approaches of extending energy optimal topology on interval graph model of wireless sensor networks with sensitivity analysis**” in *International Symposium on Applied Optimization and Game Theoretic Models for Decision Making* held at Indian Statistical Institute, Delhi centre, during February 01-03, 2023.
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Systematic study of topology control methods and routing techniques in wireless sensor networks

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Abstract

Emergence of applications of wireless sensor networks in various fields over time requires to acknowledge existing solutions which have been proposed in history to overcome various challenges/ issues related to topology formation and routing of sensing data among the sensors. Sometime data transmission requires topology establishment or its awareness and sometime it does not. In order to obtain optimal routing topology or an optimal route according to some desired aim, a wide range of topology control methods and routing techniques/ protocols come into sight. This article is the very first attempt in offering the collective and combined review on unexplored topology control methods and routing techniques, year-wise. Presenting a comprehensive study of various energy efficient topology control methods involving the graph based methodologies, explicitly designed interference models and related algorithms which have not been covered in previous year survey papers and the detail review on most efficient network structure based classical hierarchical routing protocols, topology aware particle swarm optimization and ant colony optimization based routing techniques in both static and mobile wireless sensor networks. However, the mentioned techniques don't cooperate delay sensitive routing or timely data delivery, for this purpose, we additionally provide a detail review on delay-sensitive real-time routing protocols in this survey paper. Analysis of each technique has been put forth year-wise by mentioning contribution made by the proposed work, detail of methodology opted, merits and demerits. A brief comparative study among the existing techniques, discussion about the research gaps and persisting issues along with direction of future work have also been provided.

Keywords Wireless sensor networks (WSNs) · Topology · Graph algorithms · Optimization techniques · α -approximate solution · Time complexity

1 Introduction

Wireless Sensor Networks (WSNs) can be defined as an infrastructure-less networks, made up of few tens to thousands of battery-powered omnidirectional wireless sensors. These wireless sensors work together to monitor a region to obtain data about the environment and cooperatively pass their data through the network to a main location (sink or base station) where the data can be observed and analysed. A sink or base station acts like an interface between users and the network. One can retrieve required information from the network by injecting queries and gathering results from the sink [1]. WSNs have gained worldwide attention in recent

years because of the facilitation of low-cost multi-functional wireless sensors that are small in size and communicate untethered using radio signals. Each sensor node consists of three subsystems: sensing, processing and transmission (see Fig. 1 redrawn from [2]). Each sensing subsystem has transducers in it to collect information about the environment. A processing subsystem has an embedded processor to process local sensed information and transmission subsystem has a transceiver embedded in it to exchange sensed/ processed information with external systems (base station). Some sensor nodes have two additional units for location finding and mobility so that they can carry out the tasks assigned to them [3]. So this way, sensors can be static or mobile depending on the application goals. These tiny sensor nodes, which consist of sensing, data processing and communication components, leverage the idea of WSNs based on collaborative effort of large number of sensor nodes.

Now-a-days wireless sensors are greatly engaged in tracking and monitoring tasks like indoor/ outdoor environmental

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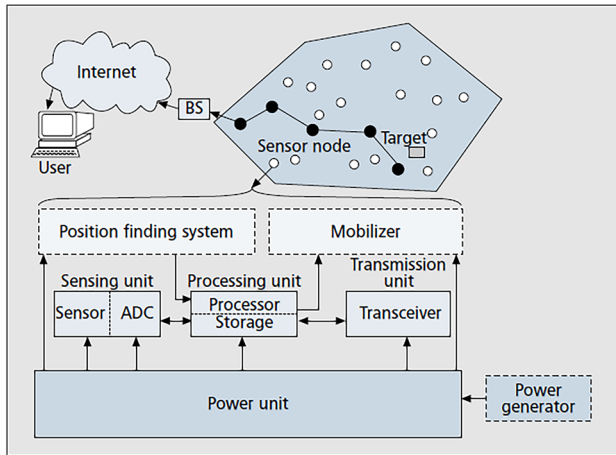


Fig. 1 Components of wireless sensors

monitoring, health and well-ness monitoring, power monitoring, inventory location monitoring, military surveillance, tracking objects, animals, humans and vehicles etc [4]. Figure 2 depicts the tracking and monitoring based applications of wireless sensors.

When sensors are deployed in a region, they are responsible for self-organizing an appropriate network infrastructure, called topology and often start multi-hop communication among themselves, called routing. Since each individual node in WSN is inherently resource constrained: it has limited processing speed, limited energy, communication bandwidth and storage capacity. Therefore more is the power expenditure of a node earlier it will die in the

network since the only source of power for wireless sensors is battery, which is usually not re-chargeable or replaceable especially when the sensor nodes are expected to operate with no human intervention for a longer period of time. So engagement of wireless sensors in various application fields retain lots of limitations and challenges related to topology as well as routing in WSNs like connectivity, coverage, network lifetime, energy consumption, fault-tolerance, transmission delay and many more [5, 6], refer Fig. 3. Some of the issues like connectivity, coverage, node failure, energy consumption, scalability, fault-tolerance etc can be dealt at topology construction phase or underlying network acknowledgment phase of WSNs, these challenges are known as topology control (TC) issues and the issues which are occurred and tackled only at routing phase like transmission delay, transmission cost, reliability, throughput, load balancing etc, are called routing issues. There can be two situations regarding topology control and routing issues, either both can be dealt simultaneously or individually. As designing an energy-aware topology of randomly deployed wireless sensors results low cost routing of data packets but meanwhile it can cause higher transmission delay [7, 8]. Similarly designing a fault-tolerant topology increases route reliability with the increment in transmission cost. Therefore one can see, integration of topology control and routing solution can be helpful only if the proposed method hits the same target or fulfilling the desired goals. Routing in WSNs are done in both ways either using topology control or non-topology control depending upon the problem objective or the desired goal, for example cluster-based or hierarchical routing protocols involve topology awareness/ establishment

Fig. 2 Applications of WSNs

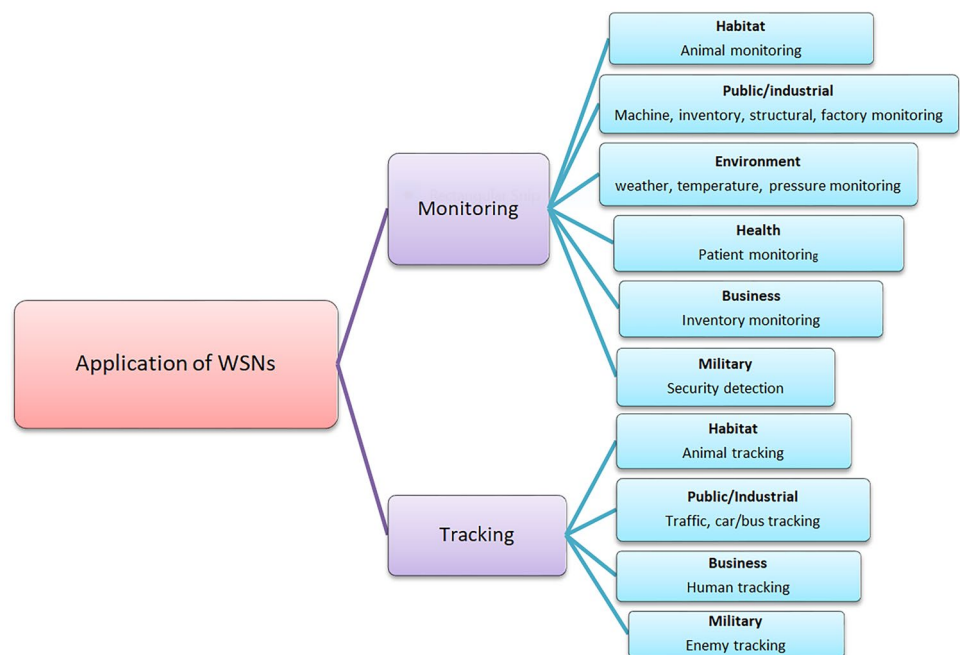
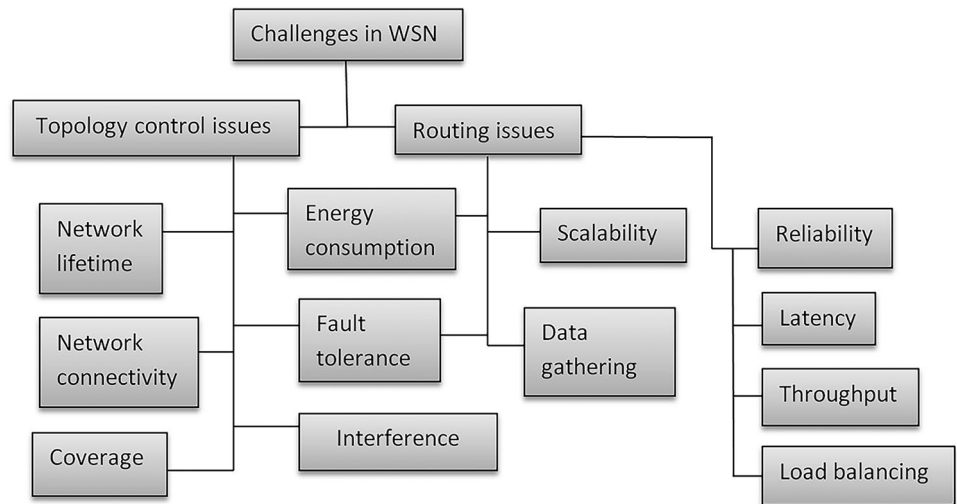


Fig. 3 Topology control and routing challenges in WSNs



to achieve energy efficiency [9, 10] while other routing techniques don't require topology setup [11, 12]. However, both topology control methods and routing techniques are used to enhance WSNs performance by saving energy of battery constraint sensor nodes in their own way, former works on topology and later is engaged to find end-to-end routing path between source to sink nodes.

1.1 Topology control and routing challenges in WSNs

Energy consumption: If the route from source sensor node to sink node is not an appropriate one for data transmission, more energy is consumed in the network. The energy required to transmit a t – bit message over a distance d is given as follows:

$$E_T(t, d) = \begin{cases} t * E_{elec} + t * \epsilon_{fs} * d^2 & \text{for } d < d_0 \\ t * E_{elec} + t * \epsilon_{mp} * d^4 & \text{for } d \geq d_0 \end{cases} \quad (1)$$

where E_{elec} , ϵ_{fs} , ϵ_{mp} be the energy required by the electronics circuit, amplifier and multi path respectively. Energy required to receive t – bit message is $E_R(t) = t * E_{elec}$. Hence as data transmission takes place between the sensors, sensors start depleting their energy according to the Eq. 1.

Network lifetime: It is measured by the lifetime of sensor who dies first in the network due to early power drainage. So objective is to maximize minimum sensor's lifetime.

Network life cycle: Lifetime of lastly alive nodes from the beginning of network execution.

Network connectivity: Connectivity refers to the wireless approach of each sensor to atleast one other sensor using their transmission range to maintain the network connectivity. Loss of connectivity causes loss of sensors

and/or forwarding data. So requirement of connected topology at every round of data transmission leads to energy expenditure in constructing and monitoring topology.

Interference: It is a kind of hindrance or obstacle existing in the topology due to collision of data packets transmitting over the same route and hence retransmission of data packets is needed. This causes more energy consumption of sensor nodes.

Load balancing: Some of the nodes are loaded more than others leading to fast energy depletion hence nodes get failed, losing network connectivity and network lifetime. So load balancing refers to uniform distribution of sensing and transmission loads between the nodes of every cluster for a uniform power consumption to take place.

Coverage: Coverage refers to the area covered by a sensor to sense and process the information. A network needs to have an optimal coverage i.e. minimum number of sensors are deployed to cover whole target region. Lack of coverage in any part of target region causes energy hole in the topology i.e. no information is being collected and forwarded from that part of region due to loss of coverage.

Data gathering: It must be done in such a way that reduces data redundancy and data retransmission so that nodes' energy could be conserved.

Energy hole or Hot-spot: Nodes near sink, have to spend extra power in data aggregation and/or transmission. So they die very early and their space leave energy hole or hot-spot in the network with degrading routing process.

Latency: It refers to an end-to end delay in data delivery. Real-time applications of WSNs require least delay in data transmission so latency needs to minimize there.

Scalability: For applications where tens to thousands of sensor nodes are deployed in a target area, routing proto-

cols should be designed in a way that they will be able to handle and respond to such greater number of events or addition or deletion of nodes.

Reliability: Reliability of a WSN is related with the probability of successful end-to-end communication i.e. data is received by the sink successfully.

Throughput: Throughput refers to how much data can be transferred from source to destination within a given time frame.

Packet delivery ratio: It is the percent of data packets delivered to the base station to those generated by source nodes.

Quality of service (QoS): QoS is related to reliability, packet delivery ratio and throughput factor with minimum error in transmitted data.

Deadline miss ratio (DMR): It refers to the fraction of data packets which don't reach destination node within a given time-frame.

Fault tolerance: It is the ability of routing protocols to respond quickly and discover new routes for the transmission of data in case there are nodes failures in the network.

The root cause of all the challenges mentioned here is limited power storage of each sensor node. Moreover, WSNs which aim to minimize latency, maximize throughput or QoS, require more nodes' power to achieve the goals. Hence this brings out the tradeoff between energy and delay in real-time application based routing. As every task from topology setup to data transmission requires nodes energy that is why topology control and routing issues become challenging to handle in WSNs.

1.2 Optimization in WSNs

Limited available energy to sensor nodes has attracted the attention of many researchers to deal with energy consumption of sensor nodes, network lifetime, network connectivity, link interference, data transmission cost, data reliability, data latency, sensing coverage, load balancing, data gathering, fault tolerance in designing the network topology and routing protocols. These pertaining issues in WSNs give rise to many optimization problems whose objectives can be (any combination of) maximize energy efficiency, maximize link quality, maximize network lifetime, maximize reliability, maximize coverage, maximize throughput, minimize delay, minimize cost, minimize interference, minimize packet error rate, minimize unbalanced energy consumption etc subject to the constraints (any combination of) network connectivity, interference, quality of service (QoS), transmit power, distance, coverage, (static/mobile) topology etc. Therefore, optimization in WSNs are of 'Network' and 'Route' optimization kinds. Network optimization is related to achieve a desired topology goals of randomly distributed sensors in a

target area that minimizes energy consumption or maximizes network lifetime subject to the constraint of network connectivity and/or coverage. Route optimization aims to find an optimal routing which results in less energy consumption fulfilling application based WSNs requirement like minimum latency, maximum throughput etc. In this respect, various topology control algorithms [13–18] targeting network optimization and routing protocols [5, 6, 19–21] targeting route optimization have been proposed in the literature for many past years. Existing literature reveals that energy conservation has been a major goal in designing topology and routing schemes in WSNs. There exists many survey papers/ review articles in history providing the study of topology control methods to optimize WSNs concerned with different objectives. Similarly, survey papers on routing protocols/ techniques with different optimization objectives and classifications are existing good in number from many past years. Among the existing survey papers/ review articles, one can either go through the study of topology control methods or the study of routing techniques at the same moment, i.e. no survey paper has provided the combined and collective study of both topology control methods and routing techniques. This motivates us to come-up with a review article offering the combined study of various energy efficient topology control methods and routing techniques along with the detail description of methodology opted, their merits and demerits along with future scope.

1.3 Contribution

Here, we are listing the contribution and focus of this review article. The major contributions of this paper are as follow:

1. This paper come-up with the combined and collective study of topology control methods and routing techniques in WSNs characterising the parameters of energy efficiency, transmission cost, nodes power consumption, network lifetime, network connectivity, coverage, load balancing, latency, QoS, scalability, fault-tolerance, solution complexity, mobility and immobility of network.
2. A taxonomy of topology control methods and routing techniques has been proposed with the aim to optimize the performance of WSNs both at topology level and routing level.
3. The basic idea to provide the thorough study of any classified category is its evolutionary techniques, well establishment and applicability in real-life scenario.
4. Year-wise representation of each reviewed technique/ protocol has been followed in this review article with the thorough mention of contribution made by the proposed work involving the beneficial part, specific approach/ methodology opted and its limitations which might be overcome in future work.

5. Study and analysis of not previously reviewed graph based approaches, explicitly designed interference model and related algorithms, is our prime concern towards topology control in WSNs.
6. Furthermore, insights on state-of-the-art network structure based and topology aware routing protocols; namely classical hierarchical routing, particle swarm optimization (PSO) and ant colony optimization (ACO) based routing, have been provided.
7. Review on real-time communication based routing protocols has been thoroughly presented, covering the articles from year 2002-2021, dealing with delay constraint, time bound, throughput, deadline miss ratio and reliability.
8. A brief comparative study among the proposed solutions has also been provided with the discussion on research gaps and open issues with new research directions for further development in this domain.

The rest of the article is organized as follows, Sect. 2 presents preliminaries, Sect. 3 introduces topology control methods and routing techniques in WSNs following the proposed classification, Sect. 4 offers the research methodology to conduct literature survey, Sect. 5 provides the summary of related survey papers from past years, Sects. 6 and 7 presents the review of various topology control methods and routing techniques respectively. Discussion part and comparative study can be seen in Sect. 8. Section 9 concludes the paper.

2 Preliminaries

Before going further in this review article, we would like to introduce general terminologies, graph related models, required definitions and abbreviations.

Topology control (TC) Topology control in WSNs is the problem of determining an appropriate topology or network structure with different topological goals like minimum interference and energy consumption with increment in network capacity like network lifetime, energy efficiency, scalability while maintaining the network connectivity and/or coverage. TC is the initial stage for any topology aware routing protocol.

Routing in WSN Data transmission is called Routing. Routing can be done either using **single-hop** i.e. direct data transmission from source to sink or **multi-hop** i.e. data is transmitted node by node to reach sink.

Routing protocols Methodology/ set of rules that has been opted to find an optimal (best possible) route/ path from a

source sensor node to the sink node. **Route optimality** may vary with objective of the problems concerned with different challenges like energy consumption, latency, reliability, throughput etc.

Centralized method A methodology which requires a master node to run and control other nodes in the network. Every other node needs to take permission or informs to master node first to work further.

Distributed method In this, each node has same authority to work so methodology can be initiated by any node independently as per the requirement.

Homogenous and Heterogenous WSN A WSN is said to be homogeneous if all the sensors have same storage, processing, battery power, sensing, and communication capabilities while a WSN which is not homogenous is said to be heterogenous.

Static and Dynamic/ mobile WSN A WSN where each sensor node even sink node is not allowed to move or kept fixed after deployment is called static WSN while in a dynamic or mobile WSN, nodes can move and if nodes are static then atleast sink node is mobile in nature.

Graphical layout of WSNs $G(V, E)$ WSN can be represented as directed/ undirected graph $G(V, E)$, with each sensor is taken as vertex (node) forming a vertex set V and communication link between sensors is taken as an edge in the plane forming an edge set E .

Weighted Graph modelling Weight on an edge can be defined as data transmission cost, link interference, Euclidean distance or delay in data transmission etc depending upon the objective of the problem which is to be solved.

Transmission cost function c A cost function c is defined as $c : V \times V \rightarrow R^+ \cup \{\infty\}$ such that $c(u, v)$ is the power emission necessary for node u to send packet to node v in directed graph where $u, v \in V$. In undirected/ bidirected graph, $c(u, v) = c(v, u) = \max\{c(u, v), c(v, u)\}$ can be taken symmetrically [15] and written as $c(uv)$ where $uv \in E$. $c(u, v) = \infty$ implies node u cannot communicate to node v directly.

Power dissipation over a link (uv) Power required for a direct transmission from u to v is a polynomial function of the distance d between u and v i.e. $c_1 + c_2 d^\alpha$, where $d = ||uv||$, α is the path loss gradient depending on transmission environment, c_1 and c_2 are some constant depending on electronic characteristics of wireless devices [22].

Power function r A power function $r : V \rightarrow R^+$ is used to assign sufficiently high transmission power to a node u so that it can send a message to node v directly i.e. $r(u) = \max_{uv \in E} c(u, v)$ where E is the set of all edges [13].

Transmit power management based TC Establishing and controlling the topology of underlying network of WSNs by minimizing the nodes power consumption either by reducing redundant transmission links or managing cost efficient transmissions depending on the required aim. This consequently results reduced and appropriate transmit power to each node in the resultant topology. Details have been discussed in Sect. 6.1 and many related approaches have been documented in Table 1.

Transmission disk $D(u, r_u)$ of node u A circular disk $D(u, r_u)$ is assumed to exist, taking node u as a centre and $r_u = r(u) = \max_{uv \in E} c(u, v)$ as a radius. Every other node which comes inside this disk is known as direct-neighbour of node u .

Coverage of a link uv and of a node u via link uv Coverage of link uv is $cov(uv) = \{w | w \text{ is covered by } D(u, r_{uv}) \text{ or } D(v, r_{vu})\}$ and coverage of a node u via link uv is $cov_{uv}(u) = \{w | w \text{ is covered by } D(u, r_{uv})\}$. Coverage illustration is given Fig. 4, redrawn from [15].

Coverage based Interference of a link uv Interference of a link uv is denoted by $IC(uv) = |cov(uv)|$. According to the illustration given in Fig. 4, $IC(uv)$ comes out to be 11.

Node interference via link The interference of a node u is the maximum interference of all links incident on u i.e. $IC(u) = \max_{uv \in E} IC(uv)$.

Sender-centric and receiver-centric interference Sender-centric interference happens to exist at the sender node u

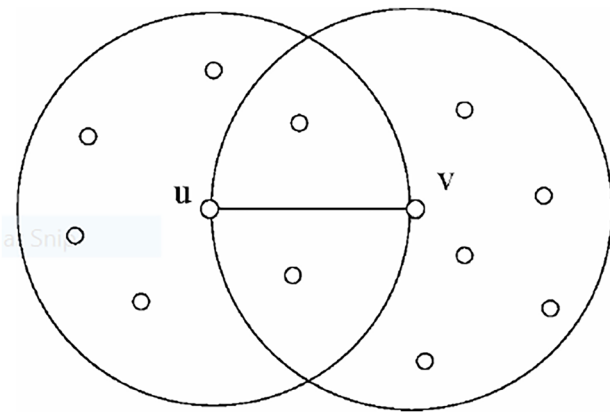


Fig. 4 Coverage of a link uv

and it is denoted by $I_S(u) = \{v \in V \setminus \{u\} | v \in D(u, r_u)\}$ while the receiver-centric interference exists at receiver node v and it is denoted by $I_R(v) = \{u \in V \setminus \{v\} | u \in D(u, r_u)\}$.

Description of abbreviations

BS	Base Station	RP	Rendezvous point
r_{min}	minimum transmission range	CH	Cluster head
r_{max}	maximum transmission range	MS	Mobile sink
SN	Sensor node	SP	Sojourn point
MN	Member nodes	ngh	neighbourhood

3 Classification of Topology control and routing techniques in WSNs

A proposed taxonomy of topology control and routing techniques is demonstrated in Fig. 5. Starting with the classification of topology control methods, Graph based approaches include those methods/ algorithms which are either depending on some graph-based model or utilizing existing concept and/or algorithms of graph theory like the concept of minimum spanning tree (MST), hamiltonian graph/ cycle, shortest path methods etc. For example, Yuanyuan et al. in [23] utilize the concept of dominating set of graph theory to obtain a connected topology, Abu-Affash et al. in [24] utilize the existence of second hamiltonian cycle to obtain low-powered connected network, Abderrahim et al. in [25] proposed Dijkstra algorithm based energy efficient technique, Yang and Cai in [26] proposed minimum spanning tree (MST) based QoS required topology likewise Cheng et al. in [27] proposed MST based algorithm to achieve Strong Minimum Energy Topology (SMET). Some algorithms are proposed to construct Relative Neighbor Graph (RNG) [28], Gabriel Graph (GG) [29], Yao Graph or Voronoi diagrams to have an energy efficient spanner topology.

Additionally, Power control algorithms include different data gathering mechanism using transmit power control and/or power mode approaches to achieve energy efficient topology. Transmit power control approaches involve the control and adjustment of nodes' power level to achieve an appropriate transmission range so that nodes' energy can be conserved. Power mode approaches involve sleep/wakeup schedule of nodes means nodes can go to sleep/inactive mode if their tasks are fulfilled for a timing or their energy level is about to go zero or below some defined threshold. Some of power control algorithms with the engagement of sink nodes are MLS [30], PCLT [31], A3 [32] and some without sink nodes are DFTM [33], DPV [34], GBP [35]. Furthermore, Interference is one of the major issue in

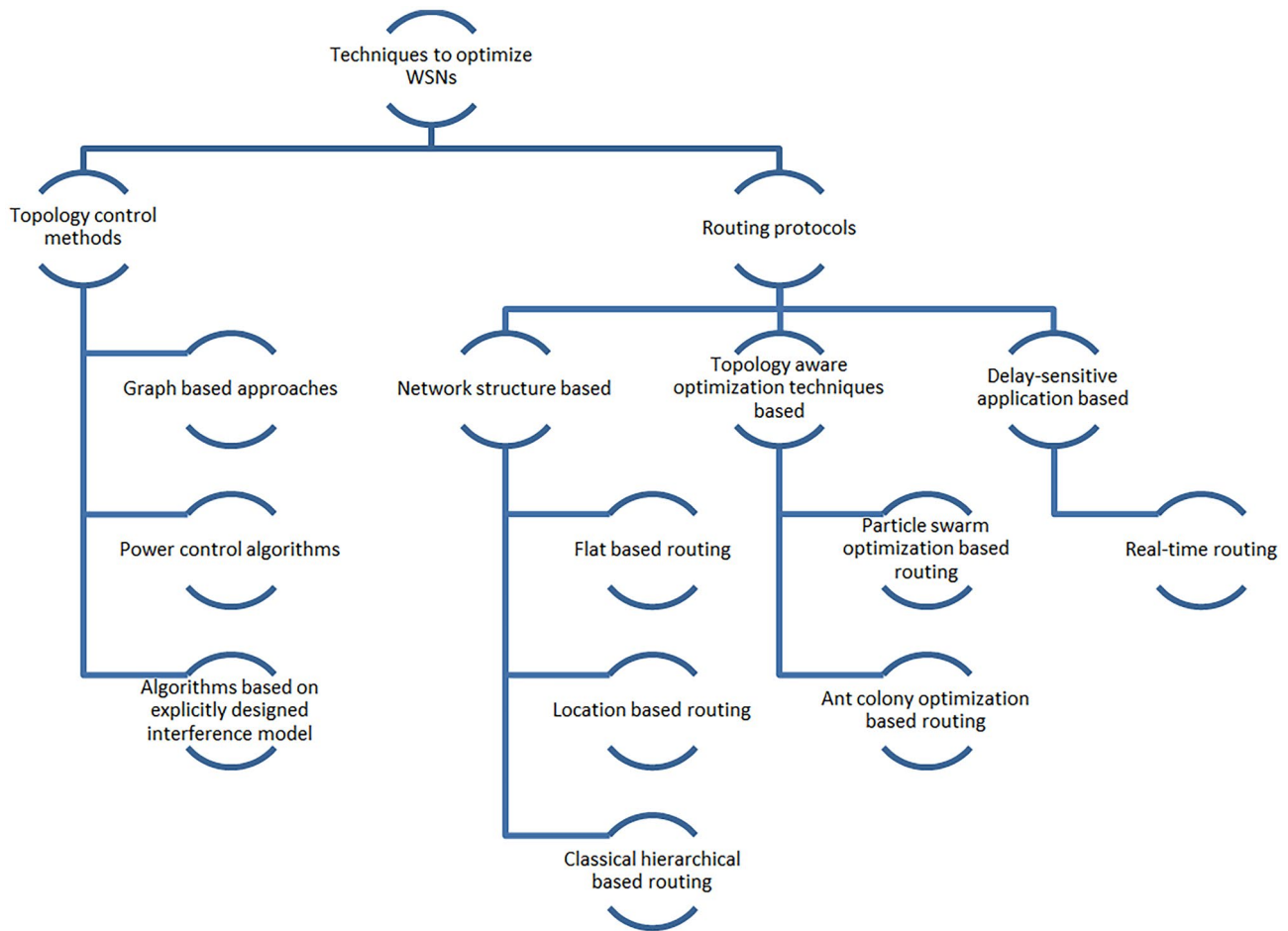


Fig. 5 Classification of Topology control and routing techniques to optimize WSNs

designing an energy efficient topology. Therefore various topology control algorithms minimizing interference explicitly based on interference models have been proposed in literature [15, 17, 22, 36–38]. Detail review on graph based approaches and algorithms to minimize transmit power and interference explicitly which have not been covered in the survey papers from past years, are presented in Sect. 6. Moving to the classification of routing protocols. On the basis of topology/ network structure formation, routing in WSNs has been classified into flat-based, location-based and hierarchical-based. Flat-based routing protocols have been proposed for homogenous WSNs where each sensor node has same functionality and operational capacity. It is also known as data-centric routing because base station sends queries to certain regions and waits for data from the sensors located in the selected regions. Some popular data-centric or flat-based routing protocols are Flooding, Gossiping [39], SPIN [40], Directed Diffusion [41], Rumor routing [42], Gradient based routing [43], COUGAR [44], ACQUIRE [45], MCFA [46]. However, flat-based routings show the limitations regarding reliable data delivery and fault-tolerance,

result routing overhead and latency with the increase in node density. In location-based routing, also called geographic routing, location information of sensor nodes are required for route discovery, data forwarding and maintenance of sensor nodes in network. Energy consumption can be estimated by calculating distance between two particular nodes that helps in reducing energy consumption and optimizing whole network. GEAR [47], GAF [48], MECN, SMECN [49], EAGRP [50], LBRP [51] are some energy aware location based routing protocols. Existing strategies tend to increase extra routing overhead in order to obtain location information of destination nodes through the location service algorithm especially in dynamic WSN. In addition, energy hole problem and location inaccuracy problem also occur in geographic routing [6]. To tackle these issues, a novel fuzzy logic-based geographic routing protocol (FLGR) [52] has been proposed. Compared with existing protocols, the FLGR protocol has reduced routing overhead and increased packet delivery ratio in a sparse network. Location-based routings are having drawbacks in large dynamic WSNs since requirement of accurate address of each sensor node makes the

routing topology costly. Also performance degrades in term of latency in large network Hierarchical routing protocols also known as cluster-based routing, operate by clustering nodes in groups, assigning a cluster head (head of a cluster group) to gather data from the member nodes (remaining nodes other than CHs) in a cluster and forward it to the destination node. This causes reduction of redundant data transmissions which minimizes congestion at the destination node and energy consumption. Classical hierarchical routing protocols are mainly operated in two phases, first one is of cluster heads (CHs) selection and cluster formation phase either as chain or tree or grid structure. The second phase is of data aggregation and data transmission [20, 53]. In cluster-based routing, data transmission happens at two levels, first is called intra-cluster routing and second is inter-cluster routing. Intra-cluster routing is concerned with data transmission within a cluster among the member nodes (MNs) and inter-cluster routing is concerned with data transmission among the distinct clusters to sink nodes either through CHs or relaying nodes. Classical hierarchical routing protocols opted different strategy of CHs selection, cluster formation, data aggregation and data routing, aiming energy efficiency, scalability, fault-tolerance, load balancing, reliability and latency. These routing protocols outperform flat-based and location-based routing protocols in saving energy, reliability, load balancing, latency, scalability and extending lifetime of mobile WSNs (MWSNs). Due to more application and advantages of cluster-based routing, we limit our review on classical hierarchical routing protocols in this article from the classification of network structure based routing protocols. Optimization techniques based Particle Swarm optimization (PSO) and Ant Colony optimization (ACO) routing techniques are actually nature-inspired routing algorithms [54]. These techniques mimic the intelligence and social behavior of natured species like swarm and ants as they develop knowledge on their own to communicate and feed themselves. These routing algorithms resolve complex clustering and routing issues very efficiently in large dynamic WSNs [55]. As these techniques are robust, adaptable and flexible with respect to topology changes so they are called topology aware optimization techniques. In this survey article, review and analysis of latest routing algorithms based on PSO and ACO techniques is presented in Sect. 7. Irrespective of most energy efficient, scalable, fault-tolerant and robust, the mentioned techniques lag where the topology and routing requirement are delay-sensitive, bound with time, concerned with data delivery ratio, throughput and quality of service (QoS). Therefore, real-time routing techniques have been proposed to satisfy QoS requirements in term of minimum latency and maximum throughput. The main concern of real-time routing protocols is to meet packet delivery deadline as well as to maximize total throughput. Therefore real-time routing aims to discover an optimal

route from source to destination meeting the real time constraints such as delay and throughput. Research in this field is on its early stage and limited to some well-known protocols like RAP [11], SPEED [12], MMSPEED [56], RPAR [57], THVR [58], EE-ARP [59], ERTLD [60], DCEMRA [61]. Emerging application of WSNs requires the real-time communication based routing protocols on priority so this survey provides the readers a whole knowledge and analysis of these protocols in Sect. 7.

4 Research methodology for literature review

Literature review on topology control methods and routing techniques in WSNs has been conducted to present articles that are published in well-established journals and conferences providing essential insights to researchers and practitioners working in this area. In our manuscript, we tried to summarize contributions, results, merits, demerits and future extension of the published work whose methodologies/ approaches are related to graph and optimization in WSNs. For the sake of understanding, flowchart of research methodology is demonstrated in Fig. 6. We searched web of science to obtain an extensive bibliography of the vast literature related to topology control methods and routing techniques in WSNs. The web of science core collection information provides us with fast and impactful access to the world's leading citation database such as ScienceDirect, IEEEExplore, Springer link online libraries, ACM digital library, Wiley, Research Gate and so forth.

The following subsections clearly outline the procedure we followed in extracting articles along with selection criteria, classification method and further explains the filtering processes.

4.1 Procedure for extraction of articles and selection criteria

We conducted a comprehensive search for articles using research questions, topic related terms and phrases such as 'Topology control in WSNs', 'Topology control algorithms/ techniques/methods in WSNs', 'Topology control and routing techniques in WSNs', 'Energy efficient hierarchical routing techniques/protocols in WSNs', 'Energy efficient and delay sensitive routing techniques', 'Graph based optimization techniques in WSNs', 'Real-time communication based routing protocols in WSNs', 'PSO and ACO techniques based routing in WSNs' and 'Topology and route optimization in WSNs'. The number of potential scholarly articles had been obtained at the initial step by using keywords described above was approximately 600, published in the year between 2000 and 2021. Many survey papers are

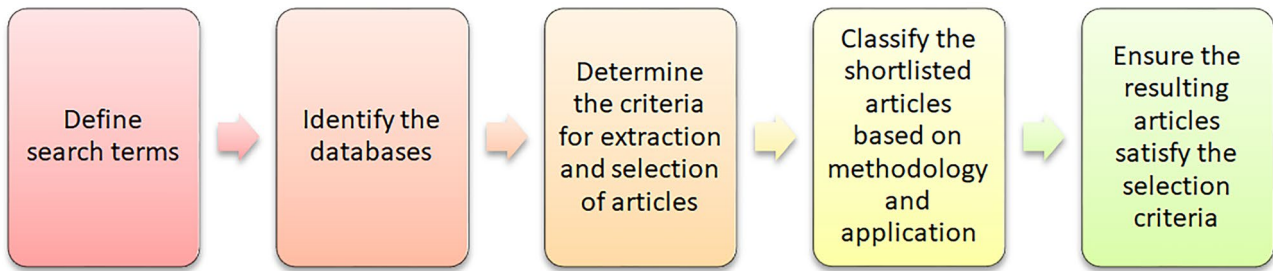


Fig. 6 Flowchart of research methodology

found from the obtained scholarly articles as well. So filtering of relevant survey papers are done first on the basis of the criteria given below.

- Only those survey papers are kept for review which have covered the literature of mentioned terms and phrases in a systematic and comprehensive manner.
- Also recent survey papers from the respected journals are kept by removing old survey papers covering the same topics.

After extracted relevant survey papers, we take note of already reviewed topology control techniques and routing protocols so that uncovered literature part can be acquainted and filtered out from the remaining scholarly articles for further review. Three criteria were determined and used to choose and acknowledge the respected journal and conference articles for additional consideration. Articles were excluded if they did not meet the accompanying selection criteria.

- From the obtained scholarly articles, only those articles are kept for review which are either containing topology control techniques or routing protocols including cluster-based/ hierarchical, real-time, PSO and ACO based routings in WSNs.
- Excluding techniques based on game-theory, fuzzy logic, machine learning, image processing related concept, entropy and statistics. Extracted articles have a basic foundation which is networking involving graph-theory based approaches, algorithmic approaches based on graph models and some optimization techniques.
- Filtered articles are crosschecked if they are already reviewed in one of the extracted survey papers in detail. Only those articles are carried forward which are not reviewed in details and left to review.

After examining the title and abstract of approximately 600 articles so obtained at the very initial step, we retrieved 145 articles to review thoroughly (including survey articles as well) which satisfies the selection criteria. These articles

undergo the next round of classification based on methodology and application area. Then finally kept them for reviewing process.

5 Related work

There has been a lot of survey papers existing in literature on topology control methods and routing techniques in WSNs. Some of them are reviewed and documented below.

5.1 Previous survey on Topology control methods

Santi in 2005 [13] provides an overview to obtain power controlled topology of wireless sensors based on homogeneous and non-homogeneous range assignment. This paper has covered the literature concerned with building and maintaining connected topology of minimum energy cost. Description about static and mobile topology in term of graph model and range assignment along with proposed solutions and complexity has been provided. Routing graphs in 2-D which are Relative Neighborhood Graph (RNG), Gabriel Graph (GG), Delaunay Graph (DG), Yao Graph (YG), OrdYaoGG have been explained and analysed in term of optimal power spanner. This paper emphasises on bounded node degree to minimize power consumption i.e. lesser the maximum node degree more power can be conserved and no routing graph works good in this respect. Rickenbach et al. in 2009 [14] present the explicit study of interference reduction in both static and mobile WSNs by analysing and comparing the various proposed sender-centric and receiver-centric interference-based models and algorithmic approaches along with their properties and complexities. It has been stated that LIFE (Low Interference Forest Establisher) algorithm builds minimum interference connected topology, LLISE (Localized Low Interference Spanner Establisher) algorithm builds a minimum interference connected and spanner topology in localized manner, NCC (Nearest Component Connector) algorithm builds minimum interference tree topology with sink nodes. Authors put forth the complexity of finding interference bounds while maintaining connectivity on

developed 1-D interference models namely the exponential node chain and the generalized highway model. Aziz et al. in 2012 [62] provide a deep discussion of energy efficient and network lifetime maximizing distributed topology control techniques in 2-D and 3-D. The classification of techniques has been done into four categories based on power adjustment, power mode, clustering and hybrid following detail explanation of each mentioned techniques respectively. MECN, SMECN, COMPOW are power adjustment based techniques, GAF, STEM, ASCENT are power mode based techniques, PACDS, ECDS, TMPO are cluster based technique and SPAN, CLUSTERPOW, LEACH are hybrid techniques of power adjustment/ power mode and clustering. Authors offer performance analysis, merits and demerits of each mentioned technique, existing issues and future scope. Yetgin et al. in 2017 [63] come-up with a review of only lifetime maximization techniques in large-scale WSNs. In this survey paper, one can see many definitions of network lifetime based on different objectives with the board classification of network lifetime techniques into resource allocation, opportunistic transmission schemes, sleep-wake scheduling, routing/ clustering, mobile relays and sinks, coverage and connectivity, optimal deployment, data gathering/ network coding, data correlation, energy harvesting and beamforming based on the defined objectives. Authors mention the constraint functions, optimization tool and achieved optimality of the most popular techniques from year 2000 to 2015. Also Nguyen in 2020 [65] provides a review on network lifetime maximizing techniques in mobile WSNs. Mathematical definition of network lifetime has been demonstrated in different category. Classification of developed techniques has been based on nodes mobility, sink mobility and event mobility. The nodes mobility category is further classified based on strategies like coverage, connectivity, energy aware, cooperative computing and localized and clustering as well as sink mobility category has been classified into random, fixed and controlled mobility. Authors cover the well established techniques till 2019 by mentioning their objectives and methodologies along with possible issues, limitations and future scope. Mohamed et al. in 2018 [64] show the study of various models of WSNs, topological designs and routing protocols based on different real-life applications of WSNs. A detail taxonomy of WSN applications considering all the main design aspects and related network features has been presented by authors. Topological designs and routing protocols have been analyzed following the applications based classification like environment application, agricultural, military and health care application etc. Authors point out the basic methodology, special features and persisting issues of well-established routing protocols in term of energy cost, scalability and network lifetime. However, the survey is majorly focused on homogenous WSNs and its related routing protocols. Singla and Munjal in 2020

[18] present topology control algorithms by categorising into centralized and distributed algorithms based on opted data gathering mechanisms. Authors classify Minimum Load Set (MLS), Probability Distribution and Competition in Same Layer (PLCT), Secured Energy Conserving Slot based topology (SECS-TMP) as centralized algorithms. A3, Dead End Free Topology Maintenance Protocol (DFTM), Interference Based Topology Control Protocol (ITCD), Disjoint Path Vector (DPV), Grid Based Topology Control (GBP), Localized Topology Control (LDTTC), Location Free Topology Control (LFTC), Hybrid AOA/TOA Topology Control protocol (HDTTC), Cone Based Topology Control (CBTC), Articulation Point Topology Control (APTC), Light Weight Redundancy Aware Topology Control Protocol (LRTCP), Lossy Link Aware Opportunistic Topology Control (LOLA-OTC) come under distributed algorithms. Working mechanism and comparison of all the mentioned protocols have been provided. Authors involve various parameters to evaluate performance that are energy consumption, network lifetime, energy cost, delay, throughput, routing overhead, time, collision ratio, maximum relative load. Only few of them show good performance against every parameter. It has been seen that there exists no survey paper in recent years, presenting proper literature on graph-based approaches and interference minimization algorithms, used in topology control of WSNs. Thereafter, this article has come-up with unexplored literature part related to power optimization in topology control using graph-based approaches, explicitly designed interference models and interference minimization algorithms.

5.2 Previous survey on routing techniques in WSNs

Bhattacharyya et al. in 2010 [66] offer the detail description of eight hierarchical routing protocols that are LEACH, TEEN, APTEEN, PEGASIS, SPIN, DD, RR and GEAR. Performance analysis of the mentioned protocols in term of mobility, network lifetime, power management, scalability has been provided along with the direction of improvements which can be done in future work. Review of hierarchical routing protocols from year 2011 to 2016 has been done by Sabor et al. in 2017 [68]. Sabor et al. classify the protocols into classical and optimized routing protocols dealing with mobile WSNs. Working details, advantages, drawbacks and performance analysis in respect of delay, network size and energy efficiency of the protocols dealing with the mobility of sensor nodes and/or mobile sinks have been provided. Authors have drawn the conclusion that centralized routing is more energy-efficient than the distributed routing so semi-distributed or semi-centralized protocols should be considered for future work. Also clustering techniques from year 2002 to 2016 are studied by Ogundile and Alfa in 2017 [69]. Ogundile and Alfa bring out the classification of routing

techniques engaged with mobile/static sink adopting either single hop or multi-hop routing to communicate to sink node. As per the classification, detail of the working protocols with respect to CHs selection, number of CHs, CHs rotation, cluster formation, intra-cluster and inter-cluster routing have been stated. Authors offer the future direction for more load balancing energy efficient cluster-based routing techniques. Guleria and Verma in 2019 [70] present a systematic study of hierarchical routing protocols from year 2012 to 2017. Moreover, a deep analysis of both classical and swarm intelligence (SI) based clustering techniques (PSO, ACO, ABC, BFO) opting different strategy and working methodology using performance metric of energy efficiency, data aggregation, location awareness, QoS, scalability, load balancing and fault tolerance has been provided. Enhancement of LEACH protocols are stated along with pros and cons. Authors bring out the long discussion of advantages, existing issues and improvement for future works in this field. Thereafter Fanian and Rafsanjani in 2019 [9] provide a comprehensive survey of cluster-based routing protocols from year 2000-2018. The aim is to cover all those methodologies which are energy efficient and help the readers to build their base knowledge about the methodologies opted in this domain. Authors classify the protocols on the basis of different methodologies as classical approaches, fuzzy-based approaches, meta-heuristic based approaches, hybrid meta-heuristic and fuzzy-based approaches. They put forth desired objectives, measuring parameters, merits and demerits of different routing protocols in each classification. A comparison of different routing protocols of each category has been shown on the basis of mobility, cluster size, network type, methodologies etc. Analysis of techniques on the basis of solution complexity is missing in the article. Chan et al. in 2020 [20] provide the insights on classical hierarchical routing protocols following the division into chain-based, tree-based, grid-based and area-based routing topology formation. Authors mention advantages, disadvantages and comparison of the routing techniques with respect to energy efficiency, load balancing, delay, scalability, network longevity and reliability. Moreover, study of recently developed versions of low-energy adaptive clustering hierarchy (LEACH) routing protocols are given along with comparison among the proposed versions. Authors put forth present challenges with direction of future work. Then Rawat and Chauhan in 2021 [71] present a survey of clustering based approaches by collecting efficient protocols from year 2000 to 2020. This is the most recent survey paper in the domain of hierarchical and/or clustering routing techniques in WSNs. A taxonomy has been given by classifying clustering approaches into homogeneous network-based, heterogeneous network-based, fuzzy logic based and heuristic based protocols. Authors explain methodologies of each clustering protocol with its objectives, key features, merits and

demerits along with future scope. Analysis of clustering protocols have been conducted by considering the parameters of CH selection, CH rotation, data transmission, mobility, topology, routing techniques and simulator used while the comparison between different techniques are based on prototype used, energy consumption, network lifetime, throughput and stability. Ketshabetswe et al. in 2019 [6] present the working details and analysis of routing protocols concerned with energy saving and network lifetime particularly. Authors follow the classification of routing protocols based on homogenous and non-homogenous network which are further categorised based on mobility of sink nodes. Insight of WSNs model in term of graph-based structure and energy dissipation model with good explanation has been provided. Based on the classification, authors cover large number of flat-based, location-based, hierarchical, QoS aware, cross-layer, nature-inspired, cooperative and opportunistic routing protocols. Analytic comparison, existing challenges, direction of improvement and new work direction are offered in this paper. Then Lin et al. in 2020 [10] cover the energy efficient strategies for static WSN from year 2002 to 2019. They also proposed the mathematical definition of energy efficiency first time in history. A review of previously existing survey is mentioned along with drawbacks. Authors classify energy efficient strategies into five categories, the Energy-Efficient Media Access Control (EEMAC) protocol, the Mobile Node Assistance Scheme (MNAS), the Energy-Efficient Clustering Scheme (EECS), the Energy-Efficient Routing Scheme (EERS) and the Compress Sensing-Based Scheme (CSS) and put forth characteristics and applications domain of each classification. One can see open challenges, opportunities and future directions related to energy efficient WSNs. Pantazis et al. in 2013 [67] present the broad classification of routing techniques on the basis of network structure, communication model, topology based and reliable routing. The study of flat and hierarchical routings are found in network structure category, location based in topology based category and QoS based in reliable routing schemes. The aim of this review article is to assemble all routing techniques which are energy efficient as well as facilitate load balancing among the sensors. Moreover, authors present merits and demerits along with the performance of the routing techniques using the parameters of scalability, mobility, robustness, route metric and periodic message type. Cao et al. in 2019 [55] bring out the new classification of routing protocols based on transmission path, network structure, working mode, next hop routing and data delivery model. The main focus of the paper is to present literature of SI based routing techniques used in application engaging mobile WSNs. Authors systematically provide the description, discussion, analysis by stating advantages, limitations and comparison of PSO algorithms, ACO algorithms, artificial fish swarm algorithm (AFSA), artificial bee colony

(ABC) algorithms, shuffled frog leaping algorithm (SFLA) proposed in the history to overcome the complex challenges of mobile WSNs related topology designs and routing. Further Zagrouba and Kardi in 2021 [72] present a board taxonomy by classifying routing techniques into 9 different categories based on application type, delivery mode, initiator of communication, network architecture, route discovery, network structure, protocol operation, next-hop selection, latency-aware and energy-efficient routing. This paper provides the literature of real-time routing protocols under the category of application type and delivery mode, data-centric routing protocols under network architecture, location-based and hierarchical routing protocols with swarm intelligence based techniques under network structure. Performance evaluation has been shown using the parameter of reliability, congestion control, energy efficiency, data aggregation, scalability, delivery delay, network lifetime, fault-tolerance and QoS. Liu in 2017 [74] is the first who provides the comprehensive survey with new classification of ACO based routing techniques in WSNs. Author offers the working details, performance advantages and limitations of some well known ACO-based routing algorithms along with comparison and future scope. Al Aghbari et al. in 2019 [5] present the study of PSO, ACO, FA, GA, ABC, BFO techniques based routing algorithms from year 2010 to 2019 in solving WSNs problems of network lifetime, energy consumption and throughput. Authors put forth concluding remarks for each of routing techniques by stating overall performance advantages and drawbacks. According to AL Aghbari, PSO technique is optimal for cluster-based routing and converge faster than any other SI based algorithm. Also PSO consumes lesser energy compared to other mentioned techniques. Secondly, ACO based routing algorithms are more reliable for multi route data transmission and maximize network lifetime. Also Chen et al. in 2020 [75] cover the literature of ACO-based routing techniques for both static and mobile WSNs. Authors provide the methodology and performance analysis of some typical ACO routing techniques using the parameter of energy utility, load balancing and complexity. Moreover issues related to the topology of mobile WSNs are discussed with direction of improvement. Rachamalla and Kancharla in 2013 [73] offer the insight of many real-time routing protocols by providing working objectives, details along with performance advantages and drawbacks for both static and mobile WSNs. Comparison among the proposed protocols have been done using parameters of energy efficiency and reliability. Further Kim et al. in 2017 [19] discuss and analyse the real-time communication based applications in WSNs and respective routing protocols. The paper covers the literature since 2010 and offers the classification of real-time routing protocols into soft, hard and firm based on two levels, first one is MAC and scheduling and second is

routing. Motivated from the previous work, this article presents the review of most popular, unexplored and latest network structure based classical hierarchical routing protocols, topology aware optimization techniques based PSO and ACO routing techniques and delay-sensitive application based real-time routing protocols. Analysis of each technique put forth the various merits and demerits related to routing issues, solution complexity, mobility/ immobility of network.

6 Review of Topology control methods

Survey papers from the previous years cover the major literature of topology control in WSNs involving the graph models, power mode based topology planning, distributed and centralized power control algorithmic approaches which we have already discussed in Sect. 5.1. However, no survey paper presents the complete study on graph based approaches in reducing and managing nodes transmit power, explicitly designed interference models and interference minimizing algorithms in TC of WSNs. This section presents the review of unexplored graph based solutions, interference minimization models and related algorithms which are used to conserve nodes power, extend network lifetime, eliminate redundant transmission and transmission cost in directed/ bi-directed connected graph models of WSNs.

6.1 Transmit power management based topology control

Topology control issue regarding energy consumption, related to minimize nodes power consumption as nodes are power constraint and depletion of their power make them die soon hence it effects network lifetime adversely. All those attempts, related to minimize and balance nodes power consumption by determining an appropriate network topology which is energy efficient and cost efficient, results an optimal power assignment to each node belonging to the topology or allow nodes to adjust their power level to design their own topology, come under 'Transmit power management based TC'. Cheng et al. in 2003 [27] formulated a 'Strong Minimum Energy Topology (SMET)' problem which is related to transmit power/ transmission range assignment to each node in the network. **SMET** states that for a given set of sensors, compute the transmit power of each sensor such that there exists atleast one bi-directional link between every pair of sensors and total transmit power gets minimized. Total transmit power is the linear sum of transmit power of each sensor node. Cheng et al. proved the NP-completeness of SMET in bidirectional graph model of WSNs and proposed two heuristics algorithms to find the approximate solution of

the problem. Santi in her survey paper [13] covers the literature related to ‘Range assignment problem’, the problem is same as SMET with the only difference is that the graph model can be directed as well as undirected/ bi-directed. In this paper, latest articles on the same problem or some extended problems from year 2012 have been reviewed and documented in the Table 1. A proper study of each technique is given in the table like the objective/ contribution of the proposed work, detail of methodology opted and limitations with its future scope. Following are the some defined terms/ definitions whose abbreviations are used in Table 1.

DPMP (Dual power management problem): Rong et al. in 2004 [76] formulated DPMP and proved the NP-hardness of the problem. It can be said that DPMP is the extended problem of SMET, dealing with optimal assignment of transmit power to each node from the discrete set of two power levels i.e. $\{H_p$ (high power), L_p (low power) $\}$. **DPMP** states that for a given set of sensor nodes and two power levels, H_p and L_p , find the optimal assignment of these two power levels to the nodes of induced graph model of WSNs such that the total power consumption gets minimized. Once the transmit power either high or low has been assigned to the nodes, it kept on fixed. Generalising the DPMP to k -distinct power levels assignment problem, named as **K -Distinct SMET** which put forth problem of optimal assignment of a transmit power to each node in the network from the discrete set of k distinct power levels such that the resultant network is strongly connected and total power consumption gets minimized. Some authors [78] come-up with fault tolerance in power management problem by introducing k -edge connectivity concept of graph theory. Going through each technique of Table 1, following are the key highlights:

- Although the developed work has provided the energy efficient topology according to the desired objective and has lesser complexity, yet is not suitable in dynamic WSNs. Topology gets disconnected with the nodes movement hence they are not adaptable to nodes mobility as once the power is assigned to the nodes, it remains fixed in the network during data transmission.
- Power assignment algorithms are limited to discrete set [24, 77, 79, 80] and the one with interval power assignment [81] is restricted to those WSNs only whose underlying graph can be an interval graph. This work needs to be generalised.
- Issues related to fault-tolerance of topology has been resolved by undertaking edge-connectivity [78]. It can be further taken to vertex-connectivity with the consideration of node failure.
- TC methods lag behind in term of network heterogeneity, scalability, fault-tolerance, mobility and routing delay.

6.2 Interference minimization approaches in topology control

Rickenbach et al. in the survey paper [14] in year 2009, present the literature of interference minimization explicitly through various interference models and algorithmic approaches. After this, no review has been conducted on these approaches. Literature from year 2009 onwards has been covered in this survey paper. Articles concerned with minimization of maximum interference, total interference and average interference have been reviewed and documented in Table 2 so that the readers can get acquainted about the contribution made by proposed work, detail of opted methodology/ approach and existing limitations. Sender-centric and receiver-centric interference models are the most opted models to minimize nodes/ links interference. Panda and Shetty in [15] come-up with new models namely SUM and MAX, to compute coverage based link interference i.e. $cov(uv)$ under SUM model is $\sum\{cov_{uv}(u) + cov_{vu}(v)\}$ and under MAX model is $\max\{cov_{uv}(u), cov_{vu}(v)\}$. Consider an illustration given in Fig. 4, link interference under SUM model comes out to be 13 and under MAX model is 7. After this, Sun et al. in [37] introduced another link interference model i.e. $I_{uv} = |\{xy \mid xy \text{ is any link, } x \text{ or } y \text{ or both } \in \text{Disc}(u, d(u, v)) \text{ or } \text{Disc}(v, d(u, v))\}|$. This only computes number of interfering links instead of nodes which come inside the transmission disc of any node. This new link interference models outperform the previous models as being more practical [37]. These interference minimizing models results minimum interference topology with the issue of network sparsity and fault-tolerance. Then Bao and Deng in [38] introduced fault-tolerant and reduced interference topology by introducing k -disjoint paths between every pair of nodes of minimum interference. However, the topology formed using these techniques is keeping high latency factor. Therefore Wu et al. in [88] propose polynomial time algorithms to result delay constraint and minimum interference topology. But the assumption that all links are having same transmission power limits the use of the work. Lakshmi et al. in [17] minimize sender and receiver interference using two distinct power levels; nodes either assign with low power level or high in such a way that the formed-MST causes minimum of maximum interference. However, the proposed techniques and algorithms lag their functionality in mobile WSNs as they are designed for static network only. Furthermore, work on interference minimization can be extended to nodes with multiple power levels or continuous range of power level like an interval, to enhance the fault-tolerance and avoid energy hole problem in interference minimization, concept of examination of link quality and nodes residual

Table 1 Overview of Graph-based approaches in ‘Transmit power management based topology control’

Reference	Contribution	Approach/Methodology	Limitation/Future scope
2012 [77]	Authors proposed a fully distributed power adjustment algorithm named as BRASP to form a connected and reliable topology of minimum cost by undertaking the factor of lossy links by computing link qualities locally. Worst case time complexity is $\leq E $ and cost is $\leq E + V $. Outperform other Deterministic TC algorithms in term of network energy efficiency and connectivity with decreased node degree.	Among the K – distinct power levels, each node starts with minimum power level and gradually enhance its power to any of the available power level to satisfy the connectivity constraint. Connectivity constraint is the value between 0 to 1 such that the value more than 0 between the pair of nodes depicts their connection and closer the value to 1, stronger is the connection. Authors used Dijkstra’s algorithm to find reliable path of minimum cost between the pair of nodes by following the restriction of hop counts means if 2-hop restriction is imposed, nodes can adjust their power to reach 2-hop away nodes only.	Defined for static network only. BRASP will consume more energy when larger threshold of hop count is used. Work on improving end-end delivery by accounting time-dependent lossy links might be undertaken in future.
2015 [78]	Introduction of fault-tolerance in DPMP through k –edge connectivity, named as k –EDP. Authors proved the NP-hardness of k –EDP in geometric case and provided a 2-approximate solution to the problem in polynomial time. Moreover, proposal of ‘Minimum-Cost-H Path’ algorithm which obtains 2-edge connected spanning subgraph.	Author formulated the problem as LPP and proposed 2-approximation algorithm using the iterated rounding method proposed in [82]. 2-approximation algorithm keeps on applying ‘Iterated rounding technique’ on LPP to convert its fractional solution into integer until a k -connected graph is obtained. Minimum-Cost-H Path algorithm successively adds H-paths with the idea of picking low power edges first to obtain 2-edge connected spanning subgraph H .	Suitable for static WSNs only. The problem can be concerned with k -vertex-connectivity undertaking the node failure in future.
2015 [79]	Proposal of 2-approximate solution to DPMP by introducing SCDPA (strongly connected dual power assignment) algorithm and an algorithm concerned with the assignment of 3-power levels which are low, middle and high power level following the similar steps of the former 2-approximate algorithm.	MST of the induced graph is found by using Prim’s algorithm and transmission range is assigned to each node using distance factor in MST. Now nodes whose transmission range is less than or equal to low power level is assigned with L_p and remaining with H_p .	Designed for static network only. No concern of topology fault-tolerance has been shown. A work to have fault-tolerant and minimum delay topology can be explored in future.
2015 [80]	Provide the study of NP-completeness of K -Distinct SMET problem. A hierarchical structure of K -Distinct SMET is introduced, named as ‘ k -Distinct-SMEHT’ problem and an approximate solution is provided which reduces total power consumption. Moreover, a polynomial time solution has been shown for the 2-Distinct-SMET problem.	Methodology of algorithm is using extension tree concept and idea of joining r -distinct spanning tree to form a single spanning tree of a complete weighted bi-directed graph model of WSNs.	Proposed solution is not optimal and suitable for static network only.

Table 1 (continued)

Reference	Contribution	Approach/Methodology	Limitation/Future scope
2016 [83]	Authors proposed topology adaptation algorithm (TAA) and preprocessing algorithm (PA) to obtain minimum power consumption and connected bidirectional routing topology in Heterogeneous WSNs with Time complexity $\leq n^3 \log^2 n$. This work is extendable to different types of nodes having different power levels. More fault tolerant than conventional sparse topology.	There are super nodes and ordinary nodes present with a sink node. Each super nodes are supposed to keep k -disjoint paths with sink nodes and ordinary node keeps a single path to sink node using graph transformation and max-flow theory. TAA algorithm follows MST initially to establish the connectedness among all the nodes and sink, then more links have been added to MST to achieve k -connectedness of super nodes. After this graph transformation takes place to convert node-disjoint paths to edge-disjoint paths using max flow theory and best k -paths have been kept. PA algorithm is used after TAA to remove unnecessary long links without disturbing connectivity if required.	Super nodes makes the network quite expensive. Proposed work is suitable in static network only.
2017 [84]	Authors proposed two algorithms for the SMET problem. first one is the improvisation of Prim-incremental heuristic in term of time complexity and second is the hybrid of Boruvka and Prim algorithms. The hybrid heuristic is more energy efficient and faster than the previous versions of incremental heuristic on the planar graph.	Methodology of first algorithm joins the nodes in greedy fashion of increasing total power consumption to form a spanning tree. There exists a set for every node to choose its closest neighbour directly based on the increased power level. Second algorithm forms MST using the hybrid methodology of Prim and Boruvka algorithms. Each vertex is taken as a single component and edge of least power consumption is selected from each component to form a new contracted graph. Selection of edges from contracted graph is done to form MST following Prim's algorithm.	No approximation bound on optimal solution has been provided. Algorithms are restricted to planar graphs only. Also nodes are assumed to be immobile in behaviour.
2017 [85]	Proposal of an integrated power adjustment and clustering based topology control method known as 'Collaborative Topology Control Protocol (CTCP)'. This approach reduces power consumption and improves stability factor in large heterogeneous WSNs. This approach causes less overhead in topology setup phase.	The methodology starts with minimum connected domination set (MCDS) theory to select dominated nodes as CHs and establish a base topology of CHs. Thereafter a power adjustment algorithm has been initiated to assign distinct transmission range to each MN within a group by its CH as each CH keeps topological map of its surrounding. Transmission range of each node can be varied to achieve desired topology as smaller range of MN causes many CHs of long transmission range which causes more energy cost and long range of MN reduces energy cost caused by CHs. with the increment of hop distance.	This method reduced hop distance at the cost of keeping dense network which causes network interference. The proposed work is not suitable when nodes are mobile since it complicates the working of algorithms in topology establishment.
2018 [24]	Authors improved approximation bound of achieving optimal solution of DPMP and proposed a polynomial time algorithm which uses the concept of second hamiltonian cycle.	The methodology is using the contracted set theory with second hamiltonian cycle effects minimum number of nodes with high transmit power on an induced directed graph model of WSNs to obtain connected subgraph.	Designed for static and directed network. The work can be extended to mobile topology with bi-directional links.

Table 1 (continued)

Reference	Contribution	Approach/Methodology	Limitation/Future scope
2019 [86]	Authors proposed two polynomial time algorithms to obtain the solution of DPMP under two cases, 1. Power levels are chosen from the given discrete set and 2. Power levels are taken as input. Lower and upper bounds on optimal number of sensor nodes assigned with high power for DPMP have been obtained. The second algorithm performs better than first to minimize total power consumption with connectivity constraint.	First algorithm starts with the finding of MST and then assignment of transmit power to each node in MST. H_p is taken maximum of assigned transmit power to maintain the connectivity and remaining transmit powers work to find L_p . Each node whose transmission range is less than H_p would be assigned with low power level L_p , so this way minimum number of sensors is assigned with H_p and total power gets minimized in the connected network. The methodology of second algorithm is to assign low power to all the nodes inducing a spanning forest which consists of one or more components. Assignment of high power is done to only those nodes which effect maximum number of connected components.	The obtained solution is 2-approximate, lacking optimality. Both the algorithms are suitable for static network only.
2019 [87]	Proposal of an virtual topology based routing protocol where nodes can vary their transmission range. The proposed algorithm is distributed in nature and improves network life, routing cost and having polynomial time complexity. This is better than the previously existing CDS based techniques.	In this, authors modelled the WSN as a disk graph and engaged minimum connected dominating set (MCDS) to have the connected topology among the nodes which keeps quick search of routes with shorter distance by keeping minimum disc radius and variable transmission range.	This approach is not much suitable for directed and mobile WSNs. Even though nodes can have variable transmission range but the effect of topology construction on nodes power level didn't show explicitly.
2019 [17]	Formulate a k -power level assignment problem and provide a near-optimal solution by proposing two polynomial time algorithms. First is MST-based and second is Incremental heuristic. Incremental heuristic performs better in minimizing total power consumption than MST-based.	The methodology of first algorithm starts with the set of k -power levels in ascending order and computation of MST. According to the weight on each edge on MST, transmit power from the set of k -power levels are assigned using divide and conquer rule i.e. if weight of node u is lying between p_i and p_{i+1} , then u is assigned with p_{i+1} . The methodology of second algorithm forms a spanning tree with one by one addition of edges in increasing order of their weights. The difference from the former algorithm is that this algorithm maintains two sets for vertices, one shows visited vertices and other unvisited.	In MST-based algorithm if weight on any edge is more than the available power level then no feasible solution exists and algorithm gets terminated. Also it is designed for static network only.
2021 [81]	Proposal of an interval graph based routing topology which minimize nodes' energy consumption and results an appropriate assignment of power level to each node while ensuring topology connectivity. This is the first approach towards WSNs to model it as an interval graph where nodes are explicitly assigned with power intervals and proposed algorithms are energy optimal and having polynomial time complexity.	Each node of WSN has been assigned with power interval ranging from minimum power level to maximum power level, required to approach different destination node. According to nodes power interval, sorting has been done to form right end interval graph model of WSN. Three algorithms have been proposed to construct an energy optimal routing topology which minimizes maximum nodes' power consumption, maximum transmission cost of worst case and total transmission cost of worst case. Methodology of proposed algorithms are based on to join maximally assigned power interval to minimally assigned power interval because transmission energy along that path is minimum.	Proposed work is suitable for static WSNs. Algorithms are centralized in nature and restricted to the sorted model of WSNs and only those WSNs whose underlying graph can be an interval graph.

Table 2 Overview of 'Interference minimization approaches' in topology control

References	Contribution	Approach/methodology	Limitation/future scope
2009 [36]	Proposal of two polynomial time algorithms MI-S and LS-IM to minimize maximum interference. Both the algorithms show good performance in some instances. Also analysis of the proposed algorithms in best and worst case scenario has been shown.	MI-S algorithm works to join edges in increasing order of their weight to form a spanning tree which minimizes maximum interference of the network. LS-IM involves local search technique to improve the solution obtained by MI-S if there is any change in network topology because of addition or deletion of edge.	Algorithms don't minimize interference optimally in every instances and is limited to 2-D topology.
2011 [89]	Proposal of first exact algorithm to minimize average interference and provide proof to the bound on maximum interference while minimizing average interference in 2-D network. Authors presented study of minimization of average interference in connected topology using no-cross property and proposed a fast exact algorithm to minimize the average interference for 1-D model in polynomial time.	Using the concept of mutual exclusion of long edges, computational geometry based on distance and transmission range, algorithm 1 obtains low interference spanning tree. Algorithm 2 considered mutual interference and right side topology for the nodes lying from left to right to construct a low interference topology.	Suitable for static nodes deployment and low dimension topology. Assumption of interference range is same as transmission range is limiting this work. Work can be extended to network where transmission range is taken more than interference range.
2011 [90]	Proposal of exact algorithms to minimize receiver-centric interference for Highway model. Multiple planar connected topologies with reduced average and maximum interference can be constructed using these algorithms.	Utilizing no-cross property to obtain connected planar topology and dynamic programming, algorithm 1 yields optimal spanning tree for a highway model.	Algorithms are designed for static network using 1-D Interference models only. Minimizing the maximum interference for the highway model is still an open problem.
2011 [91]	SBYaoGG (Smart Boundary Yao Gabriel Graph) for low interference connected network has been proposed. This optimizes transmission cost of symmetric links and resultant topology has energy efficient and low-obstacle routing.	SSYaoGG algorithm is the hybrid of GG and YG algorithms adopting smart region boundaries. This algorithm involves neighbour discovery utilizing maximum transmission range, computation of GG from the UDG yields planar topology with linear number of edges then computation of average direction vector and YG on reduced topology yields power spanner connected planar topology.	Reduced topology is sparse and every node sets to minimum transmission range reduces link reliability and quality of service (QoS).
2012 [92]	Proposal of an interference minimization technique by reducing the communication radius of nodes which therefore reduces network interference while maintaining connectivity constraint. This defined technique is applicable to asymmetric case also.	Technique works in localized way i.e. let r_{min} is the boundation set on the communication radius of each node which is actually the longest edge distance and the sensor field is partitioned into cubes of side length r_{min} which therefore results communication radius of each node inside any cube less than r_{min} .	Assumption of uniform range assignment to each node limits the applicability of the proposed technique. Also working analysis using mobile nodes has not been given.
2013 [15]	Proposal of 'Minimum Interference Strong Bidirectional Topology (MISBT)' algorithm with two new interference models, SUM and MAX, to estimate coverage-based and transmission-based nodes interference. Polynomial time MISBT algorithm under MAX model optimally minimizes maximum nodes interference and yields 2-approximate solution to min-average nodes interference.	MISBT algorithm involves the computation of complete weighted graph of the given WSN where the weight on each edge is found by adopting either the MAX or SUM model, then computation of MST yields minimum interference connected topology and reduced power assignment to each nodes.	Algorithms result magnitude of available Interference in integer number although adding the situations like signals fading and collision, interference should be taken as a real number, varying in nature. Designed for static WSNs. The work can be extended by allowing interference to result in a real number considering real-life applications.

Table 2 (continued)

References	Contribution	Approach/methodology	Limitation/future scope
2015 [37]	Presented a new link interference model which counts the actual number of interfering links with the given link by discarded the previously designed link interference model based on coverage. Author proposed a centralized greedy algorithm which construct an interference-optimal topology based on the newly proposed link interference model. The proposed link interference model is independent and can be applied to other topology control algorithms as well.	Working of the algorithm is to sort links in increasing order of their weights which is depending on link interference and distance, then insertion of links one by one to obtain an energy spanner connected topology. This reduces the chances of loop formation and redundant addition of links as one by one insertion of links is adopted.	Algorithm is defined for static WSN only. Individual focus on interference reduction makes network sparse brings reliability and energy hole problem.
2016 [38]	Proposed a fault-tolerance and interference-aware topology control (FICTC) algorithm which is having polynomial time complexity. FICTC minimizes maximum link interference in bidirectional network and meeting the fault tolerant requirement.	The algorithm works to achieve different k-connectivity between different pair of nodes by establishing k-disjoint paths between any pair of nodes. This is done according to obtain minimum vertex separation set with the use of nodes splitting technique. This splitting technique employs two nodes like v' and v'' in place of a node v in the original topology to fulfil the connectivity requirement. Then Prim's algorithm has been used to find augmented paths between the nodes on the resultant topology, obtained after splitting operation and updation of links capacity on the obtained augmented path. All the augmented paths merged together to obtain final k-connectivity topology with reduced link interference. The first algorithm MDTC is centralized in nature adopted greedy strategy like LISE in which links are sorted in ascending order of their weights. Weights are the link interference and links join themselves if they satisfy delay bound in the increasing order of their weights. Second algorithm is DMST which builds a delay-constraint MST to minimize average links interference with restricted hop counts to satisfy the delay bound. DMST adds links of minimum interference whose hop counts must be less than imposed delay. A local version of DMST is also proposed by author in which nodes make next move on the basis of neighbour information i.e node choose another node if the corresponding path has minimum interference satisfying delay constraint. A polynomial time LDBF algorithm is proposed to minimize path interference. LDBF utilizes the concept of Bellman-ford algorithm in a localized way with the inclusion of transmission delay restriction over the path.	Defined for static network only. Adoption of k-disjoint paths between every pair of nodes causes more data transmission and so more energy consumption. MDTC has higher time complexity. All links are assumed to have same transmission power. Delay is measured on the basis of hop counts only. Although delay is based on link quality and distance.
2018 [88]	Proposed one centralized and two distributed topology control algorithms which jointly work on reducing link interference and transmission delay in the connected network. Authors undertook three optimization problems: minimize maximum interference, minimize average interference and minimize path interference.		

Table 2 (continued)

References	Contribution	Approach/methodology	Limitation/future scope
2019 [93]	Adopted the strategy of additional nodes deployment in the sensor field at those position which helps in minimization of sender interference without disturbing the established nodes position. Authors proposed a polynomial time algorithm which minimizes the maximum sender interference with preserving connectivity.	Working of the algorithm named as MSD, is to connect nodes to their nearest nodes which results each edge is Gabriel and induced a spanning forest. A MST is obtained from the resultant GG such that some edges belong to MST not to the spanning forest. Then a new node is positioned at the bisection point of that edge which further reduces the range assignment of the end nodes ensuring connectivity requirement.	Designed for static network. Deployment of more nodes causes more hops to cross which may lead transmission delay.
2019 [17]	Proposal of MST-based algorithms to minimize maximum sender and receiver interference with DPMP. Algorithms yield minimum interference connected topology and having polynomial time complexity.	Working of proposed algorithms is to partition edges into two sets i.e. high power set and low power set according to their weights where weight on each edge is defined on the basis of Euclidean distance. Edges with low power are added first to construct topology following the computation of sender (or receiver) interference of each node. Then high power vertices are sorted in ascending order of their sender (or receiver) interference and added accordingly to the topology with high power assignment until a loop free connected topology has been obtained.	This work is limited to interference minimization with dual power assignment to stationary nodes.

energy can be taken into consideration so that the existing topology could be reorganized and managed. Following are some defined terms which are used in Table 2.

- **Highway model:** It is obtained when nodes are distributed linearly in one-dimension only. It is established to obtain a lower bound on network interference.
- **Unit disk graph (UDG):** UDG models WSNs in which two nodes communicate if and only if the distance between them is at most $r \leq 1$, where r is the transmission radius which is equal for all nodes.
- **Gabriel graph (GG):** GG models WSNs in which two nodes u and v communicate iff the disk obtained using the uv as a diameter doesn't contain any point.
- **Yao graph (YG):** YG models WSNs with an integer parameter $k \geq 6$ which defines k cones. In each cone, node v chooses closest neighbour node u with the distance atmost 1.

7 Review of routing techniques

A thorough review has been conducted according to the classification made in sect. 3. Review of network structure based classical hierarchical routing protocols, topology aware optimization technique based PSO and ACO routing techniques and delay-sensitive application based real-time routing protocols, is presented using tabular format to offer the reading convenience. Each routing category is the most evolutionary, well established and quite applicable. Still there are lots of persisting issues with each categorised technique which we have discussed as well.

7.1 Classical hierarchical routing protocols

This section covers the review part of classical hierarchical routing protocols with description of basic methodology, terms and abbreviation which have been used in documenting the reviewed data in Tables 3 and 4. Table 3 presents brief overview of most popular energy efficient classical hierarchical routing protocols from year 2001 to 2016 which have been extracted from previous year survey papers [2, 4, 6, 20, 67], to offer readers idea of their methodology, merits and demerits. Table 4 presents the insights on latest classical hierarchical routing protocols from year 2017 to 2021 by mentioning the contribution made by the proposed protocol which includes the beneficial part, different approaches/ methodology opted and persisting limitations which might be overcome in future work. Research on hierarchical routing protocols have overcome the many limitations of flat based and location based routing protocols in

term of scalability, energy consumption, latency, dynamic topology, algorithmic complexity, load balancing, network heterogeneity and life cycle. The basic methodology and aim involved in this type of routing is the formation of optimal number of clusters, cluster size, appropriate selection of cluster head (CH) node in each cluster, optimal assignment of member nodes (MNs) to each CH, known as cluster member, optimal intra and inter-cluster routing. A basic network structure of cluster based/ hierarchical routing is shown in Fig. 7. In hierarchical routing, cluster of nodes can take any shape like chain, tree or grid since partition of target area has been done in many shapes like square, circles, grids and concentric circles (refer Table 4).

Some authors introduced the concept of rendezvous points (RPs)/ sojourn points (SPs) selection in cluster-based routing where mobile sinks (MSs) are engaged to aggregate data from member nodes and/or CH nodes by halting at each of RP/SP [94, 95]. Basically RPs and SPs are stoppage point for mobile sink (MS) node and authors have been dealing to find optimal RPs/ SPs to improve load balancing and reduce energy hole problem specifically. Both RPs/SPs and CHs are chosen on the basis of distance factor, member node density and energy factor. Irrespective of this, intra-cluster and inter-cluster routing are need to optimize. Some authors [96] involve time division multiple access (TDMA) mechanism to intra cluster routing where member nodes transmit data in a specified time slot to CHs and keep themselves inactive otherwise. Use of code division multiple access (CDMA) [97] to inter-cluster routing where a CH sends its data only by acknowledging code associated to it. Both TDMA and CDMA avoid unnecessary data collision, transmission, congestion at CH node or BS and consequently save nodes energy. Following are the key points, we draw from Tables 3 and 4.

- These techniques form routing topology for both homogenous and heterogenous WSNs.

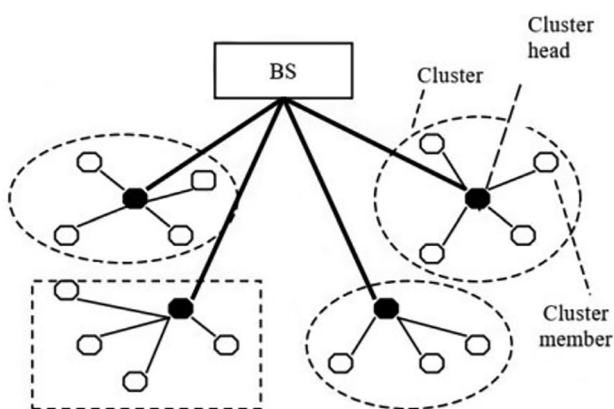


Fig. 7 Network structure of cluster based/ hierarchical routing

- Some routing protocols are suitable for both static and mobile WSNs by offering mechanism of re-clustering and re-routing and some limit themselves to static network only.
- Cluster formation makes the network more scalable and energy efficient yet they are showing limitation when timely data delivery is required. For this some author brings the concept of engaging mobile sinks (MSs) so that they themselves come to collect data from the shortest route possible but not on the urgency basis. Therefore timely data delivery is the persisting issue.
- These routing causes overhead and energy-hole problem when the hierarchy formation is chain and tree like.
- The most evolutionary part of these techniques is to have fault-recovery mechanism using beacon message exchange makes the network more reliable and fault-tolerant. However more techniques to deal with message overhead issue, fault-tolerance and energy-hole problem in dense, heterogenous and dynamic WSNs need to develop.

Moving to more stable, scalable, robust and multi-objective optimizing PSO and ACO based routing techniques.

7.2 PSO and ACO based routing techniques

From the previous section, we see that topology formed using classical hierarchical routing protocols is not robust to nodes movement and failure. Also the proposed techniques lag their performance while solving more complex multi-objective problems. So to overcome these issues and to find optimal solution of multi-objective problems, PSO and ACO based routing protocols/ techniques have been proposed for many past years. Both PSO and ACO techniques have lower complexity, more stability and robustness than classical hierarchical routing techniques. Both PSO and ACO work globally by acknowledging the whole network and setup routing topology that's why we call PSO and ACO, topology aware optimization techniques. Moving forward to look latest routing techniques using PSO and ACO. Many survey papers exist covering these techniques till the current year, so this survey paper provides the review of well established articles from year 2017 onwards and analysis of each technique is presented using tabular format by mentioning the objective/ contribution of the proposed work along with its benefits, detail of approach/ methodology opted with persisting limitations. Table 5 presents the review of PSO based routing techniques and Table 6 presents the review of ACO based routing techniques. For better understanding of PSO and ACO technique based routing schemes, an introduction of PSO and ACO technique is provided here.

Table 3 Summarization of classical hierarchical routing protocols, reviewed in previous survey papers

Routing protocol	Brief overview
Low-energy adaptive clustering hierarchy (LEACH) [98]	The first hierarchical routing protocol which is localized and distributed in nature, where nodes opt direct communication to CH and sink nodes. LEACH saves transmission energy and network lifetime upto small extent as CHs consume lot of energy if sink node is located far from CHs. Not suitable for large dynamic network as it causes data congestion at CHs and sink nodes. Only applicable to homogenous network since each node is assigned with same power level. Many modified version of LEACH have been proposed to overcome the issues concerned with energy, CHs selection and network lifetime.
Power-efficient gathering in sensor information systems (PEGASIS) [99]	Chain topology formation by sensors and sink nodes to aggregate and transfer data rather than multiple clusters formation. PEGASIS outperforms LEACH since it adapts to different network sizes and topologies, reduces extra overhead of LEACH by using multi-hop routing. Causes excessive delay due to chain topology for distant nodes and wastage of energy due to redundant transmission. To overcome the issues of delay and energy consumption, many modified versions of PEGASIS have been proposed as well.
Threshold sensitive energy efficient sensor network (TEEN) [100]	Hybrid of hierarchical and data-centric mechanism, reduces number of transmission and so energy consumption. Suitable for time-sensitive data aggregation and transmission so it is not applicable where continuous data flow are needed. Since the methodology of TEEN is threshold-based, the protocol causes routing complexity.
Adaptive threshold sensitive energy efficient sensor network (APTEEN) [101]	Extension of TEEN protocol, offers the modification in periodicity and threshold value of TEEN as per the requirement. Causes additional complexity on monitoring and altering threshold values. APTEEN consumes lesser energy than TEEN.
Virtual Line-based data dissemination (VLDD) [102]	A virtual line structure (VLS) has been constructed by the nodes so that lines can be acted as rails to define relay path between source to the location of MS. Engagement of MSs and VLS structure helps in reduce delivery time, redundant transmission and so energy consumption. Although frequent VLS formation causes extra overhead and long line structure may cause transmission delay in large-scale WSNs.
Virtual Grid Architecture (VGA) routing [103]	Fixed cluster-based routing topology formation where each sensor node keep static at their initial position. Achieves energy efficiency and maximization of network lifetime. Selected CHs are not optimal.
Base station controlled dynamic clustering protocol (BCDCP) [104]	In this protocol, BS plays the role of CHs selection based on energy level and each cluster shares the same number of member nodes. Avoids congestion at CH nodes and improves network lifetime.
Energy-balanced chain-cluster routing protocol (EBCRP) [105]	Clusters are formed by partitioning the sensing area into rectangular zone and CHs are chosen based on residual energy level. Chain connecting the CHs brings the routing path to sink node directly. CHs rotation results load balancing and enhances network lifetime. Adoption of single-hop data transmission, more energy is consumed if nodes are distance apart and so not suitable for large scale networks.

Table 3 (continued)

Routing protocol	Brief overview
Chain-based hierarchical routing protocol (CHIRON) [106]	Multiple clusters are formed by partitioning the sensing area into fan-shaped area. Nodes with higher residual energy become CHs and chain of data aggregation and routing starts from the most distant node from sink node. Adopting multi-hop intra and inter cluster routing, so this is suitable for large scale networks. CHIRON is more energy efficient, scalable and reduces delay factor in data transmission compared to PEGASIS and ECBRP.
Energy-aware data aggregation tree (EADAT) [107]	Centralized protocol since tree topology is formed by sink node where nodes with high residual energy become parent node and as the energy of parent node depletes child will switch to different parent. Overall path establishment is done on the basis of residual energy so it results increment in total energy consumption and longer time delays since distance factor has been avoided.
Balanced aggregation tree routing (BATR) [108]	A centralized protocol which constructs MST based on energy consumption and doesn't consider residual energy factor so it may lead transmission failure. Child nodes are uniformly distributed under each parent, leads to load balancing and increment of network lifetime.
Enhanced tree routing (ETR) [109]	A distributed protocol form tree-like hierarchy where each node keeps routing table containing next-hop details and path of lowest hop count is preferred. This protocol saves transmission energy until nodes have sufficient residual energy otherwise transmission failure happens.
A novel energy efficient clustering protocol (NEECP) [110]	In this protocol, nodes can adjust their sensing range to form clusters and select CHs. Adjustable sensing range is based on distance parameter between the nodes and sink node. This protocol is the solution of energy hole problem and minimizes the energy consumption.
Grid-based multipath with congestion avoidance routing (GMCAR) [111]	Grid-based cluster formation such that one CH is appoint to each grid for data aggregation and sending data to adjacent grid CH. GMCAR is good for dense and large scale routing since this offers backup CHs and multi-path data transfer if required. It is energy efficient, reliable and reduces transmission delay.
sleep scheduled and tree-based clustering (SSTBC) [112]	This protocol makes redundant nodes to sleeping state with the formation of tree based routing structure. More energy efficient than ETR, BATR and EADAT.

PSO technique PSO is an abbreviation of 'particle swarm optimization' imitating the behaviour of birds flocking in search of their food. PSO technique can solve complex multi-objective optimization problems with the feature of available swarm particles and move these particles in the search-space according to simple mathematical formula (or any multi-objective fitness function f) over the particle's position and velocity. When it comes to routing in WSNs, particle size is number of sensor nodes, say N_p and let D is the dimension of each particle, depending upon the dimension of objective function (fitness function) defined. Each particle has its own position and velocity, let X_{ij} and V_{ij} is the position vector and velocity of the i^{th} particle P_i and $1 \leq j \leq D$. Each P_i is represented as $P_i = [X_{i1}, X_{i2}, \dots, X_{iD}]$.

A multi-objective fitness function has been formulated by authors to evaluate each particle quality towards the solution. Each particle iteratively improve its solution quality to converge to an optimal solution which is globally accepted. To achieve optimal solution, each particle follows its own best, called P_{best} and the global best called G_{best} to update its own velocity and position. In each iteration, velocity V_{ij} and position X_{ij} in the dimension D is updated using the following equations, respectively

$$V_{ij} = w \times V_{ij}(t-1) + c_1 \times q_1 \times (X_{P_{best}ij} - X_{ij}(t-1)) + c_2 \times q_2 \times (X_{G_{best}j} - X_{ij}(t-1)) \quad (2)$$

Table 4 Overview of Latest classical hierarchical routing protocols

Reference	Contribution	Approach/methodology	Limitation/future scope
2017 [113]	A new distributed unequal cluster-based routing (DUCR) algorithm aiming to mitigate coverage/energy hole problem in heterogeneous immobile WSN has been proposed. DUCR algorithm utilizes energy of sensor nodes in an efficient manner, avoids the energy hole problem, improves network lifetime and tolerant capacity. It achieves constant time and message overhead.	DUCR algorithm dynamically reduces cluster's size as it requires to form more clusters near BS for load balancing and relaying. CHs adapt their cluster sizes according to their residual energies i.e. cluster's size reduces with depletion of energy over time. Nodes with higher energy selected as CHs and MNs choose their appropriate CH with the help of elementary, normalized and cost matrix. Factors involved in formation of elementary matrix are CHs residual energy, CHs distance from BS and distance of CHs from MNs. This elementary matrix is normalized for computation of cost matrix where the maximum value in the cost matrix gives the best CH to the MNs. Intra-cluster routing is done using TDMA mechanism and inter-cluster routing requires CHs to distribute their data using a cost function to the lower level CHs on receiving beacon messages. If they are not available CHs directly do the transmission to BS maintaining network connectivity. DUCR involved fault-recovery mechanism by removing dead node, re-clustering and re-routing with the help of nodes local information.	No control on number of CHs. Multi-hop inter-cluster routing causes latency. Designed for static and small scaled network only.
2017 [114]	Authors proposed a cluster-based approach to control the transmission power problem and energy hole in strip-based WSNs. The proposed work is the very first contribution to the strip-based model of WSNs which has applications in mines, railway lines, bridges etc. The method improves network lifetime and offers the optimal relationship between transmission distance and node position.	The methodology engages accurate distance based division of target region into different layers as clusters and each layer keeps a CH. CH keeps on change as per rotation of data transmission within a layer. Intra-cluster routing has been done in single-hop manner and inter cluster routing can take multi-hops. Total energy consumption of every layer has been computed and in order to maximize life of CH of the layer, closer to sink, author brings optimal length of each layers and transmission distance as layer closer to sink node should has smaller length so that the corresponding CH carries smaller transmission range to extend its life.	Assumption of static sensor nodes and initial power levels of each node must be same except sink node limits the application of this work.
2017 [115]	proposed an 'energy aware path construction (EAPC)' algorithm to further improve path efficiency for data collection, prolong network lifetime and reduce energy consumption.	The methodology starts with the formation of MST in the target area and thereafter MST has been partitioned into disjoint tree like structures with the root node is CP. There are as many trees as many CPs and each CP collects information from rest of the nodes belonging to its corresponding tree. MS builds the trajectory to visit each CP in such way that the most burdened CP must be visited first while satisfying distance constraint, buffer constraint and battery constraint.	Assumption of static sensor nodes and mobile sink having same communication range limits the applicability of the proposed work.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2018 [116]	Authors proposed a duty-cycle based clustering routing protocol. This approach minimizes transmission energy, improves network lifetime, reduces unnecessary data transmission and collision of data.	This approach is consisting of four main phases: clustering, sleep/awake scheduling of nodes, two level data aggregation and inter-cluster and intra-cluster routing. Initially weight of each node is computed based on nodes energy level and node degree and then clusters are formed using Voronoi diagram. Nodes with highest weight becomes CHs and based distance each CH chooses its group member. In each cluster, nodes except CH are set to sleep and awake cycle using MAC protocol by assigning appropriate time slots. Data aggregation is done by intermediate node as well as CHs based on the corresponding node position. Intra-cluster routing used a 'hybrid chicken swarm optimization algorithm' which reduces redundant data transmission and compress the data to transmit via optimal route. Inter-cluster routing has been done by constructing tree structure based on sink node position.	Defined for static and small network only. Authors didn't provide the complexity of the proposed work. This work can be extended to dense network in future.
2018 [117]	A clustering-based routing protocol for dynamic WSNs (CRPD) has been proposed. CRPD supports updation of network topology in case of dynamic addition of new nodes and their partial mobility in energy efficient way. This protocol has constant message and time complexity.	Network topology gets updated periodically by doing re-clustering and re-routing with the nodes mobility and energy depletion of CHs. Only nodes which is having more than 40% of their initial energy can compete for CH position and among those, the node which is having maximum degree chosen as CH. Also there can be more than one node receiving <i>youaretheadmessage</i> in the same ngh, for this, author introduced the intelligent waiting policy in which nodes have to wait for certain time before taken itself as a CH as they may receive <i>iamtheadmessage</i> from the actual CH within a given time and this makes selection sorted. Author used <i>ack</i> and <i>ACK</i> confirmation messages for reliable data transmission.	Immobility of sink causes data transmission delay. This method is defined for homogenous WSN only. The network lifetime of CRPD decreases with the increase of network size.
2018 [118]	Proposal of a 'Cluster-head Restricted Energy Efficient Protocol (CREEP)' concerning heterogeneous WSN. The focus of CREEP is to enhance network lifetime and throughput. CREEP lowers system complexity by limiting no. of CHs and performs better as compared to other existing protocols in heterogeneous environment for both stationary and mobile WSNs.	In CREEP some nodes are assume to be advanced and rest are normal. A distance component is introduced with residual energy factor in selecting CHs to ensures that the nodes far-away from BS get lesser chance of becoming a CH. CREEP approximates the square WSN field to a circular field as a disc of radius R and BS at its centre. CHs inside that disc adopt single hop routing while others use dual-hops. CREEP puts the limit on number of CHs nodes by introducing threshold on number of alive nodes inside the clusters.	CREEP has been designed for static network with BS located at centre, limits its applicability. Algorithm can be improved by including spatial distribution of nodes during CH selection process.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2018 [119]	Proposed a centralized 'hybrid energy efficient multi-path routing protocol (HEEMP)' for heterogeneous WSN. This offers multiple path availability which makes network more reliable. Also HEEMP increases network lifetime.	In HEEMP, sink does all network management task by keeping every node information, select CH nodes by calculating 'Chance of election (CE)' parameter based on normalized value of residual energy and node degree. Sink initiates multi route construction after CHs selection in which it forms a 'Route Set', containing all possible routes from CHs to itself and then reduces it to 'Legitimate Route Set', consisting of those routes in which nodes have sufficient level of residual energy. Intra-cluster routing is done either in 'Philanthropist mode' or 'selfish mode'. In philanthropist mode, node transmits to one of its neighbour node which is having maximum remaining energy whereas in selfish mode, node only aims at reducing its own energy expenditure by sending packets to the nearest progressive node. The target area has been partitioned into equal size rectangular area and distance between the nodes has been estimated with the help of received signal strength. Number of clusters and cluster size has been chosen based on distance from BS and network density. There are more clusters of small size nearer to BS to balance the burden among the CHs. The node with highest power level is chosen as CH in heterogeneous WSNs and rotation of CHs happens after reaching below a power threshold. Intra cluster routing follows TDMA schedule and Inter cluster routing follows CDMA code.	HEEMP relies on sink to compute the topology, which may result in longer delays. Assumption of immobile network and error free communication restricts HEEMP applicability.
2018 [97]	Proposal of an unequal clustering approach to mitigate energy hole problem using multi-hop routing, named UCR-H. This approach is suitable with heterogeneous energy level of nodes and minimize energy consumption with the improvement of network lifetime.	In this, network has been partitioned into square shaped cells of equal area and nodes closer to partitioned line, called intersection nodes, save the position of mobile sink node after every updation. Rest of the nodes keep the location information of their neighbour nodes including intersection nodes belonging to the same cell and based on the distance, nodes transmit their data to intersection node using minimum hops. Intersection nodes transmit data to sink node if they satisfy threshold based on Euclidean distance and residual energy. In case of failure of intersection node, a neighbour node of minimum distance, satisfying threshold is selected as an alternate intersection node using beacon messages.	BS are placed outside the rectangular target field and all the sensor nodes are assumed to be fixed, limits the applicability of UCR-H.
2018 [120]	Authors proposed a virtual-grid infrastructure and mobile sink based routing algorithm. This approach reduces hot-spot problem, end-end delay and balances energy consumption as compared to other similar algorithm. The proposed work is suitable for small as well as large WSNs. Also this doesn't need trajectory definition of MS and can operate for multiple MS.		Designed for static and uniformly distributed sensor nodes. Since the proposed algorithm is defined for 2-D model based. This can be extended for higher dimension.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2018 [94]	Proposal of a distributed data gathering technique consisting of cluster-based RPs selection algorithm and three trajectory design algorithms for MS with three different objectives: reduced energy path (REP), reduced delay path (RDP) and delay bound path (DBP). REP generates an energy-efficient trajectory that avoids the sensor nodes from using multi-hop communication. The RDP follows the same technique with delay reduction by cutting of REP traced path. The DBP algorithm determines a time restricted trajectory. These algorithms increase network lifetime.	COST and WEIGHT are defined parameters for RPs selection, the sensor nodes having highest COST among their nigh nodes, announce themselves as RPs and rest of the nodes join RPs in their nigh of lowest WEIGHT. After this, path formation for MS is initiated where MS has to visit each RPs for data collection. Author has given three types of path formation algorithms, 1. REP which determines path of minimum energy consumption. Author used Christofis's heuristic here. 2. RDP which determines delay reduced and energy efficient path. In this, some of RPs are excluded from the path if their transmission range disc intersecting with any edge of the acquired path. 3. DBP designs a delay sensitive path by restricting path length travelled by MS. If the designed path's total length exceeding the defined threshold then one or some RPs need to eliminate which falls in the middle of two adjacent edges of maximum path length sum.	RDP consume more energy in data collection and transmission to the MS. Assumption of MS has enough energy to gather data from all the RPs restricts the applicability of protocol.
2018 [121]	Authors proposed a 'Grid-Cycle Routing Protocol (GCRP)' using mobile sink to minimize hot-spot and overhead problem caused by location updation of MS. This approach improves network lifetime and timely data delivery. This approach is more scalable and stable than the existing grid-based routing protocols.	GCRP partitions the target area into square shaped grid cells, each cell is like a cluster and in each grid, grid cell head (GCH) like CH has been selected based on distance from the cell centre and residual energy. Joining of the all GCHs form exterior cycle where the boundary of each GCH is involved and interior cycle where no GCH around the GCHs and collect data from the closest GCH. Reselection of GCH has been carried out when the existing GCH energy level gets lower than the defined threshold. Only a closet GCHs update themselves with MS position and circulate the information.	Designed for static and homogenous network. Also assumption of obstacle free path between two nodes makes the proposed work impractical.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2019 [96]	Proposal of 'cluster-based fault-tolerant routing (CFTR)' protocol to mitigate frequent re-clustering which reduces overhead as well as extra energy consumption to tolerate nodes failures. CFTR is suitable for heterogeneous large-scale network, having constant time and message complexity.	Working of the protocol is to have higher energy nodes as CHs and operate multiple rounds without repetitive clustering. A cost function is defined having parameters: Energy Cost (EC) and no. of cluster members in the previous round (NCPR) for MN nodes to select appropriate CHs. There exists a case where some nodes are not able to find any CH within their communication range, CFTR sorted this situation by engaging multi-hop scheme, appointing some nodes as relay nodes and establish connection to an approachable CH. Intra-cluster routing is done using TDMA schedule and inter-cluster routing requires CHs to route their data via other CHs using a cost function that considers routes with sufficient energy for data forwarding and load is evenly distributed among the CHs. Further author introduced a low overhead fault-tolerant mechanism to cope up with CHs failures by employing remaining CHs nodes and normal nodes as backup nodes.	Designed for static WSN only. Limitation on optimal number of CH nodes and recovery strategy can be done only on CH nodes.
2019 [122]	Proposal of an energy saving routing approach by integrating geographic routing with hierarchical and using mobile sinks, abbreviated as IoGHR. This approach increases network lifetime, decreases energy consumption as well as latency factor.	Based on the nodes density, target area has been partitioned into equal size grids. Each grid consists of 4 MNs and 1 CH. CH has been selected based on highest power level or residual energy. Sink node sends its location to each CHs and moves using time schedule, this minimizes message overhead and data collision since each CH is aware of sink time schedule and so pass their data at appropriate time. Intra cluster routing can use maximum 2-hops for data transmission.	The proposed work is not suitable when nodes are mobile. The work is extensible to multiple MSs but this makes the network quite costly.
2019 [123]	proposal of a cluster-based 'energy-efficient scalable routing algorithm (EESRA)'. The proposed work extends the network lifetime in a large scale network which is a drawback of LEACH protocol.	In EESRA, each CH keeps cluster congregation (CG) nodes to aggregate sensed data from the CMs using CSMA manner and pass their data to the corresponding CH using TDMA manner. Thereafter CHs transmit data to BS. That's how EESRA forms three-layer hierarchy topology to reduce the CH load.	Assumption of all sensor nodes are homogeneous and fixed after deployment in a square two-dimensional space limits the applicability of EESRA. Moreover, the space is assumed to be free of obstacles is quite impractical.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2019 [124]	Authors proposed an 'energy-balanced tree-based data collection (ETDC)' strategy using MS. ETDC reduces transmission cost with end-end delay and prolongs network lifetime. ETDC is suitable in large-scale network as well. ETDC has polynomial time complexity.	Target area has been constructed as a tree with centre node taken as a root of data gathering tree. Nodes which are closest to the root node have the highest priority to select as child nodes. This process of child and parent formation keeps continue until all the nodes get covered to form a tree. In the process of building the data collection tree, nodes need to frequently communicate with each other by estimating signal strength and the distance between them. The constructed tree has been partitioned into distinct subtrees like clusters and MS gathers data from each of subtrees by optimizing moving trajectory of MS by using least square method.	Designed for static WSNs. This is centralized approach, operated when center node sets itself as a root of data gathering tree. Also there can be redundant data gathering possible within each subtree in case of big data.
2020 [125]	Proposed an 'Iterative Sensor Clustering and Mobile Sink Trajectory Optimization (ISCTO)' technique aiming to reduce energy-hole problem in non-uniform nodes distributed WSN with low computational complexity.	An optimization problem considering total energy consumption as objective function is formulated. To reduce computational complexity, considered problem is decomposed into two optimization problems: one is sensor clustering problem, considering the energy consumption of the SNs and other is trajectory optimization, considering the energy consumption of multiple MSs. ISCTO considers density function and residual energy for CH selection. SNs which can minimize defined density function and having higher residual energy are chosen as CHs. Authors introduce a sector boundary control method in which network is divided into several sectors to minimize the amount of energy consumed by multiple MSs in travelling and data collecting.	Specifically designed for network where sinks are mobile and other nodes are immobile. Engaging multiple MSs to eliminate hot-spot problem makes network costlier.
2020 [126]	Authors proposed a 'Joint Density-Aware and Energy-Limited Path Construction algorithm for Data Collection, called DEDC algorithm'. DEDC outperforms the existing data collection algorithm in term of network lifetime and energy consumption.	DEDC comprises of path constriction phase and path adjustment phase. The methodology consists of partitioning the target area into equal sized grids and all grids have been connected to form a route for MS to visit each grid in such a way the total route length satisfies a length constraint. Thereafter, DEDC checks the balance of each grid by assuming each sensor of each grid as an individual cluster and merge the clusters one by one to form a hierarchical clustering tree based on distance factor. Next initiates path adjustment phase which does the appropriate length segment to each balanced and unbalanced grid depending upon the amount of data existing in each grid.	Designed for static WSNs. Target area is assumed to be rectangle in shape limits the applicability DEDC. MS follows a length restricted path to collect data from each grid, might adversely effect the buffer time.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2020 [127]	Authors proposed a 'Destination Oriented Routing Algorithm (DORA)' which balances the energy consumption, double the network lifetime and reduces end-end transmission delay as compared to PEGASIS. More suitable for strip-based WSNs.	DORA works to form multiple chains hierarchy by considering accurate distance between the nodes, forwarding direction and optimal cluster size. Optimal transmission distance, optimal cluster size and data forwarding direction to sink has been obtained by estimating energy depletion during intra-cluster and inter-cluster routing, results multiple transmission paths. Each chain length of DORA is shorter than the chain length of PEGASIS	Though it is the modification of PEGASIS but still is not suitable for mobile WSNs as whole routing topology needs to reorganize with the movement of a single node.
2020 [128]	Proposed a 'Multi-objective Cluster Head based Energy-aware Optimized Routing (MCH-EOR)' algorithm. This approach improves network lifetime with the optimal selection of CHs, reduces energy consumption and hot-spot problem. MCH-EOR is the most energy-efficient optimized routing based on fitness function.	A fitness function has been formulated considering the parameters like proximity, communication cost, residual energy and coverage to form unequal clusters and to select optimal CHs. The evolutionary Sailfish optimization (SFO) approach is used to find optimal path from CH to BS for data transmission. This quickly finds the optimal solution by selecting best path. An alternate path keeps on available if the nodes over the optimal path are on verge of death. This enhances fault-tolerant capacity of network.	Designed for static and homogenous power level of nodes.
2020 [129]	Proposed an 'energy-efficient mobility based cluster head selection (EEMCS)' scheme for Dynamic WSN. EEMCS is mainly focused on improved CH selection and adaptability of network with nodes mobility in energy efficient way. Throughput of EEMCS is far better than CRPD. This mechanism offers re-clustering only when it is essential. Moreover, it is scalable, saves more energy and prolong lifetime in Dynamic network.	EEMCS uses acknowledgment messages to ensure reliable data delivery and to avoid message repetition by using time counter. For CH selection, weightage of each node is calculated based on the parameters: residual energy of sensor nodes, mobility level, node degree and node's distance from the BS and then energy level of each node is checked to satisfy the minimum energy threshold criteria to be a CH. The node with the highest weightage become a CH. Each node is allowed to join one CH based on distance parameter at a time and two clusters cannot join each other. Intra-cluster routing is done using a single hop mechanism with TDMA schedule and inter-cluster communication uses multi-hop minimum cost transmission path for data delivery to BS. Nodes mobility and failures are detected by acknowledgement messages and handled by re-clustering if required.	Designed for homogenous network only. Inter-cluster routing has latency limitation. Number of data packets delivery decreases with increasing the network area.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2020 [130]	Proposal of 'cluster-based two-level routing method' which reduces number of total hops, energy consumption as well as increases packet delivery ratio and network lifetime.	In clustering phase, nodes calculate their own score for turning to CH, based on their distance to BS, residual energy and centrality. Node with the highest score advertises itself as a CH and the node that comes second in meeting the criteria advertises itself as Backup CH (BCH). Once the energy of CH is depleted, it is immediately replaced by BCH. This reduces extra calculations in new CH selection. In routing phase, each cluster is divided into four non-empty sections with respect to CH position in center. Intra-cluster routing requires nodes to transmit directly to CH only if it has the highest score otherwise node has to transmit highest scorer nodes which further transmit to CH. Inter-cluster routing involves layering of CHs according to the distance from sink as first layer CHs are only one-hop away from sink, second layer CHs are two-hop away from sink and likewise until all CHs are covered and formed layers. Routing of data are done from higher layer to lower layer of CHs. Lower layer CHs use single-hop to route to sink.	Designed for static homogenous network only. Node has to choose highest scorer node in intra-cluster routing causes unnecessary transmission delay. Random selection of CH or BCH when there are more than one choice for MNs, should be replaced by optimal selection in this work.
2020 [131]	Proposed an 'energy-aware cluster-based routing protocol (ECRP)' to reduce excessive overhead due to frequent CH renewal process. ECRP has linear message and time complexity. It reduces and balances energy consumption of SNs.	This method avoids unnecessary re-clustering by executing setup phase once in the first round and then network continues to function for rest of the rounds until it is out of service. ECRP supports topology changes with the help of acknowledgement messages and fault tolerant mechanism. In this, energy is the only factor for CH selection and MNs join CH on the basis of cost function made up of residual energy and distance parameters. Intra-cluster routing involves TDMA schedule and inter-cluster routing involves the concept of choosing appropriate relay nodes for data forwarding.	Designed for homogeneous and static network only. No control on number of CHs requirement.
2020 [132]	Proposed an energy-efficient clustering protocol for heterogeneous WSN in which nodes have been classified as normal and power nodes based on energy level. This protocol maximizes network lifetime.	A probability model and threshold function is formulated depending on sensors energy for CH selection. Nodes satisfying the threshold criteria becomes CHs otherwise the process repeats again.	Designed for static WSN. There is no optimal path choice in inter-cluster routing for delay concern.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [133]	Proposal of 'Hierarchical Data Dissemination Strategy (HDDS)' for a large-scale dynamic WSN to optimize energy consumption with coverage preservation and fault tolerability of the network. This distributed algorithm forms a load-balanced routing path and diminishes the hot-spot problem by means of an unequal clustering strategy. It is more reliable, extend the coverage lifetime and supports the addition of new nodes or the partial movement of existing nodes. HDDS has linear Time complexity.	HDDS operates in five phases, 1. Bootstrap: Involves nigh discovery and acknowledgement of local information by nodes, 2. Clustering: Involves division of network area into non-overlapping clusters and selection of CH nodes based on intelligent waiting time which is computed using residual energy, coverage significance, distance and transmission energy. A node which results a low waiting time considered as an appropriate CH. Adaptability of network come up with the variation in transmission radius of CH nodes as closer the CH to the edge server smaller its radius for energy conservation purpose. MNs choose to join suitable CH by computing cost function having parameters, residual energy and transmission range. The CH resulting in higher value indicates a better association for MN. 3. Hop count calculation: CHs communicate with minimum hop counts, 4. Next Hop selection: CHs choose relay nodes based on residual energy, transmission energy and node degree. 5. Steady phase: It involves intra-cluster and inter-cluster routing, route recovery mechanism to handle fault at cluster or routing level.	Designed for homogeneous network. Authors should consider latency factor with coverage preservation and fault tolerance in clustering.
2021 [134]	Proposed a 'Tree-structured data aggregation' method. This approach significantly reduces energy consumption, provides more reliable and error free end-end communication.	The method consists of three phases. First phase is the binary tree formation in which nodes are organized in a sequential manner with the help of status given to nodes i.e. nodes are either visited or unvisited mode and get connected based on energy level. Second phase involves the process of secure data aggregation in which each parent has shared a key with all its children. Each node has to satisfy valid key allotment to the received data aggregation request for further processing. Third phase consists of reliable data aggregation process in which modified cyclic redundancy code (CRC) is utilized to find the error after every hop i.e. CRC code is computed on every bit of sensed data by each node and then data has been passed on to one-hop away node. If error is detected, request of retransmission is processed.	Process of retransmission causes data latency. Designed for static homogenous network only.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [135]	Proposal of a 'secure hybrid data aggregation method (SHSDA)' based on the combination of star and tree structures. SHSDA is reliable and fault tolerant with low overhead in route discovery. The stable path created between the source and the BS improves packet delivery ratio and reduces energy consumption.	In this method, whole network is divided into four sections and a star-like structure is formed in each section with the assignment of root node by BS on the basis of highest energy level and shortest distance from the center. Every child node is assumed to know its parent and grand parent based on the local information they received from BS and so each node selects the closest and highest energy node as its parent in each section with the sharing of a authentication key between every parent and child node. Then the phase of tree construction initiates connecting the selected root nodes to the BS which lies at root of tree. Both star and tree structures are made to be regular and fault-tolerant as when a parent node dies, child nodes are supposed to connect with the grandparent node with the new authentication key restoring the structure. For secure data gathering, leaf nodes send encrypted data to the parent node, parent node decrypts data, aggregates it with its own data and forward it further with the sharing of authentication key.	Designed for static homogenous network only. This method employs hop by hop data transmission may cause latency.
2021 [136]	proposed a 'multi-hop routing algorithm based on path tree (MHRA-PT)'. This algorithm optimizes network energy consumption and improves network lifetime and load balancing as compared to other tree related hierarchy routing.	Working of MHRA-PT comprises of CHs selection, working nodes assignment and transmission paths generation. Nodes closer to BS and having higher energy level than the defined threshold have been selected as CH nodes. Nodes except CHs, having residual energy more than average is chosen as working nodes and cluster partition happens by considering maximum sensing range of working node so that the whole cluster can be sensed by the working node. Data transmission happens by first sorting CHs based on distance to BS and CHs closer to BS called parent node. Each CH node except parent node, transmits aggregated data either via parent node or directly to BS by estimating residual energy and energy consumption of the relaying node and opting path respectively. This results multi-hop tree structure for routing.	Designed for static and homogenous WSNs. Only suitable for square-shaped network where sink is fixed at center.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [137]	put forth an energy efficient path planning strategy using multiple sinks. Author proposed stable election algorithm (SEA) which minimize message exchange, SP determination algorithm and multi-objective evolutionary algorithms (MOEAs) which optimally trace the route connecting all SPs. The proposed work is most suitable for Internet of things application of WSNs as it minimizes hot-stop problem, unbalanced energy consumption and improves network lifetime with connectivity.	The proposed work have been executed on three different types of network. First is homogenous WSNs, second is two-tier heterogeneous WSNs in which two types of nodes exist based on low and high energy level and third is three-tier heterogeneous network in which three different types of nodes exist based on low, medium and high energy level. The target region has been partitioned into equal area having same side length and no. of partition depends on number of MS. SEA algorithm initiates to form optimal number of clusters and selection of CHs based on node degree and residual energy, determine next CH rotation until the existing CHs' energy level gets lower than the defined threshold and data aggregation from CM by CH has been done in single hop. CHs transmit data to BS when BS traces path going through each SPs using MOEAs where MOEAs is objectifying path cost, travelling delay and distance.	Engagement of multiple mobile sinks makes the network costlier. Sensor nodes are assumed to be fixed after deployment.
2021 [138]	Authors proposed a 'Competition-Based Unequal Clustering Multihop Approach (CUCMA)' which maximizes network lifetime in large WSNs. This approach is distributed in nature and results optimal number of CHs and load balancing.	In CUCMA Nodes compete for CHs by calculating their own distance and residual energy and broadcast a message of being CH within a specified time threshold. Thereafter clustering happens on the basis of cluster radius, distance to BS, node degree and distances to around nodes. New CHs in other round are selected by involving one more factor i.e. distance to the existing CHs.	Assumption of target area is square in shape and nodes including BS are fixed restrict the application of the proposed work.
2021 [139]	Proposal of a 'novel non-uniform hierarchical clustering with dynamic route adjustment (NHCDRA) scheme using two MSs. This scheme is most suitable for a heterogeneous WSNs. NHCDRA significantly reduces the data delivery delay and improves network lifetime when compared with state-of-the-art.	Heterogeneous network carrying two different types of sensors named as α -sensor and β -sensor where the energy level of type- $\alpha < \text{type-}\beta$. The target region has been partitioned into concentric circles of different radius and each ring region has been further divided into sectors by using straight lines which is based on communication range of type- β sensor, results non-uniform clusters formation. CH is chosen in each sector based on highest residual energy and distance from center of cluster. Additionally there are boundary CHs (BCHs) lying over the boundary of each clusters. All the CHs accumulate and transmit data to BCH of adjacent cluster. Thereafter two MSs which are placed opposite to each other, move back and forth around the clusters boundary and collect the data from BCH if BCH is closer to MS else BCH relays its data to nearest node to pass on its data.	Designed for static network and engagement of two MSs makes the network costly. Assumption of obstacle free path between two sensor makes the work impractical. There is no defined path for MSs to visit BCHs.

Table 4 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [140]	Introduce 'divide-and-rule sectorization (DRS)' based hierarchical routing scheme. DRS approach brings balanced energy consumption of nodes and mitigate energy-hole problem, has better stability and network lifetime as compared to available state-of-the-arts.	The target area has been partitioned into 3 concentric circles where inner ring is depicted as I_C , middle ring as M_L , upper ring as M_U and outer region as O_R . Each ring is further partitioned into 8 sectors except I_C where 10 nodes are uniformly distributed in each sector. Nodes in I_O transmit their data to the forwarding node (FN) of M_U . FN further transmits to the FN of M_L , likewise data transmits to O_R and finally FNs of I_O transmit to BS. FNs are randomly selected and repetition of FNs continue to happen until new FNs are going to exist. This balances load distribution.	This approach is suitable only for static and homogenous network where BS is placed at the center of target region and remain fixed after deployment.

$$X_{ij}(t) = X_{ij}(t-1) + V_{ij}(t-1) \quad (3)$$

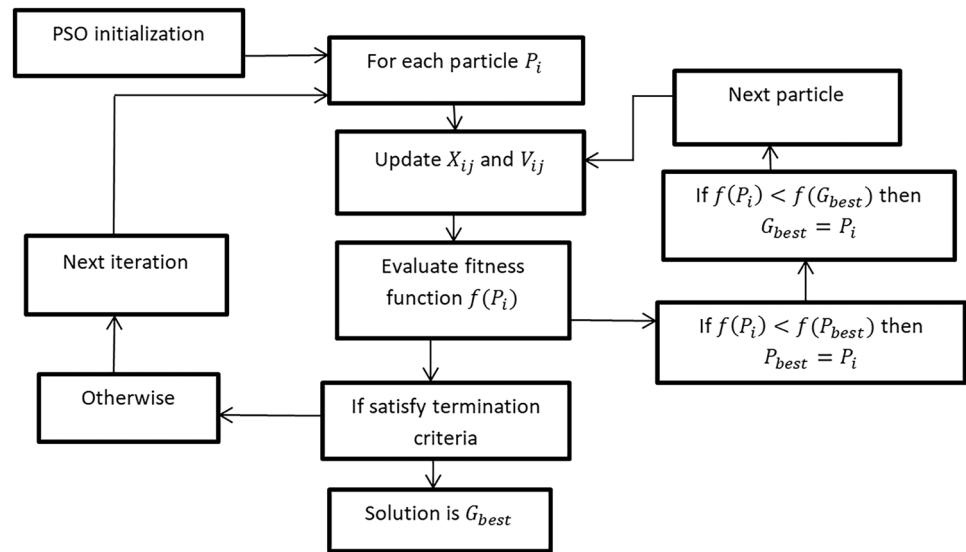
where w is the inertia weight, c_1 , c_2 are two non-negative constants, q_1 , q_2 are two distinct randomly generated number in between $[0, 1]$. The update process is iteratively repeated until either an acceptable G_{best} is achieved or a maximum number of iterations is reached. A general mechanism of PSO technique is demonstrated in Fig. 8.

ACO techniques: ACO is the abbreviation of 'ant colony optimization'. This technique is introduced imitating the foraging behavior of ants seeking a path between their colony and source food. When ant moves in search of food, it leaves pheromone (a kind of liquid marker) along the path which evaporates over time. So other ants follow the same path by smelling the pheromone concentration. Moreover shortest path carries more pheromone quantity as more ants opt that path. With the movement of ants, quantity of pheromone updates everytime, this way a reliable and shortest path can be achieved using ACO technique. ACO has been used for a long time to solve route optimization problems in WSNs. Formulation of pheromone update function depending on distance factor, nodes residual energy and/or node degree has been done to discover optimal route/path in both static as well as mobile WSNs. A general mechanism of ACO technique is demonstrated in Fig. 9, taken from [141].

Following are some key points, we draw from Tables 5 and 6:

- **Optimal clustering by PSO technique:** PSO technique is having good performance in finding optimal location for CHs, RPs, SPs, anchor points (APs) and optimizing cluster size with optimal coverage. PSO routing techniques are based on to formulate one or more fitness function as first one is used to find optimal CHs as well as cluster size and second one is usually to find optimal data routing when there is a fixed BS or gateway node.
- **Optimal routing by ACO technique:** ACO technique is having best performance in bringing out optimal route between sensors with minimum energy cost and maximum network lifetime with the help of pheromone value. ACO technique gets upgraded with time like assignment of three different types of ants, one is used to find forward path, second is used to provide feedback and third is used to maintain the route. This promotes network connectivity and fault-tolerance. Also improved ACO techniques bring pheromone updation based on many factors like nodes residual energy, distance between the nodes, nodes degree, hop count, etc. This provides optimal next hop and enhances network life-cycle.
- **Hybrid of PSO and ACO technique:** Hybrid techniques of PSO and ACO, engage PSO to setup initial cluster topology and ACO to find the shortest route for data transmission and/or data aggregation. These tech-

Fig. 8 General mechanism of PSO technique when the problem is of minimization



nique increase the convergence rate of ACO technique and results better network lifetime, reduced energy consumption and load balancing.

- **Engagement of MSs/SPs/RPs/APs:** Some PSO and ACO techniques engage MSs and SPs/RPs/APs for faster data aggregation and load balancing purpose. PSO-based technique utilizes fitness functions to find optimal number and position of SPs/RPs/APs and to obtain optimal trajectory for MS to visit each SPs/RPs/APs in an energy efficient and cost efficient way. ACO techniques update pheromone value along the route based on RPs load and distance between RPs and MSs.
- **Transmission delay and latency issue:** Both PSO and ACO handle contradicting multi-objectives in a problem and provide a balanced solution. Yet transmission delay/

latency is the persisting issue in heterogenous and large WSNs in a cost efficient way. To reduce latency factor, some authors proposed solutions by engaging more than one MSs so that data aggregation and transmission time gets reduced. This reduces inter-cluster transmission delay only and makes the network expensive as well. Some ACO based routing techniques cause lesser end-to-end delay than PSO routing but in homogenous WSNs.

- **Mobility and robustness:** For routing in mobile WSNs, engagement of multi power level nodes are come into picture and CHs selection is done by formulating fitness function which consider nodes residual energy, nodes degree, nodes speed and distances to other nodes. Undoubtedly this makes the topology more robust and stable with lots of expense.

Fig. 9 General mechanism of ACO technique

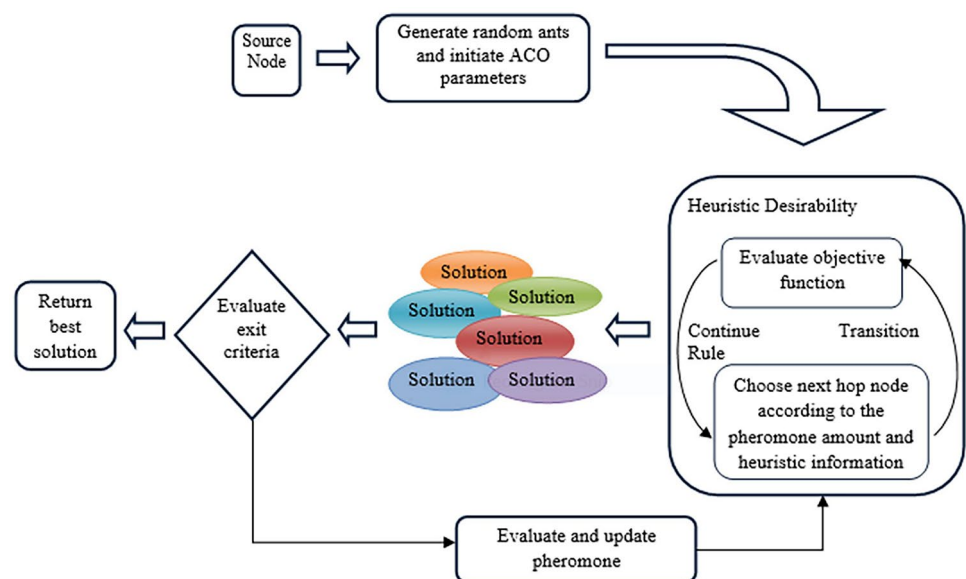


Table 5 Overview of latest 'PSO based routing techniques' in WSNs

Reference	Contribution	Strategy/methodology	Limitation/future scope
2017 [142]	Proposal of a 'PSO-based energy-efficient routing and energy balanced clustering algorithms' for WSNs. The algorithms are able to cope-up with the failure of CH nodes. Algorithms save and balance energy consumption with prolonging network lifetime.	Authors formulated two multi-objective fitness functions with normalized parameters. One fitness function evaluate G_{best} route for energy efficient data transmission from gateway nodes to BS considering low energy consumption in transmission and nodes residual energy. Second fitness function brings out the optimal clustering by considering residual energy of each gateway nodes with even distribution of MNs to each gateway.	All nodes including gateway nodes are assumed to be static in the field. The proposed algorithm is centralized in nature and may cause routing delay since low energy path can be chosen, so not suitable for real-time application.
2018 [143]	Proposal of a 'PSO-based unequal and fault-tolerant clustering (PSO-UFC)' protocol. This protocol attempted to minimize energy-hole problem and load imbalance in intra-cluster and inter-cluster routing with connectivity assurance.	The target area partitioned into clusters of unequal sizes. Process of selection of master CHs initiated by BS, so a fitness function is formulated considering the parameters of highest residual energy, closeness to BS and lesser routing cost. After the clusters formation, PSO-UFC protocol constructs the multi-hop routing tree among the elected MCHs by formulating a fitness function with the parameters of residual energy of the next hop, distance of a nex hop from a MCH, distance of next hop from BS and node degree of next hop. Moreover, concept of surrogate CH (SCH) has been introduced as a backup of MCH incase of failure. MCH itself selects SCH in each cluster for optimal data routing.	Designed for static WSNs. There is lot of beacon message exchange among the nodes during the whole process, this causes overhead and energy expenditure as well.
2018 [144]	Proposal of 'a multi-objective particle swarm optimization (MOPSO)' algorithm which evaluates energy efficient path for MS. MOPSO successfully balances the trade-off between transmission distance and intermediate hop counts. MOPSO builds an effective approach of data aggregation through MS against hot spot problem.	MOPSO algorithm involves particle initialization using PSO technique and derivation of multi-objective fitness function for RPs based MS path design problem with the aim of minimizing both maximum average transmission distance and maximum number of total hop counts at any RP. Algorithm works to find RPs location and construct an efficient trajectory for MS connecting all RPs using the formulated fitness function. After maximum number of iterations, a global solution is obtained depicting the energy efficient path designs for MS where MS periodically travels and halts at each RP, for the purpose of gathering the sensory data.	Energy is not considered in examining objectives so some nodes may die prematurely.

Table 5 (continued)

Reference	Contribution	Strategy/methodology	Limitation/future scope
2019 [145]	Proposal of 'PSO-based uneven dynamic clustering multi-hop routing protocol (PUDCRP)'. This paper introduced a connecting line aided route construction method for energy efficient routing. Moreover, PUDCRP achieves balanced energy consumption of whole network, good scalability and reliability.	PUDCRP brings clusters formation as circles of distinct sizes based on distribution of nodes. Cluster size changes dynamically with the nodes count and distance from BS. Optimal no. of circles has been obtained by cyclic calculation and a multi objective function has been designed for optimal CHs selection considering nodes residual energy, no. of neighbor nodes and distances from nodes to the BS. Two fitness functions have been modelled to determine optimal positions of CH nodes and circular areas. first one is based on two important factors: 1.coverage rate 2.intersection over universal (IoU) and second depicts the absolute difference between actual number of nodes covered by the circular area and no. of ideal coverage nodes. If the distance between CH nodes and BS is less than some fixed threshold, CH nodes send data directly to the BS via single-hop routing otherwise a multi-hop route is established by the connecting line aided route construction method and the node with the smallest weight is selected as the next hop. An improved PSO algorithm is used to find high energy centers so that nodes near the energy centers can be selected as CHs. EC-PSO provides optimal solution by designing fitness function considered with residual energy of those nodes which are two-hop away from the position of energy center. A low energy protection mechanism has been used to avoid weak nodes to become relay nodes by fixing threshold on residual energy. For inter-cluster communication, a mobile agent is utilized to gather the information from CHs which is placed at the center of the network initially. The collector always moves to the energy center with highest average energy.	Designed for homogenous WSNs only. PUDCRP faces transmission delay in large-scale network.
2019 [146]	Introduced a special clustering method for heterogeneous WSNs targeting hole problem known as 'Energy Centers Searching using Particle Swarm Optimization (EC-PSO)'. EC-PSO shows good performance in terms of energy saving and improving network lifetime.	EC-PSO is only applicable to the WSNs where nodes are evenly distributed and initiated with same power. Engagement of mobile agent makes the network costlier.	

Table 5 (continued)

Reference	Contribution	Strategy/methodology	Limitation/future scope
2019 [95]	'A travel route planning schema with a mobile collector (TRP-MC)' to find a shortest route covering maximum no. of sensors, has been proposed. Energy consumption of TRP-MC is much less than LEACH. TRP-MC minimizes coverage overlapping with maximum coverage rate and improves data gathering efficiency of mobile collector. 'Sojourn points (SPs)' are used to define stoppage location of mobile-collectors and 'anchor points (APs)' are used to denote SNs to estimate their distance from SPs.	TRP-MC divides the target field into regular hexagon with each hexagon is supposed to have a SP. Only one node need to keep active while other nodes go to sleeping mode momentarily in each division. PSO algorithm finds optimal position of SPs which cover maximum number of APs with minimum coverage overlapping. Authors formulate a fitness function as the ratio of overlapped coverage rate and total coverage rate and works to minimize it. Particles solutions are improved with iterations following P_{best} and G_{best} where speed and position of particles are restricted to some bounds so that particles don't move fast and go away from field. After max no. of iterations, G_{best} shows the optimal solution for SPs selection. After the SPs selection, ACO-based shortest path algorithm is applied to find shortest traveling route covering all SPs in single round. PSOBS works in a centralized way as sink runs the PSO algorithm, specifies RPs and determination of routing path. A fitness function is formulated based on anticipated RPs number, maximum tour length of maximized tour and computation of path length through RPs. G_{best} provides optimal RPs.	Assumptions that nodes are static after deployment, sensor field is a rectangular area and a static sink is set at the corner of sensor field restrict the applicability of TRP-MC.
2019 [147]	Proposal of 'particle swarm optimization based selection (PSOBS) algorithm' which finds optimal and near-optimal RPs location for MS. PSOBS gains network lifetime, reduces energy consumption and packet loss rate.	PSOBS works in a centralized way as sink runs the PSO algorithm, specifies RPs and determination of routing path. A fitness function is formulated based on anticipated RPs number, maximum tour length of maximized tour and computation of path length through RPs. G_{best} provides optimal RPs.	PSOBS is limited to RPs selection only, lacking in optimizing clusters number and size. No optimal selection of CH nodes.
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Table 5 (continued)

Reference	Contribution	Strategy/methodology	Limitation/future scope
2019 [148]	Proposed a PSO-based routing algorithm to improve network lifetime. This approach reduces power consumption with improvement in load balancing.	A novel fitness function with three objectives has been designed by applying weighted sum approach to cope up with trade-off among the objectives. Objectives are minimization of maximum distance between gateways and BS, minimize number of relay nodes for data forwarding and minimize relay load factor in the network. PSO-based routing provides an optimal routing path between each gateway to the BS following the proposed fitness function and dimension of each particle is equal to no. of gateway nodes.	In order to achieve load balancing, hop-count increases which causes transmission delay.
2020 [149]	Proposed a 'PSO-based dual sink mobility (PSODSM) technique' to reduce the energy expenditure of SNs in the heterogeneous WSNs. Use of two sinks make communication more energy efficient by reducing number of hops in large area networks, reduces overhead, transmission delay, improves stability and network lifetime.	Nodes with different energy level has been differentiated into normal, super and advance. Formation of fitness function for optimal CH selection, undertook factors like ratio of remaining energy to the initial energy, node degree, node centrality, distance between sink and node, CH count and energy consumption rate. Author placed two sinks at opposite corners and sinks moved towards each other to gather data from selected CHs against hotspot problem. Also cluster MNs have been allotted with TDMA slots for sending data to the closest sink.	Assumption of target area is square shaped restricts the applicability of algorithm.
2020 [150]	Proposed a 'PSO-based Energy Efficient Clustering and Sink Mobility (PSO-ECSM)' algorithm for optimal CH selection and to optimize sink mobility such that hotspot problem can be reduced in clustered based routing. PSO-ECSM decreases no. of dead nodes with respect to no. of rounds and prolongs network lifetime. Also throughput increases with load balancing.	Heterogeneity at nodes energy level differentiated nodes as normal, super and advance. Author formulated two fitness functions independently for the two optimizing purposes which has to be minimized comprising of multi-parameters that are nodes residual energy, distance factor between sink and other nodes, nodes degree and energy consumption rate. PSO-ECSM provides the optimal CHs and optimal inter cluster routing using fitness functions.	Assumptions of static nodes, target area is square shaped and sink has no energy constraint, restrict the applicability of the algorithm.
2020 [151]	Proposal of 'Energy-Balanced Cluster-Routing Protocol (EBCRP)' with five mutation operators in PSO algorithm to optimize number of clusters and cluster centroid. Reversing probability operator, Decreasing probability operator, Increasing probability operator, Swapping probability operator and Transforming probability operator are the five mutation operator which improve the performance of PSO algorithm by increasing search diversity and enhances the probability of finding better solution.	Any of the five mutation operators is randomly selected to apply to the position of i -th particle. Particles begin to search for the best clustering solution through iterations. These solutions converge towards the desired target by shorting the distance between nodes and their centroid while balancing the no. of nodes between clusters with the evaluation of P_{best} and G_{best} through fitness function. Also adoption of rotation of CHs based on the highest residual energy effectively balanced the energy consumption between nodes.	EBCRP algorithm dynamically selects CH for each cluster in each round, which reduces the stability of the network system. It is applicable to homogenous network only. This work can be extent to large sensing region where some relay nodes can be selected to assist the communication between CHs.

Table 5 (continued)

Reference	Contribution	Strategy/methodology	Limitation/future scope
2020 [152]	Introduced an 'Optimal Coverage Multi-Path Scheduling Scheme with Multiple Mobile Sinks (TSCR-M)' for large-scale WSNs. This enhanced network lifetime and Reliability.	Author introduces single hop trajectory scheduling i.e. sensors transmit their data directly to MS in just one hop. TSCR-M finds optimal parking slots where MSs stop for data collection subject to the constraints of maximum sensors are covered within the transmission disk of MS with minimum coverage overlapping. Author utilizes improved PSO algorithm for optimal search of parking slots where the fitness function evaluates the percentage of overlapped sensors.	Involvement of multiple MSs makes the topology quite expensive. Assumption that each sensor initialised with same power restricts TSCR-M applicability.
2020 [153]	Proposed a 'Mobile Double Cluster Head-Particle Swarm Optimization (MDCH-PSO)' algorithm to enhance the network lifetime and load balancing in hybrid WSN. By hybrid, it means SNs can be dynamic as well as static in nature. MDCH-PSO achieves good energy conservation of CHs, reliability and network longevity.	The author introduced the concept of female node (super power node), male node (CH) and child node (remaining MNs) in clustering. All nodes can move except female node and it is located at the centre of each cluster, do monitoring, clustering and data aggregation task by taking note of each child node ID, residual energy, one hop neighbors and distance to itself. This makes CHs live long in the network. Moreover male node selection i.e. CHs selection is done by PSO method by defining multi-objective fitness function considering residual energy, node density, mobile speed and distance to female node. Also female node predicts the mobility of child nodes using time stamp and so routing path is immediately updated on the movement of nodes.	Scalability will be the demerit due to high cost in deploying female node.
2021 [154]	Proposal of a PSO-based energy efficient clustering (PSO-EEC) protocol. PSO-EEC is designed for the heterogeneous WSNs. The proposed work improves the network lifetime and stability as compared to MDCH-PSO, MOPSO and HSA-PSO protocol.	Authors engaged two types of sensor nodes, namely normal and power nodes where power nodes are having extra power than normal nodes. A fitness function has been formulated for the optimal selection of CHs based on the factors like energy ratio (i.e initial energy to residual energy), node degree, distance between nodes and CH. Further relay nodes are selected by employing fitness function considering the residual energy and distance factor of member nodes. Intra cluster routing uses TDMA schedule and in inter-cluster routing, CHs adopt multiple hops depending on the distance.	Designed for static WSNs only and multi-hop data transmission in large WSNs might cause latency.
2021 [155]	Developed a hybrid approach GAPSO-H where GA method is used to select optimal CH and PSO algorithm is used to trace optimal route for MS for data gathering or transmission. GAPSO-H is found to be computationally optimized and improves the network stability.	GAPSO-H method categorises nodes into normal, advance and super nodes on the basis of their power level. It undertakes hot-spot issue by integrating 5 major parameter in a single fitness function for optimal CH selection that are nodes residual energy, distance factor, average node energy, no. of surrounding neighbour nodes and energy consumption rate. Also for optimized sink mobility, three factors that are discovery of lowest energy CH, distance to CH and cluster length are considered.	Sink mobility makes network more costly. Also this work could extent to minimize end-end delay.

Table 5 (continued)

Reference	Contribution	Strategy/methodology	Limitation/future scope
2021 [156]	Proposal of a mobile data collector (MDC) based energy efficient clustering routing protocol for large WSNs. The focus of this approach is to reduce energy consumption and latency during transmission. Algorithm runs in polynomial time. Engaging MDC improves network lifetime and end-end data delivery.	The approach is the combination of improved variant of PSO i.e. variable dimensional-PSO (VD-PSO) and ACO methods. The VD-PSO method starts finding optimal data collection points (CPs) by iterating through variable dimensions so that each CP must surround with maximum CHs within a hop. So a fitness function has been formulated considering node's residual energy, neighbour centrality and coverage significance to choose optimal CP location using VD-PSO. After finding optimal CPs, ACO algorithm is used to form an optimal trajectory of the MDC to visit each CPs only once and get back to BS. Pheromone updation with time results an minimum length trajectory for MDC. Data aggregation starts with member nodes by CHs using TDMA and MDC uses CDMA to collect data from each CH by posting themselves at CPs.	Defined for static network. Involving MDC makes the network costly.

Moving to real-time routing protocols which are better helpful to minimize end-end delay and to obtain better QoS.

7.3 Real-time routing protocols

Real-time routing protocols are particularly concerned with delay-constraint or time-sensitive data transmission. These protocols are designed to minimize deadline miss ratio (DMR) and end-end data delivery with the maximization of throughput and reliability factor to support real-life wireless communication in a more efficient way [19, 72, 73]. A thorough review of well established real-time routing protocols from year 2002 to current, has been conducted and analysis of each technique has been documented in a tabular format where the analysis of some articles are done again and other are explored recently. Table 7 presents the objective/ contribution made by the proposed work, detail of methodology opted and associated limitations which might be overcome in future work. The definition of estimating end-to-end delay is distinct according to the proposed work like if the methodology is based on to determine packet velocity to travel, distance becomes the prime concern; some authors count number of intermediate nodes called as hops from source to destination; when methodology uses duty-cycle then total time spend during switching over sleep to awake, is considered for delay estimation. In Table 7, we have taken all the aspects to estimate delay where sometime delay is taken as propagation delay + transmission delay + queuing delay and so on. Real-time routing protocols are well adaptable to dynamic WSNs which makes them more applicable to real-life scenario. Most of them don't refer to establish topology and topology awareness like hierarchical routing. RAP [11], SPEED [12], MMSPEED [56] which are the very initial real-time routing protocols, determine velocity of data packets based on delivery deadline, urgency factor and distance from the destination node at the time of data transmission without establishing initial topology. However, their network lifetime doesn't last long as nodes die soon. After this, some authors [170–172] proposed delay constraint techniques which consider nodes residual energy as well while prioritising data delivery or to determine shortest route for data transmission. This reduces earlier node failure. Author [173] proposed a technique by involving location awareness and link quality to minimize routing error and packet loss at destination node. This maximizes throughput with reduced end-end delay. Some authors [8, 61, 174–176] proposed delay minimization techniques by opting cluster formation but these techniques are only suitable for static network. Technique in [177] engages MSs to collect data to minimize redundant waiting time, makes itself suitable for dynamic and delay-harsh network but by ignoring energy saving factor. Technique in [60] is non-topology aware protocol minimize end-end delay

Table 6 Overview of latest 'ACO based routing techniques' in WSNs

Reference	Contribution	Approach/methodology	Limitation/future scope
2017 [157]	Introduced an improved ACO based routing method which reduces energy consumption and extends network life cycle.	For optimal route evaluation, authors formulate a fitness function involving nodes residual energy and route length simultaneously with updation of pheromone concentration after each iteration. Larger the fitness value, more pheromone quantity gets update and the more optimal the route is. In this improved version, authors not only consider transmission distance of every next node but also distance between node and sink to save more energy. Also to reduce algorithmic complexity and energy consumption, authors limit the search space of ants by introducing the search direction angle which should be smaller than 90° between the line connecting a node and sink and the boundary line.	Assumption of a square field where nodes and sink become static after deployment limits the applicability of algorithm. Author ignores the effects of other energy levels (e.g., average energy, minimum energy). Extension to mobility and realtime constraints can be done in future work
2017 [158]	Proposed an 'ACO Energy Aware (ACOE) routing algorithm' which offers multiple transmission path from source to sink. ACOEA, optimizes the ant colony algorithm in terms of angle and distance between nodes to obtain better multiple paths. This achieves balanced energy consumption and extended network life cycle.	In ACOEA, three types of ants are engaged, 1. Search path ants: search multiple energy efficient paths from source to sink by computing transition probability (having pheromone and distance parameters) at every next move, 2. Back-off ants: sent by sink node to source along the same path for updation of pheromone value and take record of nodes residual energy, 4. Repair ants: do the routing maintenance task and in case of node failure, path is repaired by replacing dead node with one of its suitable neighbour node. Authors introduce 'angle factor' so that any node which forms the largest angle between sink and source node taken as the next hop node. This helps in reduction of energy consumption and optimize the path performance. To search more paths, remaining links that are unused, selected successively to receive ants using the transition probability formula.	Authors assume the static model to design the algorithm. Dis-connectivity and hole problem can be the issue due to nodes replacement to repair path since these factors are ignored.
2018 [159]	Introduced a 'new data gathering method (ACO-MSPD) based on ACO technique' for non-uniform data producing sensors at different time period. ACO-MSPD finds the optimal no. of RPs and optimal trajectory (tour) for MS which needs to visit each RP to collect data within an imposed constraint of maximum tour length. ACO-MSPD minimizes average MS tour length, reduces the complexity to find suitable RPs. It adopts re-selection RPs dynamically during the change of weights of RP nodes. Therefore, it is not only minimizes the energy consumption but also balances the energy consumption among the RP nodes.	The proposed algorithm mainly consists of four phases, 1. Directed Spanning Tree (DST) formation: each SN chooses one forwarding node from its neighbour nodes of maximum load, 2. Optimal selection of RPs: RPs selection is done by improved ACO method where each ant chooses the next node as an RP for which the defined probability function (depending on pheromone value and distance) returns the highest value with the updation of pheromone value between the current node and the selected RP node, 3. MS path formation: an optimal tour by MS visiting each RPs to collect data using ACO technique and 4. Re-selection of RPs: If MS tour exceeds defined constraint length and/or when nodes generate non-uniform data then re-selection of few RPs takes place.	Complexity of Algorithm increases with increase in ant population and iterations. This method is designed for those network where sensors become static after deployment and a single MS has to engage.

Table 6 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2018 [160]	Proposal of a 'Hybrid ACO and PSO' based energy efficient clustering and tree-based routing protocol named as ACOPSO. This method prolongs network lifetime.	The algorithm first clusters the nodes based on their remaining energy. Then the hybrid ACOPSO method improves the inter-cluster data aggregation using pheromone factor which is deposited by ants moving from one CH to other. Involved ACO technique discovered a minimum cost spanning path between CHs and sink with the updation of pheromone after every iteration. Then PSO method initiates to optimize the cost of obtained structure further with the fitness function as minimum distance between the nodes.	Designed for static network where each sensor and sink remain static after deployment. Due to consideration of distance factor only, it may cause transmission delay when path of low energy is taken.
2019 [161]	proposed ACO based QoS aware energy balancing secure routing (QEBRSR) algorithm. This algorithm improves network lifetime, end-end delay, reliability and load balancing.	QEBRSR used improved models for calculating end-to-end delay based on lesser number of hops counts during transmission and trust worthy nodes has been chosen based on energy level and distance factors for data transmission to reduce packet dropping. While moving from source to sink, these ants store the information about the quality of nodes traveled such as energy consumption, residual energy, packets generated, packets received, packets transmitted and end-end delay and so update the pheromone level on their way back from destination to source. After maximum iteration, pheromone level keeps on updating and the path with maximum pheromone quantity becomes an optimal path for data transmission.	Designed for static and homogenous WSNs.
2019 [162]	Proposal of a novel 'Energy-Efficient Load Balancing Ant-based Routing Algorithm (EBAR)' for WSNs. This balances energy consumption, prolongs network lifetime and speed up the convergence of route discovery under the constraints of limited energy supply.	This new algorithm involves three components to achieve its aim: 1. An improved pheromone trail update method, based on energy levels of SNs and path length information, 2. An effective heuristic strategy which reinforces low energy cost routes in the route discovery phase based on expected cost metric and 3. An opportunistic broadcast scheme is adopted to control the packet overloading and congestion. Authors adopted both, global pheromone and local pheromone update strategies. To avoid the concentration of pheromone in the path that can be too high or too low, the pheromone concentration is set to range from some min to some max which in turn accelerate the convergence of algorithm. Updation of pheromone happens after every transmission along the path. A probability function has been defined to choose next move based on pheromone quantity. When the data needs to be transmitted, the optimization mechanism runs to calculate the current communication quality and distance of neighbour node.	Assumptions of links are symmetric and sensors are isomorphic in functionality and capacity, restrict the applicability of the algorithm.
2019 [163]	Proposed an improved ant colony algorithm to optimize mobile WSNs in term of life cycle, imbalance energy consumption of sensor nodes and data gathering. Proposed method is having good dynamic adaptability and load balancing. A low power low error geographic routing based on ACO to enhance data reliability is seen.	Authors adopted both, global pheromone and local pheromone update strategies. To avoid the concentration of pheromone in the path that can be too high or too low, the pheromone concentration is set to range from some min to some max which in turn accelerate the convergence of algorithm. Updation of pheromone happens after every transmission along the path. A probability function has been defined to choose next move based on pheromone quantity. When the data needs to be transmitted, the optimization mechanism runs to calculate the current communication quality and distance of neighbour node.	Opted multi-hop communication routing may cause latency issue while maintaining load balancing and stability factors.

Table 6 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2019 [164]	Proposal of an 'ACO Optimized Self-Organized Tree-Based Energy Balance (AOSTEB)' algorithm in which an energy optimized intra-cluster route is selected using the ACO algorithm by MNs to communicate with nearest CH. AOSTEB algorithm involves rotation of CH when its energy becomes low, offering balanced load-distribution and helps in improving network lifetime.	The ant's movement creates multiple paths between MNs and their CH, optimal path is selected dynamically among the multiple paths on the basis of minimum energy consumption during transmission along the path. AOSTEB improves CH selection by introducing a parameter called as Probability of Election (PE) based on residual energy and neighbor nodes so a node which is having maximum PE value selected as a CH. Also selected CHs keep minimum distance between each other so as to have a balanced distribution of clusters inside the network. Data transmission are done when CHs form a tree-like structure in which only one CH on behalf of all CHs communicates with the BS whose residual energy is greater among all.	AOSTEB is designed for static network only. Initial random route selection causes redundant data transmission. Moreover, only one CH transmits data to BS directly, it may cause congestion at CH node as well as transmission delay.
2020 [165]	Proposed a 'distributed data gathering technique (GTAC-DG)' where the decision making skill of game theory are used to find optimal set of RPs and enhanced ACO technique is used to find optimized route among the RPs for MS visit. GTAC-DG helps to reduce redundant data transmission, energy consumption and delay in data delivery. A reliable and intelligent trajectory of MS has been resulted, extending the lifetime and energy conservation. As selected RPs directly transfer data to MS, this leads to better balance in energy consumption and reduces communication overhead.	The technique starts with game theory Based RP-selection algorithm where nodes involve in a local-game with its neighbors competing with each other to be a RP depending upon the relative energy and load of player nodes. Larger the relative energy and load, more is the probability of being a RP. After the suitable RPs selection, trajectory design phase is executed i.e. travel path for MS visiting all RPs, same as TSP problem. So a modified ACO method is proposed by authors, formulating a tradeoff function between considered parameters that are load factor of RPs, pheromone value and distance between RPs and MS to have an optimized path. Also RPs with higher load are prioritized first to visit by MS to reduce retaining data to buffer for a longer period of time.	Assumptions of sensors are immobile and have same communication range restrict the applicability of algorithm.

Table 6 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [166]	Proposal of 'Multi-sink placement and any-cast routing (MPAR)' and 'Extended multi-sink placement and any-cast routing (EMPAR)' involving two level architecture where lower level comprises of all cluster groups and upper level comprises of sink nodes and their related activities. With the increase of clusters numbers and decrease in cluster size, algorithms can balance nodes energy consumption, reduce transmit power consumption and consequently prolong network lifetime.	Methodology of the proposed algorithms is as follows, 1.Sensors cluster themselves based on Euclidean distance where MPAR selects CHs whose energy level is maximum while EMPAR considers both residual energy and distance factors along with the history of sensors selected as CHs, 2. Formation of shortest path tree at lower level through which each sensor transmits its data to the corresponding CH, 3. This engages improved PSO algorithm to choose optimal locations for multiple sinks to place where a fitness function considers distance between the CHs and sinks with data length of each cluster and 4.Construction of high level any cast routing tree connecting all the CHs with multiple sinks for low cost data transmission has been done by improved ACO algorithm using pheromone based probability function. The employed probability function jointly considers residual energy and Euclidean distance for selecting next hop CH.	With the increment of sensors, time complexity increases. Involvement of multiple sinks makes network costly. As the number of sinks increases, finding the best location for the sinks becomes more complicated.
2021 [167]	Proposed an energy-constrained and delay-sensitive data aggregation approach. This approach gives shortest possible MS traversal route in polynomial time. Latency is minimized by employing one hop communication between CHs and MS and shortest path traversal of MS.	This method finds SPs such that tradeoff between energy preservation and data transmission delay gets balanced. This formulates the optimization problem of minimizing no. of SPs with one-hop connectivity to CHs. Since more SPs causes delay and less SPs causes energy hole issue so this approach aims to find optimal SPs and shortest path visiting each SPs by MS. SPs nomination is based on density factor i.e. density of nodes around CHs and coverage overlapping. An ACO method has been employed to find shortest travelling path for MS, using transition probability function which is further based on pheromone concentration and distance. CHs use TDMA schedule for data transmission among themselves.	Applicable where nodes remain static. Obtained traversal route for MS can be less reliable since nodes residual energy has been ignored.

Table 6 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [168]	Proposal of 'Energy-efficient cluster-based routing protocol using unequal clustering and improved ACO techniques (ECRP-UCA)'. ECRP-UCA is able to achieve global optimal solution and prevent quick convergence to local solution. Moreover it treats connectivity, fault tolerant issues of large network, balances energy consumption and improves network lifetime.	This method uses a new parameter i.e. no. of backward relay nodes in previous round with distance from the sink, residual energy and node degree to determine cluster radius. This measure gives unequal cluster formation. MN selects its CH according to the multi-objective cost function which is based on residual energy and distance parameters. Also to manage connectivity in large-scale network, ECRP-UCA refers higher energy nodes as relay nodes for ensuring multi-hop connectivity between isolated nodes and distant CH nodes. Strategy like replacement of dead nodes by already backup nodes without reconfigure whole network is adopted to ensure fault-tolerant network. For energy efficient inter-cluster routing, author formulates improved ACO (IACO) method with an addition of important parameter called NPPRN (No. of preferred probable relay nodes) with the energy of next hop node and distance from the current node to the next node.	Designed for static homogenous network only. Computational time increases with the increment in the value of parameters.
2021 [169]	Proposal of an extended ACO-based mobile sink path construction for event-driven WSNs, abbreviated as ACO-MSPD. This helps to improve network lifetime, hot-spot issue, load balancing and throughput. The present work outperform its previous version and EAPC.	The first task is to construct a connected directed spanning tree (DST) of the given network. Thereafter ACO is used to generate the RPs location by updating the ant's pheromone quantity iteratively. A probability function has been constructed to trace delay-aware path for MS to collect data from each RPs and MS prioritized RPs based on available data size. Concept of virtual RPs (VRPs) is the new approach used in this work. VRPs are those SNs which are more closer to MS than the RPs so they directly transfer their data to MS when comes under its communication range. This reduces the buffer time, redundant transmission and delay as well.	Nodes except sink node are supposed to be static in nature after deployment. So the proposed work is not suitable in mobile WSNs. Also the preferred DST model of this work is less fault-tolerant.

Table 7 Overview of 'Real-time routing protocols' in WSNs

Reference	Contribution	Approach/methodology	Limitation/future scope
2002 [11]	Proposal of a new packet scheduling policy i.e. 'RAP' which minimizes packet miss ratio within a deadline. RAP is most suitable for large-scale WSN as it minimizes communication overhead of large network.	In RAP, transmission of data packets at a node are prioritised according to the requested velocity. This velocity assignment depends on the distance of node from the destination and deadline given to the required data packets to reach to its destination. Shorter the deadline and larger the distance more is the velocity requirement. So packets with higher requested velocity are prioritised first. Author investigates two types of velocity assignment: Static velocity monotonic (SVM) and Dynamic velocity monotonic (DVM). SVM, velocity is calculated once at the initial node and remain the same while DVM, velocity may vary at the intermediate nodes accordingly.	RAP doesn't consider residual nodes energy in prioritising and determining velocity as it may cause earlier death of nodes and hence detruncate network lifetime.
2003 [12]	Proposal of 'SPEED' protocol which reduces end-to-end delay, congestion and voids. SPEED helps to maintains desired speed of data packets in transmission. It supports feedback control and saves energy by reducing number of control packets.	SPEED protocol consists of beacon exchange scheme, delay estimation scheme, stateless non-deterministic geographic forwarding algorithm (SNGF), neighbour feedback loop (NFL) and back-pressure re-routing. SNGF used by SPEED to select highest speed node as relay nodes, calculation of relay speed is based on distance of ngn nodes and local information exchanged by beacon messages. Delay estimation between the sender and receiver nodes is done with the exchange of acknowledgement messages. NFL is used to maintain single hop relay speed with the knowledge of nodes miss rate. Back-pressure re-routing is utilized to reduce the congestion inside the network while transmission. Each node keeps the detail of its neighbour nodes and calculates packet progress speed of each neighbour node to forward its data to the node whose progress speed is greater than the predefined lowest speed limit.	If a node cannot find a neighbour node with progress speed more than predefined speed, packet will dropout, hence less reliability. This protocol ignores energy metric in speed determination and routing.
2004 [183]	Proposal of an 'energy-aware delay constraint routing algorithm' for both static and dynamic sink engagement. This approach provides the least cost path meeting end to end delay constraint, reduces average delay and packet miss ratio. Also this limits unnecessary re-routing which helps to save energy.	Extended Dijkstra's algorithm is used to find k disjoint paths in ascending order of cost where the cost function for each link is based on communication distance, nodes residual energy and transmission error rate. Appropriate transmission range for each node is calculated, satisfying time deadline. The method is extended to support sink mobility in the network by employing automatic re-routing approach. If the MS lost connectivity due to its movement, new nodes are directed to connect with the sink to relay packets and routes are updated by adding new hops. If the sink moves close to the source nodes, it over-hears the current transmitting data traffic, reduces unnecessary relaying and end to end delay. Re-routing only happens when already established routes become less energy efficient.	This does not support scalability factor. Utilization of class-based, priority queueing mechanism rises complexity and makes the network costlier for the resource-constrained sensors.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2006 [56]	Proposal of a 'Multi-Path and Multi-Speed (MMSPEED) protocol' considering timeliness and reliability factor. MMSPEED achieves guaranteed end-to-end delay and reliability requirements in a localized way. Suitable for more scalable and large dynamic WSNs.	MMSPEED replicates the SPEED protocol in designing multiple SPEED layers. This protocol works on two important notions: virtual isolation among the SPEED layer and dynamic compensation of local decisions. Virtual isolation aims to classify incoming packets into an appropriate speed layer based on their speed, this reduces transmission delay of lower speed packets by differentiating them from higher speed packets from the queue. The dynamic compensation is the adjustment in speed at intermediate nodes while transmission to meet end-to-end deadline. Delay is estimated at each node after every hop using elapsed time concept for updation of speed if necessary. To enhance the reliability level, MMSPEED adopts multiple path data forwarding. Number of forwarding path is depending on packet reliability level as low reliability packet is delivered using a single path while a high reliability packet is delivered via multiple paths.	MMSPEED requires more complex calculations for multiple path selection and dynamic compensation, it may lead to more power consumption. This is designed for short-lived network only.
2006 [170]	Proposal of an 'energy-aware dual path real-time routing scheme' for delay sensitive data packets. This approach focused on achieving reliability and improving network lifetime. It also reduces congestion at intermediate nodes by minimizing redundant transmission.	Packets are prioritised to deliver on the basis of urgency factor depending on path distance and packet deadline. More is the urgency factor is for a packet at a node, earlier the packet has to deliver. Also on the basis of urgency factor, the value of distance traveled from the source node towards the destination node is decided. Intermediate nodes are prioritised using the calculation of energy and delay so that packets can reach final destination on time. This scheme involves dual transmission of data packets from the source node i.e. source transmits same data packets to first priority node as well as second priority node in its surrounding and after that only one path is taken to route data upto final destination. Additionally, Author engaged MAC layer to minimize delay on its part by stopping post backoff for real-time data packets.	This approach ignores nodes energy consumption to achieve QoS and delay requirement.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2007 [171]	Proposal of 'LR-ENR and HR-ENR' real-time routing algorithms with the aim to have energy efficient delay constraint routing with achieving optimal lifetime in the connected network.	At first, author implement a LPP objectifying maximum of minimum network lifetime avoiding delay factor for energy efficient routing. A distributed algorithm is proposed which iteratively solve this LPP by giving least cost path routing where the cost of each path is either the sum or the maximum of the cost of the nodes on that path. Due to ignorance of delay factor, author extends the approach further by incorporating delay guarantee which has been named as Level restricted energy efficient routing (LR-ENR) and Hop restricted energy efficient routing (HR-ENR). LR-ENR limits the number of iterations and HR-ENR limits the length of path by minimizing number of hop counts to meet the deadline.	It does not consider link's reliability. An unreliable link in any route may cause longer delay because of higher retransmission probability, which in turn increases energy consumption.
2008 [173]	Proposal of a 'real-time routing protocol for load distribution (RTLDD)'. RTLDD distributes task of data forwarding, hence prolongs network lifetime. RTLDD reports high performance in term of delivery ratio, control packet overhead and power consumption.	This method is location aware and uses geo-cast with directional forwarding to engage multiple paths for data delivery. Routing consists of forward metrics calculation, forwarding mechanism and routing problem handler. Packet reception rate (PRR), Packet velocity and residual energy are forwarding metrics, defined to calculate optimal next node and data forwarding is done using either unicast or geo-directional cast routing depending on the direction and distance of receiver node and its surrounding. Average delay in data routing after every hop is calculated to ensure packet delivery before the deadline, this increases throughput. When there is no forwarding node found to route data to destination then routing problem handler is used which has two recovery method: fast recovery, it can be done when a node's transmission range covers the whole hole and slow recovery is done by using feedback control to acknowledge backward nodes about the hole.	Designed for static network only. Inspection of link quality based on PRR is time taking and leads to energy consumption.
2008 [184]	Proposed a new routing algorithm undertaking energy, delay and reliability factors simultaneously. This increases sensors lifetime and works for low mobile sensors also.	Approach is the extension of AODV protocol where nodes discover routes in request-response cycles with the difference of opting energy efficient paths on which transmission delay is lesser than the allowed delay.	Delay is measured on the basis of hop counts only. Interference and distance parameters should be taken into account for transmission delay.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2009 [58]	Proposed a new real-time routing based on two-hop neighbourhood information named as 'THVR'. THVR eliminates topology hole problem while routing and reduces DMR. Achieved a better tradeoff between load-balancing and transmission delay.	The approach of calculating and requesting required velocity for every incoming packet at each forwarding node is similar to SPEED protocol but with the difference of considering two-hop away nodes from the source node for next move. Also in THVR, a joint metric of required potential velocity and nodes residual energy is considered to determine next forwarding node. Delay estimation at each forwarding node involves MAC delay and transmission delay is done to predict packets deadline. Initiative Drop Control method is conducted when there is no two-hop forwarding node available with the required speed. Then dropping of packets with the request of retransmission takes place.	Data reliability is low and with the dropping of packets, chances of overhead and energy consumption increases.
2012 [178]	Proposed a 'Fast Time-Dependent Shortest Path (FTSP) Algorithm' for end-end delay reduction in WSN. FTSP algorithm achieved least latency. Moreover, another version of FTSP is also proposed, named as 'FTSP-M' to deal with mobility of sensors. Both the proposed algorithms have polynomial message and time complexity. FTSP-M runs concurrently at multiple nodes if there are multiple node updates.	FTSP algorithm works similar to Bellman-Ford algorithm with the inclusion of β -synchronizer to bound the message complexity for time-varying network. FTSP consists of two major steps: first is construction of spanning tree and second is computation of shortest path from every node to sink, layer-wise i.e. nodes which are lying in the nearest layer to sink are computed first and so on. Control messages play the task of doing acknowledgements about distance between nodes. This algorithm is further extended to dynamic networking named as FTSP-M. FTSP-M updates shortest path only for updated nodes without disturbing whole network and the process of spanning tree formation accomplished with the help of control messages.	This cannot solve the void path and routing loop problem which affect the real-time communication.
2013 [60]	Proposed an 'enhanced real-time routing protocol with load distribution for mobile WSNs (ERTLD)', extension of RTLD. ERTLD achieves higher throughput and reduces end-end delay in dynamic WSN. Also ERTLD solves hole problem and saves calculation time by utilizing RSSI as optimal forwarding metric.	This method engages corona mechanism instead of geocast-directional routing. Corona mechanism involves the formation of concentric circles centred at sink, more the distance of SNs from the sink, farther they lie in the concentric circles having more radius. Corona level increases parallelly with the radius of concentric circles and so data travels from higher corona level to lower to finally reach the sink. Optimal forwarding parameters like RSSI (received signal strength indicator), delay per hop and residual energy are simultaneously considered to make choice for next move. Backup corona mechanism is used against network hole while data forwarding. In this, a backward control packet is send to stop transferring data on the same route and re-routing is done.	This method does'nt consider lifetime and energy saving factor explicitly.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2014 [179]	Proposed two distributed query-driven data forwarding approach for delay sensitive network and bring out good tradeoff between energy efficiency and congestion avoidance. Development of 'First Forwarding Nodes (FFNs)' and 'Second Forwarding Nodes (SFNs)' enhance network connectivity and mitigate void path problem in real-time communication. Proposed methods can be applicable to mobile network also and establish an energy balanced delay sensitive routing.	The concept of FFNs and SFNs are introduced so that each node first uses FFNs as relay nodes until FFNs go out of energy, afterward SFNs work as relay nodes. Flooding of query messages by BS makes every node to get their FFNs and SFNs on the basis of hop counts. Data routing is done by choosing minimum cost link which is depending on residual energy and channel congestion degree in such a way that it constitutes a directed acyclic graph avoiding looping of data transmission. Proposed first algorithm utilizes routing messages for every transmission causes energy consumption so the modified algorithm is proposed to reduce this by employing the prediction of link quality and local information based on previous round. A new energy-cost function is designed with the delay estimation including link, queuing, transmission and propagation delay to find the least cost route satisfying delay constraint. DCEM does network organization task including clusters, route formation and data transmission i.e. multi-hop inter cluster routing. DCEM operates the same task after every round. Two parameters α , related to CHs' residual energy and β , related to distance between CHs and sink, are engaged to establish the balance between energy consumption and end-end delay.	Assumption of communication link is error free makes this method quite impractical. The problem of nodes isolation and voidness persists in sparse network.
2016 [174]	Proposal of a 'delay-constrained energy multi-hop (DCEM)' algorithm to find optimal solution of TED ('Tradeoff between Energy and Delay) optimization problem in real-time cluster-based routing.	This work involves the identification of slow data packets by calculating expected delay whose transmission deadline to reach destination are expired at any intermediate node and removal of those expired or much delayed packets from the queue of that node to eliminate its transmission. In this way, a queue maintains data packets of sufficient residual deadline only. Also nodes can vary their transmission power based on distance using two-hop ngngh information to reduce dropping of data packets and retransmission.	Designed for static network only. Consecutive network organization causes energy consumption.
2016 [59]	Proposal of an 'energy-efficient adaptive routing protocol (EE-ARP)' for real-time communication. EE-ARP ensures energy efficiency, better routing performance and increases network lifetime with least latency and few packets dropping.	DELAR consists of two phases: cluster setup and routing phase. An optimization problem is formulated in which link cost is taken as an aggregated function of energy consumption and transmission delay to find the least cost path from the set of all paths from SNs to sink satisfying the delay constraint. Two heuristic algorithms are proposed, algorithm 1 is to find the optimal multiplier α which belongs to some interval $[a, b]$ which is further engaged in algorithm 2 to find optimal path of least cost satisfying end-end delay.	Designed for homogenous, static and dense network only. Dis-connectivity and voidness can be a problem in less dense network.
2017 [172]	Proposed a 'Delay-constraint energy efficient routing protocol based on Lagrange relaxation (DELAR)'. This heuristic algorithm has polynomial time complexity. Reduces average delay and energy consumption. This method has been extended to find multiple shortest path.	Analysis complexity increases with the increment of number of SNs. Defined for static network only.	

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2017 [17]	Proposes an energy efficient link delay aware routing algorithm. This approach is very well suited for real-time wireless communication as it reduces end-end delay and balances the tradeoff between energy consumption and delay factor.	The routing mechanism comprises of four phases, i.e., neighbour discovery, metric calculation, score calculation and node selection. Exchange of 'Hello' msg tracks the neighbour list. Residual energy, end-end delay and transmission counts are considered metrics and PRD value is the required score based on the given metrics. PRD demonstrates the predicted remaining counts of successful deliveries within one unit of delay. Next relay node has been selected for routing using PRD results best link quality, shortest end-to-end delay and highest residual energy.	Designed for directed and homogenous network. Authors didn't provide the complexity of the proposed work with the change of network density.
2018 [182]	Proposed a distributed 'Minimum-Delay Energy-efficient flooding Tree (MDET)' algorithm to find the optimal solution to the multi-objective problem of minimizing delay and energy consumption in flooding based data transmission. MDET shows a very good balance between flooding delay and energy efficiency with minimization of flooding cost and delay.	MDET is designed to bring out the balance to energy-delay tradeoff in low duty cycled network with unreliable links by constructing energy-optimal tree with bounding flooding delay. It involves the process of Expected Transmission Count (ETX)-based shortest path formulation and corresponding nodes enhance energy efficiency locally by choosing its parent node with best link quality and loop free transmission. A biased back-off mechanism is adopted in flooding tree construction to reduce control message overhead.	Designed for static network with low duty cycled and unreliable wireless link. Reduction of delay is usually achieved at the price of truncate network lifetime.
2019 [177]	Proposed a shortest path strategy for MS for data aggregation while ensuring connectivity in a typically connected/disconnected network. This approach reduces end-end delay with load balancing. The proposed work is most suitable for delay-harsh applications where the delivery delay is much more important than energy efficiency.	The methodology consists of two phases: first phase determines the anchor points (APs) where MS visits to collect data. APs are selected such that some are close to network center and some are far from network center. So the target area has been partitioned into grids having APs and joining of 4 grids having distant APs from center results quadrilateral like structure. Second phase adds new APs to the existing structure until there exists an AP in every segment. The new APs must be near the current path as much as possible. DAAP considers disconnected segments of the network in an decreasing order of their minimum joining cost to obtain a final and shortest path for MS.	The proposed solution is localized and approximate one. This approach is more inclined to minimize delay so energy consumption is still persisting issue.
2019 [8]	Proposed a 'delay aware energy efficient reliable routing (DA-EERR)' technique for a large scale heterogeneous WSNs. The proposed work deals with hot-spot problem, improves network lifetime and reduces packet loss. This technique achieves a good balance between energy efficiency and fast data delivery in the large densely deployed heterogeneous WSN having mobile sink.	In DA-EERR, the network area has been partitioned into multiple rings with three types of nodes based on energy level i.e. energy of type A > type B > type C. Type B and C nodes sense the data packets and type C nodes collect and forward data while carrying the information of mobile sink node. For quick and easy data transmission, search space towards the destination node has been reduced. Every source node chooses relay node of highest energy level within minimum distance to transmit its data to mobile sink.	Defined for static network. The proposed work depends on excess beacon messages transmission.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2019 [61]	Proposal of a new 'Delay Constraint Energy-efficient Multi-hop routing algorithm (DCEMRA)' involving a new rule-based clustering and routing which builds balanced intelligent clusters and low cost routes. The major advantage of DCEMRA algorithm is the improvement in reliability, packet delivery ratio, network lifetime and reduction in energy consumption and delay.	Clustering process consists of three major steps: CH selection, cluster formation and tradeoff of energy and delay (TED) calculation. Selection of CH is based on nodes energy, RSSI and agent computed value to acknowledge ngh. Updation of CH need to be done when it doesn't able to satisfy energy threshold. TED factor adjusts mobility parameters over the CHs energy expense and distance of CHs from MNs to bring out the balance between energy consumption and end-end delay. DFS algorithm is utilized to find low cost path satisfying the delay constraint during inter-cluster routing. The whole routing process consists of link and route cost calculation of all possible paths, end-end delay calculation of selected low cost shortest paths and multi-hop data communication.	Designed for static homogenous network and work in a centralized way by sink node which is located at the centre of the field. Engagement of intelligent agents makes the process costly.
2020 [175]	Proposed a 'Delay-aware Green Routing Protocol (DGRP)' to deal with hot-spot problem in WSNs by engaging mobile sink node. Engagement of mobile sink nodes reduces the energy hole problem, end-end packet delivery, energy consumption and outperform general ring routing protocol (RRP) in large WSNs.	DGRP partitions the target area into optimal number of rings, by considering the length of the sensor field and communication range of the sensor nodes. The nodes which are closer to sink node in every ring chosen as high tier nodes which keep the location information of sink nodes and then high tier nodes, belonging to the same partition, reshape the ring by joining themselves. Second type of nodes are called low tier nodes which pass their data in one hop to high tier node belonging to the same ring. Thereafter high tier nodes pass the information either directly to sink node or minimum length neighbour high tier node by setting an acute angle towards the direction of sink node.	Homogeneous and static sensor nodes having same radio range limits the application of the proposed work.
2020 [181]	Proposed a 'transmission delay minimization based on adjustable duty cycle (DMADC)' routing scheme. DMADC makes better use of node energy and reduces network latency than previous methods. DMADC scheme can reduce the end-to-end data transmission delay by 10.25% – 26.37% while maintaining the network lifetime and increase the energy utilization rate by more than 25%. The DMADC scheme improves network lifetime by about 30% while maintaining end-to-end latency.	DMADC is Basketball Net topology (BNT) based approach and each node is assumed to have two state: sleep and awake. DMADC comprises of two algorithms; Algorithm 1 computes the shortest path by computing minimum transmission delay of each relay node from source to sink node i.e. source node use a relay node of minimum transmission delay where transmission delay is the sum of sleeping delays along the path. Algorithm 2 is the duty-cycle adjustment approach. When there is no node in the active state to forward data, parent node informs the child node to adjust its duty-cycle from sleep to awake. Therefore nodes which are lying far from sink node are kept on higher duty cycle and nodes near the sink node are kept on low duty cycle. This helps in reduction of energy consumption of nearby nodes and avoidance of hot-spot problem.	Designed for static and homogenous network, makes the work quite impractical.

Table 7 (continued)

Reference	Contribution	Approach/methodology	Limitation/future scope
2021 [176]	Proposed a 'Delay-Sensitive, Reliable, Energy-Efficient, Adaptive and Mobility-Aware (DREAM)' routing protocol, incorporating nodes mobility and network heterogeneity. DREAM is the cluster-based solution to the multi-objective problem which is simultaneously dealing with energy consumption, network lifetime, latency, throughput, reliability and scalability.	In DREAM, target area has been partitioned into sectors where nodes are randomly spread and BS at the center. From each sector, a CH is selected based on the highest power among the competing nodes which are eligible to be CHs on the basis of residual energy and distance factor. MNs from each sector send their data to the CH and CH, depending on its distance from BS, can transmit data directly to BS or use other CH as a relay node to pass the data to BS.	This work utilizes single and two-hop routing in 2-D model of WSNs persisting redundant transmission issue. The work can be extended to 3-D model incorporating multi-hop routing and data redundancy issues.
2021 [180]	Proposed a real-time application based routing scheme to minimize end-end delay using two-hop information. This approach establish a good balance between energy efficiency and QoS level.	Every node keeps the record of its hop count from BS as height i.e. node with one hop away from BS has height 1 and so on. Data transmission has been carried out by estimating every two-hop away node's residual energy, delay and link quality and so a PRI (Potential relay information) function has been formulated to estimate the possible forwarding nodes based on these parameters. After computation of PRI value, each node computes its neighborhood state index (NSI) based on PRI which depicts the quality of its surrounding nodes. Higher the value of NSI, higher the chance of being selected as a relay node during data transmission.	Designed for static and homogenous network. The proposed scheme utilizes two-hop information at the expense of computational complexity.

by considering RSSI (received signal strength indicator), delay per hop and residual energy simultaneously to make next move. [7] considers more factors like packet delivery, nodes energy, nodes degree, distance, retransmission etc to determine next routing hop to balance energy consumption with delay minimization. Authors in [58, 179, 180] consider hop counts to determine minimum delay path while authors in [181, 182] estimate end-end delay based on sleep/wakeup delay. After going through each technique of Table 7, we draw some conclusion:

- Prioritizing end-end delay minimization effects the nodes energy adversely in such a way that they die soon in the network so saving nodes energy should be taken equally important. This improves overall network performance.
- Topology aware real-time routing techniques balance end-end delay and energy consumption in an optimal way but they are lacking their performance in dynamic network. Also some are only designed for homogenous network.
- Indeed proposed techniques in this field enhance packet delivery ratio, improve throughput, reliability, reduce DMR and end-end delay at the cost of more energy consumption than usual or by making network costly by opting MSs or backup nodes.

8 Discussion and comparative study

In this section, we discuss about prime benefits, persisting issues and future scope of each of the reviewed topology control and routing technique along with the comparison between the existing solutions.

8.1 Topology control techniques

Graph based range assignment and power control Among the graph based approaches which are mainly focussed on range assignment problem and transmit power control, some of the work [15, 84] has adopted arbitrary and fixed transmit power assignment to nodes and some [17, 24, 78, 79, 86] bring out the strategy of discrete power assignment to have energy efficient routing topology. So when it comes to power assignment to nodes, work has been limited to discrete power management in topology control. Although some TC techniques [62] involve dynamic adjustment of transmission power level of nodes but in an implicit way to reduce power consumption and extend network lifetime. Authors in [81] come-up with interval graph based polynomial time algorithms to explicitly deal with nodes having power levels in the form of intervals. However this work is limited to static WSNs and algorithms are centralized in nature. Therefore more work on TC with continuous range assignment can be explored further. Also existing solutions to

achieve minimum cost routing topology by minimizing nodes' power consumption or engaging smaller transmission range enhance transmission delay in large network so work to obtain minimum delay energy efficient topology can be promoted in future work.

Interference minimizing models and approaches Approaches in [15, 17, 36, 37, 88, 89, 91, 93] are good enough to minimize nodes energy consumption and maximize network lifetime by explicitly reducing network interference but topology formed by them are less fault-tolerant and non-dynamic. Bao and Deng in 2016 [38] introduced the fault-tolerant topology with low interference. Most of the existing interference models work with static transmission range of nodes in minimizing interference and energy consumption [14, 15]. Appropriate estimation and reduction of interference in mobile WSNs is still an existing issue in this field.

8.2 Routing techniques

Classical hierarchical routing protocols These protocols or routing techniques have received maximum attention for reducing network energy consumption with incorporation of scalability factor. LEACH, Extension of LEACH and many more multi-hop cluster-based routing protocols [20] have been designed undertaking the issues of energy consumption, scalability, network lifetime, reliability and load balancing yet the energy-hole problem has been mitigated by proposing unequal clustering protocols namely DUCR [113], HEEMP [119], ISCTO [125], HDDS [133] and fault tolerant issue by CRPD [117], CFTR [96], EEMCS [129], ECRP [131], SHSDA [135]. Yet SHSDA [135] is the most secured, reliable and stable among the existing cluster-based routing protocol for static and homogenous WSNs. Further, HDDS [133] has better network lifetime and coverage than DUCR and CRPD. CFTR [96] conserves more energy, more stable and fault-tolerant than DUCR protocol. EEMCS [129] does 20 – 30% better energy utilization than CRPD and outperforms LEACH and MOD-LEACH in load balancing, network stability, network lifetime and throughput in large-scale network. Also EESRA [123] performs better than LEACH and its variant in term of energy consumption, load balancing and network lifetime. The SEA [137] approach outperforms LEACH, SEP, TEEN and DEEC protocols in term of CHs selection, CHs rotation and network lifetime compared to . Tree-structure based data aggregation method given in [134] offers the best network lifetime and reliability with reduced energy consumption and average delay over the existing tree-based data aggregation and clustering approaches. Mostly available works have addressed energy optimization, coverage preservation and fault tolerance aspects independently. Also developed protocols are lacking in real-time application based WSNs because of persisting transmission delay while establishing network topology. For

heterogenous large scale mobile network, more energy and fault-aware routings need to develop.

PSO-based algorithms Algorithm in [149] outperforms many Genetic Algorithm (GA) based on clustering schemes, namely GADA-LEACH, GABEEC, DCH-GA in term of minimizing energy consumption, maximizing throughput and scalability and has better applicability to real-time communication. Also inadequate clustering technique of PSOBs [147] has been overcome by Kaur et al. in [149]. Kaswan et al. [144] through their algorithm MOPSO overcome the issue of unbalanced energy consumption and load distribution of 'Tree cluster based data gathering algorithm (TCBDGA)' and 'energy aware sink relocation (EASR)'. PUDCRP [145] balances the energy consumption of sensor nodes more effectively, has good scalability and consumes less energy per round than the existing PSO-based protocols namely PSO-ECHS and PSO-SD. EC-PSO [146] minimizes energy consumption and maximizes network lifetime better than VD-PSO. MDCH-PSO [153] outperforms the standard LEACH-M, LEACH-C algorithm by improving the average residual energy by 13.9% and 27% respectively. Also average delay has been reduced by 29.6385% and 35.26% than LEACH-M and LEACH-C by MDCH-PSO algorithm respectively. GAPSO-H [155] outperforms some existing PSO-based algorithms (PSOECMS, PSO-UFC, PSOBs) and GA-based algorithms (GADA-LEACH, GABEEC, DCH-GA) in term of stability, network lifetime and throughput.

ACO-based algorithms ACO-based EMPAR and MPAR schemes by Jari and Avokh [166] have better performance than the ECDA algorithm in terms of energy consumption and network lifetime. EMPAR reduces energy consumption by 5.98% and 12.20% and increases the network lifetime by 12.26% and 30.38% compared to MPAR algorithm and energy-aware CS-based data aggregation algorithm respectively. Although, EMPAR performs better than MPAR. Improved ACO-based algorithm [163] is better to extend network lifetime and residual energy than the randomly distributed WSN strategy and ACO strategy. Further AOSTEB algorithm [164] reports the enhancement of network lifetime by more than 200% as compared to improved-ACO (IACO). Roy et al. in 2021 [167] put forth an ACO-based technique which comes out to be the most efficient in terms of network lifetime, energy consumption, packet delivery ratio and end-to-end delay as compared with the existing ones concerned with the issue of non-uniform and uniform nodes densities in target area.

Hybrid PSO and ACO techniques A hybrid algorithm of ACO and PSO in [160] outperforms Genetic algorithm (GA) and Artificial Bee Colony (ABC) algorithms in saving nodes energy and improving throughput. More work using PSO and ACO techniques can be conducted to optimize energy

consumption and bring network stability with delay minimization in heterogeneous large-scale dynamic network. It has been observed that hybrid algorithms of ACO and PSO are having better performance therefore, research on bringing hybrid algorithms with distinct multi-objectives can be more helpful in this field. Most of the PSO and ACO based routing techniques engage mobile sinks while the other nodes can be static or mobile, this makes the network more costlier so work on same objectives with different approach to obtain a low cost network should be undertaken in future.

Real-time routing protocols WSNs face an inherent conflict between energy efficient communication and desired quality of service such as end-to-end communication delay, timely data delivery and/or deadline miss ratio. The well established MMSPEED protocol performs better than SPEED and SPEED is better than RAP in the domain of timely packet delivery and reliability but lacking energy efficient communication as protocols are not concerned with nodes residual energy and loss of energy while transmission. THVR [58] which focuses on minimizing delay factor with load balancing, outperforms SPEED protocol in lowering end-to-end deadline miss ratio and utilizing energy efficiency. EE-ARP protocols in [59] improves the deadline miss ratio of THVR and PATH protocol. ERTLD [60] which is the extension of RTLD protocol minimizes average end-end delay upto 65% and 31% compared to RTLD and MMSPEED respectively. Moreover ERTLD has better packet delivery rate as compared to RTLD and MMSPEED. DCEM algorithm of Huyunh et al. [174] overcomes the latency effect of cluster-based Gen-LEACH and multihop-HEED protocols. MDET [182] outperforms tree-structure based data flooding schemes in minimizing transmission delay with load balancing. DCEMRA [61] has better packet delivery ratio with the reduction in energy consumption and transmission delay compared to LEACH and PEGASIS. Real-time routing protocols meet the delay and QoS requirements by ignoring energy saving and lifetime prolonging factors. Therefore work on increasing network lifetime with minimum deadline miss ratio needs to undertake since node's battery gets drained so fast. Real-time routing protocols can utilize multiple paths to achieve energy efficiency and delay minimization simultaneously in future work. Also work on improving energy efficiency is achieved with using two-hop forwarding method only, it can be extended further.

9 Conclusion

In this paper, a detail review of 145 articles from the respected journals and conferences representing the topology control methods and routing techniques in WSNs have been provided. This review article offers the literature of

graph-based methods utilized in range assignment problem, algorithmic approaches to minimize interference explicitly in topology control and various routing protocols/ techniques related to classical hierarchical routing, real-time routing, PSO and ACO-based routing with parallel mentioning of contribution made by the proposed solution, detail of methodology opted and limitations which might be overcome in future work. A brief review on previously existing survey papers has also been conducted with the observation that the existing review articles on topology control methods skipped many models and algorithms, particularly related to interference minimization in topology control. Also no survey paper has provided the combined and collective study of topology control methods and routing techniques. Therefore, reviewing unexplored and latest topology control methods and routing techniques in WSNs, dealing with various topology control and routing challenges, and placing them together systematically is the accomplished task of this review paper. Moreover, research methodology to conduct literature survey, from selection criteria to papers filtering, has been offered for readers reference. Henceforth, this review article acknowledges readers about the application of WSNs, existing issues related to topology control of WSNs, routing challenges, accomplished objectives of the existing solutions along with the detail of methodology opted, results, merits and demerits, a brief comparison among the proposed solutions, research gaps/ open issues and direction of future research work.

Declarations

Conflicts of interests The authors of the paper declare that they do not have any conflict of interest.

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Interval graph based energy efficient routing scheme for a connected topology in wireless sensor networks

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Abstract

Energy conservation has always been a major issue in wireless sensor network since wireless sensors spend lots of energy while sensing, receiving and transmitting signals, meanwhile network lifetime decreases. Excessive power usage in transmission among the sensors can be reduced by designing an energy efficient routing topology which reduces nodes' energy consumption by abandoning those links which are highly energy expensive. In this paper, energy optimizing algorithms are taken over those wireless sensor network whose graphical layout formation can be a 'connected bidirectional interval graph', in which each node has been assigned with an appropriate power interval to hold network communication. Two new polynomial time algorithms are proposed to optimize energy consumption over the network subject to constraint that the resultant network topology remains strongly connected with minimum number of required links. Both the algorithms work to evaluate an energy optimal path cover whose path components combine to result an energy efficient connected routing structure (spanning path/tree) in an interval graph, reduces power usage over links and nodes in wireless sensor network. Optimization of maximum transmission cost in worst case scenario over a maximum weighted bidirectional interval graph and reduction of maximum power consumption limit of nodes in an interval weighted interval graph to the minimum possible value are also discussed.

Keywords Wireless sensor networks (WSNs) · Topology control problem · Transmission range assignment · Optimization · Interval graph · Path covering

1 Introduction

Wireless communication exists only when sensors have sufficient amount of power stored in them so that they can receive, sense and transmit data to the required destinations. Since sensors are powered by battery, they have limited power and so their life in WSNs. This requires energy-aware routing topology which can significantly save nodes power. Many authors like Li et al. [15], Panda and Shetty [23] believe that power consumption of nodes increases at least quadratically with distance while

transmission, so they proposed topology control algorithms to conserve energy by abandoning long-range communication links with the adoption of multi-hop data transmission. Li and Halpern [14] reveal that it is not energy efficient to set every node at maximum transmit power in multi-hop WSN. As maximum transmit power of a node implies longer transmission range of that node which arises more unnecessary communication links with other nodes within its transmission disc hence causes higher interference. Therefore maintaining small number of communication links with reduced transmission range provided the whole network remains connected will speedup the routing protocol and simultaneously leads to energy saving [18, 19]. Reduced transmission range omits unnecessary interference since it reduces collisions of data packets along the route which in turn lowers node's energy consumption [12].

Panda and Shetty in [23] defined a power function $r: V(G) \rightarrow \mathbb{R}^+$ to assign a transmit power to each node in WSN as $r(u) = \max_{uv \in E(G)} c(uv)$ where $V(G)$ is the set of

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nodes, $E(G) = \{uv : c(uv) = |uv|^2\}$ is a set of bidirectional links and $c(uv)$ is a power emission cost taken symmetrically on the link uv to transmit data from u to v and v to u forming a complete graph $G(V, E)$. This follows when a node u is assigned with fixed power level $r(u)$, it forms a transmission disc with range $r(u)$ taken as a radius and u as centre. So whenever node u starts transmitting, data is received by all nodes which belong to its transmission disc. Hence this works good in case of broadcasting of data packets but in case of data transmission by node u to some particular nodes v such that $c(uv) < r(u)$, then this causes unnecessarily data transmission to the remaining nodes which come inside the disc of radius $r(u) = \max_{uv \in E(G)} c(uv)$. However this problem of unnecessary transferred data can be reduced to some level by allowing nodes to decrease their transmission range according to the priority/order of data transmission.

In wireless sensor network, transmission power level of each node can be tuned between a minimum and a maximum value [23] to establish/demolish communication link with other nodes directly. Let $r : u \rightarrow [r_{min}, r_{max}]$ be the power function which assigns a power interval to an arbitrary node u of a WSN so that node can vary its transmission range between r_{min} to r_{max} where r_{min} is the minimum power required to establish/maintain at least a single connection in the network and r_{max} is the maximum power required to communicate the farthest node directly by u .

The work of this paper is dealing with wireless sensor network where each sensor is static and assigned with unique power interval like $[r_{min}, r_{max}]$ and incident links extract power ranging in this interval. This communication network topology formed by sensors has an interval graph layout when two sensors communicate directly via a link if and only if their power intervals have non-empty intersection. Not every WSN forms an interval graph layout, it can be checked in linear time [5] but if network is complete i.e. each sensor can directly approach to every other sensor by adjusting its transmission range then the wireless network can definitely form an interval graph layout. Allowing nodes to adjust their transmission range in an assigned power interval is quite helpful in saving energy when large number of nodes are deployed in a region having a complete communication network instead of assigning fixed transmission range to each node i.e. $r(u) = \max_{uv \in E(G)} c(uv)$.

In this paper, wireless sensors are assigned with interval of transmit power levels forming a bidirectional interval graph and goal is to find an energy efficient routing topology, covering all nodes with minimum number of required links in an interval graph. Problem related to high energy consumption by links while transmission in WSN is

under taking by newly proposed algorithms and resulting to minimize the upper limit of power consumption of maximum assigned power interval, minimizing maximum node's power consumption, maximum transmission cost over the link where links are using maximum power available to them i.e. worst case scenario. The aim of proposing new algorithms is to evaluate an energy efficient spanning path/path cover on an interval graph by selectively choosing minimum number of links based on power usage so that energy can be saved and result to an energy efficient connected topology (i.e. no loss of any sensor in the resultant network). Algorithm 1 finds an energy optimal path cover in a complete interval graph and Algorithm 2 is the generalisation of 1st algorithm, finds an energy optimal path cover in an arbitrary interval graph. Achieved energy optimal path cover when it contains more than one path component, needs to turn into a connected energy efficient routing structure (spanning path/tree) with the joining of distinct path components by applying Algorithm 3, motivated by [16].

To perform our task, following assumptions are made:

1. WSNs whose graphical layout are connected bidirectional interval graphs are considered.
2. Communication links are assumed to be symmetric and bidirectional in behaviour.
3. Each sensor node becomes static after deployment.
4. Whenever interval graph is mentioned it must be understood as connected bidirectional interval graph.
5. Sensors are position-aware of their neighbourhood nodes in term of co-ordinates.

The rest of the paper is organised as follows, Sect. 2 provides the summary of existing related works, Sect. 3 explains about graphical layout for WSNs, definition and discussion about interval graph, formation of maximum weighted interval graph in worst case situation, problem definition, explanation of proposed Algorithm 1 and corresponding results. Section 4, discussion about interval weighted interval graph and optimization by Algorithm 1 in general case. In Sect. 5, proposed Algorithm 2, formation of connected routing structure by Algorithm 3 and some corresponding results are seen. Performance evaluation of proposed algorithms are given in Sects. 6, 7 shows the simulation work and Sect. 8 concludes the paper.

2 Related work

Various topology control and routing algorithms have been proposed in history [27] to reduce power consumption of nodes in order to enhance network lifetime satisfying the connectivity requirement of the resultant topology. In 2003, Cheng et al. [8] proposed graph theory based heuristics named as Minimum Spanning Tree (MST) and Minimum Incremental Power (MIP) which approximately minimizes total transmit power of nodes while maintaining the network connectivity bidirectionally. MIP which is a greedy heuristic performs better than MST-based heuristic. Three more heuristics can be seen in [13] to obtain power-aware and strongly connected topology. The main idea of the first two heuristics is to remove a maximum power such that network still remains strongly connected and third one is based on minimum spanning tree. These developed heuristics are able to find the solution approximately. Further Aneja et al. in 2010 [2] established the lower bound to the optimal minimization of total power consumption and proposed a branch-and-cut algorithm to provide a solution to the Linear programming formulation aims to minimize total power consumption while maintaining strong connectivity in unidirectional network. Authors in [22, 20] extends the Incremental Power greedy heuristic based on Kruskal and Boruvka algorithm respectively. A hybrid heuristic of Boruvka and Prim MST algorithm is proposed in [21] which saves more energy and has lower complexity than the existing versions of Incremental Power heuristics. Moving to Topology control algorithms based on Discrete power management i.e. an appropriate selection of power level from a Discrete set of power levels. Lakshmi et al. in 2019 [10] put forth two algorithms to deal with power assignment to nodes from the set of two power levels such that the total energy consumption gets minimized in the bidirectional connected network. Also Lakshmi et al. in [11] considered k -power level assignment problem which aims to assign a power level to each nodes from the set of k -distinct power levels which minimizes the total power assigned to the network and proposed two graph-theory based heuristics which run in polynomial time.

Some other works regarding power optimization in connected topology includes interference minimization approaches to save nodes energy and extend network lifetime. Burkhat et al. [6] proposed a centralized algorithm (LIFE) which results to a spanning forest having minimum interference. Authors in [18] proposed greedy algorithms which optimally minimizes the maximum link, node interference and approximately minimizes total interference. Sharma et al. in [26] proposed two algorithms MI-S and LS-IM. MI-S algorithm works to join edges in

increasing order of their weight to form a spanning tree which minimizes maximum interference of the network. Panda and Shetty in [23] proposed Minimum Interference Strong Bidirectional Topology (MISBT) algorithm which involves the computation of complete weighted graph of the given network where the weight on each edge is found by adopting the newly proposed MAX or SUM model, then computation of minimum spanning tree yields minimum interference connected network and resultant power assignment to each nodes. Authors in [12] studied the problem of minimizing maximum sender interference in a connected WSN where nodes are meant to assign either high or low power from the set of two power levels. These developed algorithms significantly save nodes energy and enhance network lifetime with the appropriate power assignment to each node in the connected network. But once the power is assigned to the nodes, it remains fixed in the network during data transmission. Most of the work mentioned here adopted single and fixed transmit power of nodes and rest brought out the strategy of Discrete power assignment to have energy efficient routing topology. Although some topology control techniques [1] involve dynamic adjustment of transmission power level of nodes but in an implicit way to reduce power consumption and extend network lifetime.

Motivated by the previous works, this paper is dealing with appropriate power assignment to each nodes explicitly in the form of power interval in which nodes can vary their transmission range. The aim is to optimally reduce power consumption of nodes and links transmission cost while preserving connectivity bidirectionally in an induced interval graph. To the best of our knowledge, the proposed work is the first attempt towards the modelling of WSN as an interval graph and providing the energy efficient routing topology on an interval graph. The designed WSN model and the proposed algorithms are interval graph theory oriented which makes this work different from the existing literature.

3 Graphical layout for WSNs

Let $N = \{s_1, s_2, \dots, s_n\}$ be a set of sensors, also called nodes in wireless sensor network. We model a weighted interval graph $G(V, E)$ with each node is taken as a vertex forming a vertex set $V(G)$ and each communication link is taken as an edge forming an Edge set $E(G)$. Let $\{r_{s_1}, r_{s_2}, \dots, r_{s_n}\}$ be the corresponding power assignment to each node respectively in the form $r_{s_1} = [r_{\min}(s_1), r_{\max}(s_1)]$, likewise each node is assigned with this kind of transmit power interval (or power interval can be said) such that no two vertices have equal assignment, at least they must be

differ by epsilon(ϵ) quantity. That epsilon difference is very minute does not have any effect on the established network but helps to satisfy an interval graph definition. Adjacency of any two vertices implies a non-empty intersection of power intervals assigned to both the vertices and viceversa, description is given in Sect. 3.3.

In WSNs, each sensor can detect the location of every other sensor either through global positioning system (GPS) or manual pre-programming during deployment [17] and so Euclidean distance among themselves [4]. Since we are dealing with non-mobile sensors deployed in any region to detect and transmit signals simultaneously, knowledge of position and Euclidean distance of neighbourhood nodes must be preserved by each wireless node and this works enough to design an efficient routing structure connecting all sensors.

3.1 Power dissipation theory in WSNs

Let $G(V, E)$ be the underlying graph of a WSN. Let u, v be any two nodes belonging to vertex set $V(G)$ and uv be any bidirectional link belonging to edge set $E(G)$. Amount of power assignment may vary node to node because varied power extraction is done by corresponding links incident on the node to have successful communication. According to reference [18], power assignment to a node u must include maximum amount of power needed by node u to communicate the farthest node v directly. Power dissipation comprises mainly because of 3 tasks:

1. Node u consumes energy to process/decode the received coded data.
2. To transfer data successfully to its destination, node u needs sufficient power to compensate the path loss of signal along the way from u to v .
3. Then energy is expended by node v to process the signal and acknowledge back to node u .

Therefore, the minimum power required to establish a connection between the node u and v separated by an Euclidean distance d is $c_1 + c_2 d^\alpha$, where $d = ||uv||$, α is the path loss gradient depending on transmission environment, c_1 and c_2 are some constant depending on electronic characteristics of wireless devices [18]. This $c_1 + c_2 d^\alpha$ corresponds to the transmission energy cost of link (uv) denoted by $c(uv)$. The value of c_1, c_2 is not known and also the exact value of α ($2 \leq \alpha \leq 6$) [25], only Euclidean distance between the nodes is known in the network therefore transmission energy cost $c(uv)$ is approximated by the value $||uv||^\alpha$. Thus, transmission cost function $c : V(G) \times V(G) \rightarrow \mathbb{R}^+$ [23] gives an approximated minimum value which in practical can be more depending on the various environmental factors. Moreover, Power assigned to a node u would be $r(u) = c(uv)$, $r(u)$ varies because of variation

in transmission cost of different links incident on u i.e. $r_{min}(u) = \min_{uv \in E(G)} c(uv)$ and $r_{max}(u) = \max_{uv \in E(G)} c(uv) \Rightarrow r(u) = r_u = [r_{min}(u), r_{max}(u)]$ is a power interval assigned to an arbitrary node u in a WSN. This power interval of node u , makes node u to communicate all its neighbourhood nodes because incident links' transmission power usage belong to this power interval of u . Hence each node can be assigned with an appropriate power interval.

3.2 Interval graph and its properties

A graph G is called an interval graph if there exists a one-to-one mapping between graph vertices and a family I of intervals on real line such that adjacency of any two vertices corresponds to a non empty intersection of their intervals. I is called an Intersection model for G [16]. An interval graph can be obtained by just knowing an Intersection model of it and viceversa.

Fig. 1 and 2, redrawn from [16], show an interval graph G of five vertices a, b, c, d, e and corresponding Intersection model $I(G)$, where each vertex is weighted with assigned power interval, represented by $I_a = [a_1, b_1], I_b = [a_2, b_2], I_c = [a_3, b_3], I_d = [a_4, b_4], I_e = [a_5, b_5]$ respectively, has a unique plot on Power line ($\mathbb{R}^+$) representing continuous range of power levels. For any adjacent vertices like a and b , their corresponding power intervals overlap and for non-adjacent vertices like b and e , their power intervals don't overlap, demonstrated in Fig. 2 that means sensor nodes are directly accessible to each other if and only if they have non-empty intersection of their power intervals.

Let's move towards the right end ordering of an interval graph since right end ordered interval graph is required throughout this paper. A right end ordering of an interval graph G can be obtained by sorting intervals in its Intersection model at their right ends in time $\mathcal{O}(|V(G)| + |E(G)|)$ [24]. Looking at the Intersection model $I(G)$ in Fig. 2 where $I_a = [a_1, b_1], I_b = [a_2, b_2], I_c = [a_3, b_3], I_d = [a_4, b_4], I_e = [a_5, b_5]$ are the power intervals assigned to nodes a, b, c, d, e of G respectively. Every interval has two extreme values so a right end value is the upper bound of the power consumption, therefore each vertical dotted

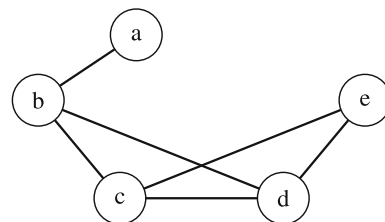


Fig. 1 Interval graph G

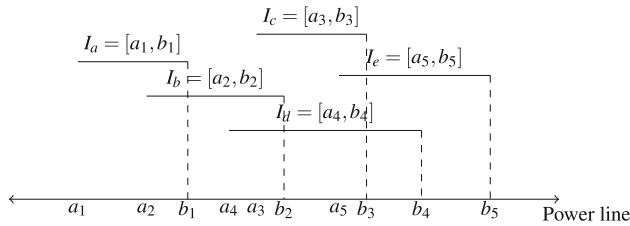


Fig. 2 Intersection model of interval graph G , $I(G)$

line in $I(G)$ shows the right end value of its corresponding power interval. Relabelling of nodes of G are done as $u_1, u_2, \dots, u_5$ according to the fall of right end on the power line going from left to right i.e. node corresponds to first dotted line labelled with u_1 , node corresponds to second dotted line with u_2 and like so on until all dotted lines are counted and all nodes get labelled. Fig. 3 represents a right end ordered interval graph G and Fig. 4 shows its corresponding right end ordering.

Moreover, a right end ordered interval graph makes easy to compare the upper limit of power interval of each node. This is because upper limits/ right end values of power intervals are in increasing order with the subscript of vertices and it can be seen in Fig. 2 that the upper limit of power interval of $u_1 < u_2 < u_3 < u_4 < u_5$. Hence vertices' subscript plays an important role in this paper. A vertex whose subscript is the minimum number is called *leftmostvertex(LMV)* of G and a vertex whose subscript is the maximum number among all vertices' subscript of an interval graph G is called *rightmostvertex(RMV)* of G .

$$V_{\text{leftmost}(G)} = \{u_j | j = \min_{v_i \in V(G)}(i)\}$$

$$V_{\text{rightmost}(G)} = \{u_j | j = \max_{v_i \in V(G)}(i)\}$$

leftmostvertex and *rightmostvertex* are abbreviated to *LMV* and *RMV* respectively.

A right-end ordered interval graph has some properties [16] which help to build a routing tree with an obtained path cover which is having more than one path component. Considering connected graphs only, every path in a right end ordering graph of an interval graph G must intersect with atleast one other path.

Property 1: For every path p on a right end ordering graph, there must be another path $q \neq p$ such that p and q are intersecting.

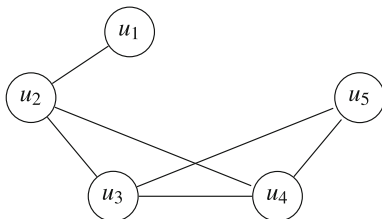


Fig. 3 Right end ordered interval graph of G

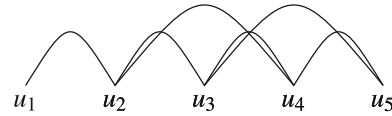


Fig. 4 The right end ordering of G

Property 2: Let G_1 and G_2 be two intersecting subgraphs of an interval graph G . If $\text{leftmost}(G_1) < \text{leftmost}(G_2)$, then $\text{leftmost}(G_2) < \text{rightmost}(G_1)$.

3.3 Formation of maximum weighted interval graph by wireless sensors

Let's take a WSN with the knowledge of number of nodes and the Euclidean distance among nodes where each node can access to every other node. Euclidean distance between the nodes a and b is taken as $\|ab\|^2$ in bidirectional topology and $c(ab) = \|ab\|^2$ (i.e. $\alpha = 2$) is the symmetric transmission cost for a link(ab) [23]. In this paper, transmission cost for any link, based on Euclidean distance only, is taken as a minimum guess/ approximate value to assign power interval to the corresponding end nodes and allowing transmission cost to increase in a particular range/ interval available at that link in order to enhance quality and quantity of signals to travel and reach destination successfully. For better understanding, let us consider a complete wireless network of 5 nodes, given in Fig. 5, redrawn from [23].

Consider node ' a ' of Fig. 5, mentioning all the links incident on ' a ' with Euclidean distance i.e. $\|ab\|^2 = 252$, $\|ac\|^2 = 14$, $\|ad\|^2 = 589$, $\|ae\|^2 = 34$. Minimum and maximum weight value among all the links incident on ' a ' is 14 and 589 respectively. Therefore, $r_a = [14, 589]$ is the range of power assigned to node ' a ' so that all required communication can take place by node ' a ' and likewise $r_b = [75, 252]$, $r_c = [12, 428]$, $r_d = [75, 589]$ and

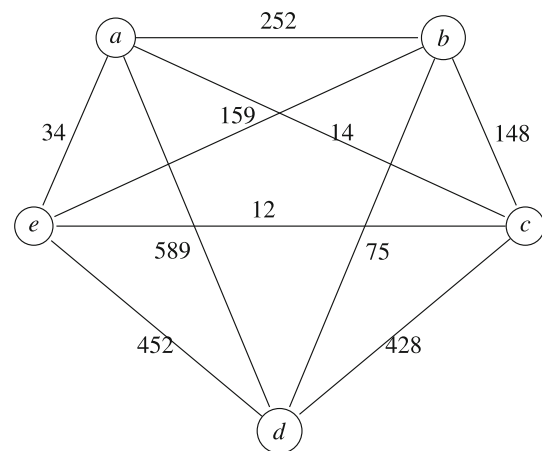


Fig. 5 weighted complete graph G'

$r_e = [12, 452]$ are computed and power intervals are assigned to the corresponding nodes. One can see how a complete weighted graph turned into a complete right-end ordered interval graph from Fig. 6(a,b,c). All power intervals in a complete graph are intersecting to each other because every node is directly approachable to every other via a link. So a non-empty intersection exists because common link contributed its weight value to its end nodes' interval. Hence adjacent nodes in WSNs always have non-empty interval intersection and this non-empty interval intersection contributes weight to the existing links.

Let's move forward to see maximum weighted complete interval graph where each link is using maximum power in transmission. Let $u_1 = [a_1, b_1]$, $u_2 = [a_2, b_2]$, $u_3 = [a_3, b_3]$, $u_4 = [a_4, b_4]$ and $u_5 = [a_5, b_5]$ be the assigned range of power to the nodes forming a complete interval graph G .

Note: Mentioned interval graph must be taken as right-end ordered interval graph from now onwards.

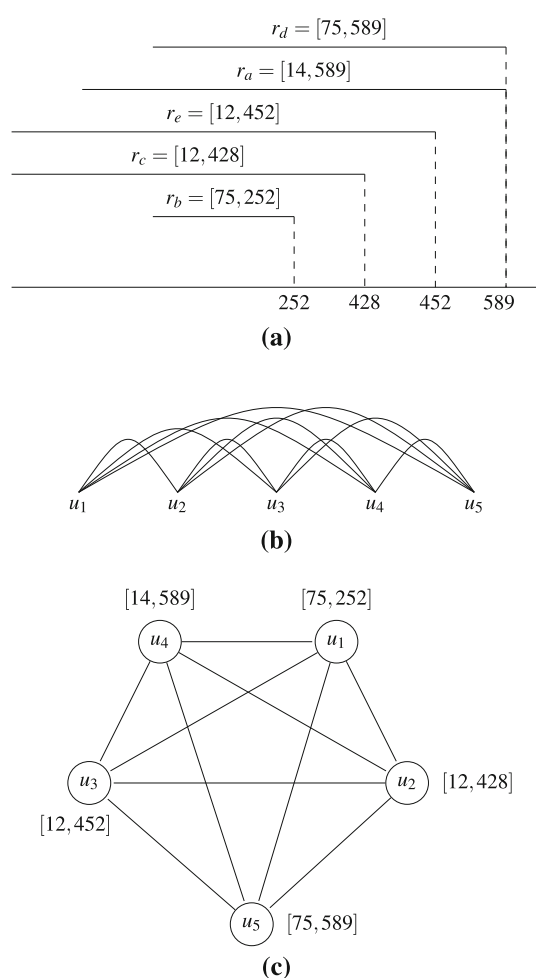


Fig. 6 **a** Intersecting model of G' **b** right end ordering of G' **c** required right-end ordered interval graph

Since labelling of vertices are subscripted with integers $1, 2, \dots, |V|$ using right end ordering, therefore we have $b_1 \leq b_2 \leq b_3 \leq b_4 \leq b_5$ (more often $b_1 < b_2 < b_3 < b_4 < b_5$). Larger the value of b_i , more is the power emission capacity of that node. Nodes u_1, u_2, u_3, u_4, u_5 are subscripted as increasing order of their power emission capacity in the network and adjacency of every two nodes implies a non-empty intervals' intersection with each other. This non empty intervals' intersection is again an interval of power level which is used by the common link corresponds to that adjacent nodes. Thus power used by the link written as a weight of that link in the interval graph formed by wireless nodes. Let I_{12} be a non-empty intervals' intersection of adjacent nodes $u_1 = [a_1, b_1]$ and $u_2 = [a_2, b_2]$, then $I_{12} = [\max\{a_1, a_2\}, \min\{b_1, b_2\}] = [\max\{a_1, a_2\}, b_1]$ ($\because b_1 \leq b_2$). Let e_{12} be the link between u_1 and u_2 and weight of e_{12} , represented by $w_{e_{12}}$ is one of the value from I_{12} . Considering a worst case scenario of WSNs where every link is using maximum power available to them (more than they actually need), $\therefore \max w_{e_{12}} = \supremum\{I_{12}\} = b_1$. Likewise, we computed maximum weight on every link i.e. maximum power usage by links in transmission. Thus a maximum weighted bidirectional complete interval graph is formed, see Fig. 7.

b_4 is the amount of maximum power consumption in this worst case corresponding to the link e_{45} over a complete network of 5 vertices. Hence without the loss of generality, maximum transmission cost always find on link $e_{(n-1)n}$ in an interval graph of n vertices when worst case happens. The first goal is to minimize this maximum transmission cost of worst case by Algorithm 1 i.e. energy could be conserved even in worst case where links are using possibly maximum power available to them.

3.4 Problem definition and proposed Algorithm 1

Min-MaxTCP (Minimization of maximum transmission cost problem): Given a bidirectional complete interval graph with n nodes, finding an energy optimal connected routing structure (spanning path/tree) such that the maximum transmission cost gets minimize over the link in the worst case.

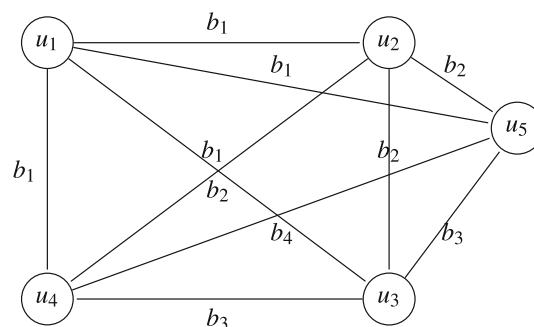


Fig. 7 Maximum weighted complete interval graph

For the requirement of a connected network i.e. no loss of node in the resultant graph, Algorithm 1 computes a path cover (covering all the vertices of a complete interval graph) which optimizes network transmission cost. Algorithm 1 can only be applicable to complete interval graph and the path cover obtained in complete interval graph is itself a routing/spanning path since a single path component spans all the nodes. Sometimes word ‘Spanning path’ is used in place of path cover.

The basic theme of Algorithm 1 is to first find a node with maximum assigned power which is the *rightmostvertex*, join it with the minimum assigned power node which is the *leftmostvertex* of G , then minimum assigned power node is joined to second maximum assigned power node, second maximum assigned power node to second minimum assigned power node and so on until all the vertices get covered in path. These joining are nothing but the links which are using maximum power from the available power interval over that link. These available power intervals are non empty intersection of power intervals of their end nodes like one of a link has an available power interval which is the intersection of maximum assigned power interval and minimum assigned power interval. Upper bound of maximum assigned power interval is the supremum of all intervals’ upper bound i.e. it is the maximum limit of power consumption in the network and upper bound of minimum assigned power interval is the infimum of all intervals’ upper bound so this node has minimum limit of power consumption (that’s why they are called maximum assigned power and minimum assigned power intervals). A link which is a joining of maximum assigned power interval with the minimum assigned power interval has minimum limit of power consumption in transmission. So joining by this way preserves those links which can form a spanning path with low power consumption by abandoning high power usage links.

Algorithm 1 Finding an energy optimum path cover in complete interval graph

Input : n , number of vertices of a complete interval graph G .

Output : A path cover/spanning path of G which minimizes the maximum transmission cost over the link.

```

1: Define  $V = \{u_1, u_2, \dots, u_n\}$   $\triangleright$  (A right end ordered vertex set of  $G$ )
2: Path is empty initially i.e.  $P = \emptyset$ 
3: Take  $i = 1$  and  $j = n$   $\triangleright$  ( $i$  and  $j$  denote the subscript of vertices  $u_i$  and  $u_j$  respectively)
4: while ( $j \geq i$ ) do
5:    $leftmostvertex = u_i$ 
6:    $rightmostvertex = u_j$ 
7:   if  $leftmostvertex \neq rightmostvertex$  then
8:      $P = P \cup rightmostvertex \cup leftmostvertex$   $\triangleright$  ( $\cup$  shows the joining of vertices in path  $P$ )
9:   else  $P = P \cup rightmostvertex$ 
10:   $i = i + 1$  and  $j = j - 1$ 
11: return  $P$ 

```

Step by step description of Algorithm 1

Algorithm 1 takes an input of n number of vertices of a right end ordered complete interval graph.

Step 1. Defines V , set of n vertices, in ascending order of their subscript.

Step 2. Initially path P has covered no vertex so it is taken as empty.

Step 3. Two distinct variables i and j are taken to denote the subscripts of vertices, starting with the opposite ends of V i.e. i starts denoting from leftmost and j from rightmost of V .

Step 4. Repeation of steps from 5 to 10 until the subscript number j becomes less than subscript number i .

Step 5. Vertex with the subscript i becomes *leftmostvertex*(LMV).

Step 6. Vertex with the subscript j becomes *rightmostvertex*(RMV).

Step 7. It checks whether LMV is same as RMV if it is not then

Step 8. Path P starts with RMV and join it to LMV .

Step 9. Otherwise path P visits RMV only.

Step 10. i adds one to its subscript number and j subtracts one from its subscript number.

Step 11. Returns the Spanning path P after the termination of step 4.

Working illustration of Algorithm 1:

Take an interval graph G of 5 vertices (Fig. 7) where $u_1 = [a_1, b_1]$, $u_2 = [a_2, b_2]$, $u_3 = [a_3, b_3]$, $u_4 = [a_4, b_4]$ and $u_5 = [a_5, b_5]$. First evaluate the *leftmostvertex*(LMV) and the *rightmostvertex*(RMV) of G , in Fig. 7 $RMV = u_5$ and $LMV = u_1$. Path is initiated with $RMV = u_5$, u_5 looks all the way to choose minimum among the maximum weighted links which is e_{51} , so u_5 maintains single link with u_1 i.e. RMV is joined to LMV . Path becomes u_5u_1 , u_5 and u_1 are covered in path so it is assumed that u_5 and u_1 are eliminated from the vertex set of G since covered vertices are not considered in further evaluation. Therefore RMV and LMV of G are updated to u_4 and u_2 respectively and path becomes $u_5u_1u_4u_2$. Now only one vertex u_3 is left to cover in path and after updation $RMV = LMV = u_3$, thus final path = $u_5u_1u_4u_2u_3$, covered all the five vertices of G (see Fig. 8).

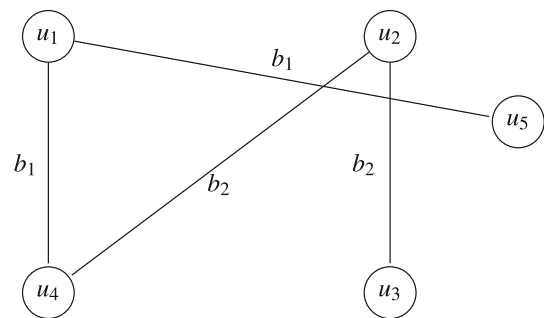


Fig. 8 Resultant path cover obtained by applying Algorithm 1 on the network given in Fig. 7

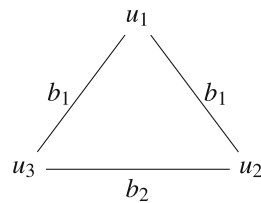
In other words, a path computed by Algorithm 1 is nothing but a joining from *RMV* to *LMV*, *LMV* to second *RMV*, second *RMV* to second *LMV*, second *LMV* to third *RMV* and so on until all vertices get covered in path with minimum number of required edges. But instead of mentioning second *RMV*, third *LMV* and all, updating *RMV* and *LMV* is used.

Fig. 8 shows the resultant path cover which is a spanning path, obtained by applying Algorithm 1 on Fig. 7 and maximum transmission cost minimizes from b_4 to b_2 .

Theorem 1 Algorithm 1 gives an optimal spanning path in term of minimizing the maximum transmission cost where each link is using maximum power available to it.

Proof The proof is trivial for $n = 1$ and 2, let's start proving for $n = 3$. Find all possible spanning paths traced in a complete graph of 3 vertices. These are as follow:

$$\begin{aligned} u_1u_2u_3 &\rightarrow b_1 + b_2 \\ u_1u_3u_2 &\rightarrow b_1 + b_2 \\ u_2u_3u_1 &\rightarrow b_2 + b_1 \\ u_2u_1u_3 &\rightarrow b_1 + b_1 \\ u_3u_1u_2 &\rightarrow b_1 + b_1 \\ u_3u_2u_1 &\rightarrow b_2 + b_1 \end{aligned}$$



□

Algorithm 1 provides spanning path ' $u_3u_1u_2$ ' whose maximum transmission cost is b_1 and b_1 is the minimum value can exist in any path of G . Hence maximum transmission cost minimizes to the optimal value for $n = 3$.

Let's prove the optimality of Algorithm 1 for $|V| = n$ by contradiction method. Spanning path traced by Algorithm 1 is:

$$\begin{cases} u_n u_1 u_{n-1} u_2 \dots u_{\lfloor \frac{n}{2} \rfloor} u_{\lfloor \frac{n}{2} \rfloor + 1}, & n \text{ is odd} \\ u_n u_1 u_{n-1} u_2 \dots u_{\lfloor \frac{n}{2} \rfloor + 1} u_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is even} \end{cases} \quad (1)$$

and sum of maximum transmission cost over each link:

$$\begin{cases} 2b_1 + 2b_2 + \dots + 2b_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is odd} \\ 2b_1 + 2b_2 + \dots + b_{\lfloor \frac{n}{2} \rfloor}, & n \text{ is even} \end{cases} \quad (2)$$

Maximum transmission cost value over the link is $b_{\lfloor \frac{n}{2} \rfloor}$ for the case $|V| = n$. Let this spanning path travelled is not optimal, then there exists atleast one another spanning path whose max value is strictly less than $b_{\lfloor \frac{n}{2} \rfloor}$. Let P' be that spanning path whose maximum transmission cost over a link is minimum. Therefore no edge in P' has value equal to $b_{\lfloor \frac{n}{2} \rfloor}$ and values over every edge must be less than $b_{\lfloor \frac{n}{2} \rfloor}$. But number of edge count must be equal in both the spanning paths, it means in path P' , b_i ($1 \leq i < \lfloor \frac{n}{2} \rfloor$) repeats more than twice to cover the place of $b_{\lfloor \frac{n}{2} \rfloor}$ in Eq. (2) since every b_i ($1 \leq i < \lfloor \frac{n}{2} \rfloor$) already contributes twice in P' . Therefore in

order to have equal count of edges, one/two of the value in $\{b_i, 1 \leq i < \lfloor \frac{n}{2} \rfloor\}$ has to come thrice in Eq. (2), depending on whether n is odd or even, which implies that one of the vertex, belonging to $\{u_i, 1 \leq i < \lfloor \frac{n}{2} \rfloor\}$ repeats more than once in P' which contradicts the definition of path. Hence, our assumption is wrong that P' is optimal spanning path. Algorithm 1 provides optimal spanning path in term of minimizing maximum transmission cost over the link.

Proposition 1 Optimal spanning path obtained by Algorithm 1 minimizes the total of maximum transmission cost over each link i.e. optimizes the total power consumption in the worst case of WSNs.

4 Interval weighted complete interval graph

In previous section, optimization of energy consumption has been done where links are conditionally weighted with maximum power from an available power interval (worst case scenario) and so a maximum weighted complete interval graph is formed with crisp power value at all links. This is one of a particular case of the general theory followed from this section. This general concept of interval weighted complete interval graph come into picture when links are considered to use any power level ranging in their available power intervals in a complete network according to their necessity. This available power interval is a power consumption interval of links. Links need to vary their power level while transmission according to different surrounding characteristics, quality and quantity of data packets to deliver and rate of delivery. So allowing wireless transmission links weighted with their complete available power interval creates an interval weighted interval graph. Proposed algorithms work well in optimizing power consumption in interval weighted interval graph also. For better understanding of this type of graph, let's initiate with an example of complete network and energy optimization by Algorithm 1.

Example 1 Consider a right-end ordered complete interval graph G' given in Fig. 6(c).

Each edge of Fig. 6(c) is a wireless link, extracting power from its corresponding end nodes, end nodes which are adjacent contributing a power interval to their common link and allow link to use any power level ranging in that available power interval according to its transmission necessity.

Table 1 shows the available power interval on each link of G' . Available power interval on each link is taken as edge weights in G' .

Interval weight on edge e_{12} is a non-empty intersection of power intervals of node u_1 and u_2 i.e.

Table 1 Available power interval to each link of G'

$w_{e_{12}} \in [75, 252]$	$w_{e_{24}} \in [14, 428]$
$w_{e_{13}} \in [75, 252]$	$w_{e_{25}} \in [75, 428]$
$w_{e_{14}} \in [75, 252]$	$w_{e_{34}} \in [14, 452]$
$w_{e_{15}} \in [75, 252]$	$w_{e_{35}} \in [75, 452]$
$w_{e_{23}} \in [12, 428]$	$w_{e_{45}} \in [75, 589]$

$w_{e_{12}} \in [\max\{75, 12\}, \min\{252, 428\}] = [75, 252]$, likewise $w_{e_{13}} \in [\max\{12, 75\}, \min\{252, 452\}] = [75, 252]$ and similarly all the available power intervals are computed and weighted on its corresponding links. In this manner, network given in Fig. 6(c) becomes an interval weighted complete interval graph where each link's interval weight is given in Table 1.

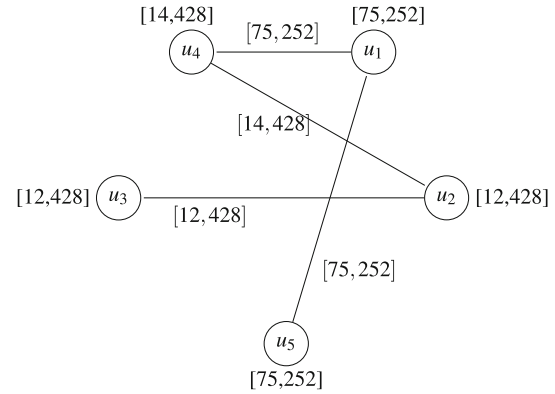
When Algorithm 1 is applied on the network (given in Fig. 6(c)), it results to a spanning path = $u_5 u_1 u_4 u_2 u_3$ (given in Fig. 9) in which interval weight of each edge is shown.

This resultant path has connectivity among all the nodes with minimum number of links. Abandoning unnecessary links lead to conservation of power of each node, hence enhancement to lifetime of wireless devices because more links create more interference and hindrance in networking.

Power interval of a node u must contain intervals of all the links incident on it, therefore resultant $r_u = [\min\{\text{lower bounds}\}, \max\{\text{upper bounds}\}]$ where lower bounds and upper bounds are taken over every incident link's power interval. Let's see the resultant power consumption interval of each node in the traced spanning path given in Fig. 9, for a node u_1 : edges e_{14} and e_{15} are incident on u_1 and interval weights on e_{14} and e_{15} are same = $[75, 252]$, therefore $r_{u_1} = [75, 252]$, for node u_2 incident links are e_{24} and e_{23} having weights $[14, 428]$ and $[12, 428]$ respectively, therefore $r_{u_2} = [\min\{12, 14\}, \max\{428, 428\}] = [12, 428]$ similarly $r_{u_3} = [12, 428]$, $r_{u_4} = [\min\{14, 75\}, \max\{252, 428\}] = [14, 428]$ and $r_{u_5} = [75, 252]$. u_5 which is maximally assigned power interval with $[75, 589]$ in the original network (Fig. 6(c)) reduces to $[75, 252]$ in the spanning path (Fig. 9) traced by Algorithm 1.

Remarks

1. Maximally assigned power interval of a node is the interval which is assigned with maximum extreme power and one of the incident link has that much power usage. It is also observed that extreme values of assigned power interval to each node is the power consumption of one of the incident link in an original interval graph. That's why algorithm works to eliminate those highly power usage links.

**Fig. 9** Resultant path cover/spanning path by Algorithm 1 with interval weight on each edge on the network given in Fig. 6c

2. Comparison among the power intervals are done on the basis of each interval maximum power consumption capacity which is the upper bound of each interval. This is because nodes' power consumption can be reduced by eliminating those links whose available power interval are maximally upper bounded and consequently reduces power consumption interval of nodes from the right side. Diminished power interval won't lead to high power consumption while transmission. That is why right extreme values of assigned power intervals are compared.

Theorem 2 *Upper bound of the assigned power interval to the rightmost vertex of a complete interval graph which is a maximum value of power consumption in the network reduces to the minimum possible value, being an upper bound in the resultant path cover obtained by Algorithm 1.*

Proof An interval $[a_i, b_i]$ is a range of power emission capacity assigned to a node i . The value of b_i reveals the maximum power emission capacity/limit of power consumption of a node i in transmission. Since in a right end ordered interval graph, leftmost vertex $u_1 = [a_1, b_1]$ is minimally upper bounded by b_1 and rightmost vertex $u_{|V|} = [a_{|V|}, b_{|V|}]$ is maximally upper bounded by $b_{|V|}$ which means one of a link incident on $u_{|V|}$ consumes maximum power equal to $b_{|V|}$ but the path traced by Algorithm 1 initiated by a joining from rightmost vertex to leftmost vertex terminate that link from the network and this joining has weight $\in [\max\{a_1, a_{|V|}\}, b_1]$, and since *rightmost vertex* has only single link incident on it in the resultant path hence $r_{u_{|V|}}$ is upper bounded by b_1 which is the minimum among all the upper bounds. So maximum limit of power consumption in the network reduces to minimum value possible to save the energy. $\square$

Theorem 3 *Path cover obtained by Algorithm 1 minimizes the maximum node's power consumption optimally.*

Proof Path cover obtained by Algorithm 1 consists of single path component covering all vertices so let P be that spanning/routing path obtained by Algorithm 1. Assume that it is not optimal and P' is an optimal routing path. Consider a link with maximum power consumption interval in P , say l . Then link l does not belong to P' (otherwise path P would have been optimal). Also the power consumption intervals of links in P' is less than l 's. This means a connected path can be constructed using links whose power consumption interval is less than the power consumption interval of link l . In that case our Algorithm 1 would have terminated before processing l and this is contradiction. $\square$

Theorem 4 Algorithm 1 takes $\mathcal{O}(n)$ time, where n is the number of nodes.

Proof Step 1 takes $\mathcal{O}(n)$ time. Step 2 and 3 can be done in constant time. Steps inside the while loop take constant time since *leftmostvertex* and *rightmostvertex* can be directly assigned by using subscript and while loop repeats at the most n times. So total time complexity can be $\mathcal{O}(n)$. $\square$

5 Arbitrary interval graph and proposed Algorithm 2

Till now, discussion has been carried out on complete interval graph only, what if underlying graph of WSNs form an incomplete connected interval graph. For a communication network topology whose graphical layout is an arbitrary connected interval graph, a methodology in the form of algorithm (say Algorithm 2) has been proposed to optimize energy consumption of the established network. Algorithm 2 presents a general method for providing an optimal path cover with respect to low power consumption in an induced arbitrary interval graph. Obtained path cover results to an

energy efficient routing structure connecting all nodes with the minimum number of required edges. That routing structure can be a spanning path or tree, depending upon the path components obtained in path cover. If there is a single path component covering all vertices then the routing structure is a spanning path else it can be a spanning tree. When Algorithm 2 applies to a complete interval graph, it gives the same optimal path cover/spanning path as Algorithm 1 does and some similar results are also seen with an arbitrary communicating network whose graphical layout need not to be a complete interval graph.

The working of Algorithm 2 is started with *RMV* of an arbitrary interval graph same as Algorithm 1 and joined to *LMV* among its neighbourhood vertices. Since graph can be incomplete so *LMV* among the neighbourhood vertices need not to be a *LMV* of the whole graph. Hence every vertex keeps two sets, say S and T , consisting of unvisited/uncovered neighbourhood nodes. Let u be the vertex, first set S contains those neighbourhood vertices of u whose power interval is greater than the power interval of u and second set T contains those neighbourhood vertices of u whose power interval is less than the power interval of u . Algorithm 2 prioritizes set S first and let the vertex u to join with maximum power interval from set S i.e. *RMV*(S). T is considered when S of u is empty, then u selects vertex with minimum power interval from T i.e. *LMV*(T) to extend the path further. Since initially chosen *RMV* which is a *RMV* of the whole graph has its S empty so it is joined to *LMV* of its T set. This is how two sets S and T play role in computing path cover by Algorithm 2 using *RMV* and *LMV* in an arbitrary interval graph. For better understanding, an illustration (example 2) has been provided following the steps of Algorithm 2 given on next page.

Algorithm 2 Finding an energy optimum path cover in any arbitrary interval graph

Input : An arbitrary right-end ordered interval graph $G = (V, E)$ with $|V| = n$ and $|E| = m$.

Output : A path cover/spanning path of G which reduces power consumption of the network optimally.

```

1: Define an array of  $n$  vertices,  $V = \{u_1, u_2, \dots, u_n\}$ 
2: Let path cover as  $opc = \phi$  initially.
3: Let  $uncoverV = V$  initially.
4: Take  $num = 1$ .
5: Path is started with  $u_n \in V$  (rightmostvertex of  $G$ ) i.e.  $Pnum = u_n$ 
6:  $uncoverV = uncoverV / \{u_n\}$ 
7: while  $uncoverV \neq \phi$  do
8:   calculate  $S = \{u_k \mid u_k \text{ is an unvisited neighbour node of } u_n \text{ and } k > n\}$  and  $T = \{u_k \mid u_k \text{ is an unvisited neighbour node of } u_n \text{ and } k < n\}$ .
9:   if  $S$  and  $T$  of  $u_n$  is empty then
10:     $[v = \text{rightmostvertex}(uncoverV); num = num + 1; Pnum = v]$ 
11:   if  $S \neq \phi$  then
12:     $[v = \text{rightmostvertex}(S); Pnum = Pnum \cup v]$ 
13:   else
14:     $[v = \text{leftmostvertex}(T); Pnum = Pnum \cup v]$ 
15:    $u_n = v$ 
16:    $uncoverV = uncoverV / \{u_n\}$ 
17: return  $opc = \{P_1, P_2, \dots, Pnum\}$ 

```

$\triangleright (\cup)$ shows the joining of vertices

Step by step explanation of Algorithm 2

Algorithm 2 takes an input of an arbitrary right end ordered interval graph of n vertices and m edges.

Step 1. A vertex set V of n vertices is defined in ascending order of their subscript.

Step 2. Initially Path cover opc is empty i.e. no vertex is visited/covered by any path component.

Step 3. Define another set $uncoverV$ which consists of unvisited vertices of V so initially $uncoverV$ is taken same as V .

Step 4. Significance of taking num variable is to obtain distinct number of path components of opc as $opc = \{P_1, P_2, \dots, P_{num}\}$. So num is starting with 1 to start with P_1 .

Step 5. P_1 starts with the *rightmostvertex* of $uncoverV$ so $P_1 = \{u_n\}$.

Step 6. Since u_n is covered by P_1 so it is removed from $uncoverV$.

Step 7. Repeattion of steps from 8 to 16 until all vertices get covered.

Step 8. Computation of set S and T for each u_n . S comprises of those unvisited neighbour nodes of u_n whose subscript is greater than n and T comprises of those unvisited neighbour nodes of u_n whose subscript is less than n .

Step 9. Checks emptiness of S and T . If it is then

Step 10. P_1 is terminated with the initiation of P_2 which starts with *rightmostvertex* of $uncoverV$ and jumps to step 15.

Step 11. Checks the non-emptiness of set S if the condition of step 9 is not satisfied and P_1 keeps on covering unvisited vertices.

Step 12. If set S is non-empty then P_1 extends to cover v which is a *rightmostvertex* of S by joining it with u_n .

Step 13. Otherwise

Step 14. P_1 extends to cover v which is taken as *leftmostvertex* of T by joining it with u_n .

Step 15. Now variable u_n starts representing vertex v .

Step 16. Since $u_n = v$ is covered it is removed from $uncoverV$ and Algorithm jumps to step 7 to repeat the whole procedure again for $u_n = v$.

Step 17. $opc = \{P_1, P_2, \dots, P_{num}\}$ is returned after the termination of step 7.

Example 2 A path cover computation by Algorithm 2 on an arbitrary (incomplete) right-end ordered interval graph given in Fig. 10 is as follows.

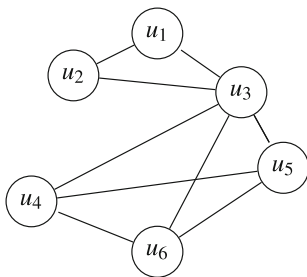


Fig. 10 An arbitrary right-end ordered interval graph

*Refer Algorithm 2 first to understand defined terms and abbreviations.

► $opc = \{\}$ initially.

► $uncoverV = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.

► $num = 1$, so first path component is represented as P_1 .

► P_1 is started with u_6 , u_6 is the *rightmostvertex*.

► $uncoverV = \{u_1, u_2, u_3, u_4, u_5\}$ is updated with the deletion of u_6 .

► Entering in the while loop since $uncoverV$ is non-empty.

► Since $S(u_6) = \{\}$ and $T(u_6) = \{u_3, u_4, u_5\}$, step(14) computes $P_1 = u_6u_3$.

► u_6 is replaced by u_3 and $uncoverV = \{u_1, u_2, u_4, u_5\}$.

► $S(u_3) = \{u_4, u_5\}$ and $T(u_3) = \{u_1, u_2\}$.

► So step 12 extends $P_1 = u_6u_3u_5$.

► u_3 is replaced by u_5 and $uncoverV = \{u_1, u_2, u_4\}$.

► The whole procedure repeats for u_5 and extends $P_1 = u_6u_3u_5u_4$.

► Now u_5 is replaced by u_4 and $uncoverV = \{u_1, u_2\}$.

► S and T of u_4 come to empty so step(10) selects u_2 from $uncoverV$, makes $num = 2$ and $P_2 = u_2$.

► Steps inside while loop repeats for u_2 and extends $P_2 = u_2u_1$.

► Then u_2 is replaced by u_1 and $uncoverV$ become empty, hence while loop stops running and path cover is returned as $opc = \{u_6u_3u_5u_4, u_2u_1\}$.

Remark Path cover obtained by Algorithm 2 may contain more than one distinct path components to cover all the vertices on an arbitrary interval graph as in example 2 so in order to join distinct path components to form an energy efficient connected routing structure (spanning tree), a polynomial time algorithm (say Algorithm 3) has been designed, motivation taken from [16]. Algorithm 3 first checks the path cover obtained by Algorithm 2 to run after it. When path cover contains single path component covering all vertices then that single path component is itself an energy efficient connected routing path/tree, in that case Algorithm 3 doesn't run after Algorithm 2.

5.1 Formation of connected routing structure by Algorithm 3 if the obtained path cover contains more than one distinct path components.

Motivated by the methodology of Algorithm given by Li et al. in [16], a polynomial time algorithm, say Algorithm 3, has been designed to form an energy efficient routing structure (spanning tree) with the joining of distinct path components of path cover obtained by Algorithm 2 over any arbitrary right-end ordered interval graph. Algorithm 3 runs after Algorithm 2 only when path cover obtained by

Algorithm 3 Provides an energy efficient routing structure with the joining of distinct path components of path cover by Algorithm 2

Input : A path cover of an interval graph G consisting of more than one distinct path components i.e. $opc = \{P_1, P_2, \dots, P_{num}\}$ where num is some finite natural number, obtained by Algorithm 2.

Output : A connected routing structure in the form of spanning tree of G .

```

1:  $T \leftarrow \{P_c | P_c \text{ is a path component of } opc\}, opc = opc / \{P_c\}$ .
2: while  $opc \neq \emptyset$  do
3:   choose a path component  $q$  from  $opc$ , where  $q$  is intersecting with  $T$ . ▷ (subsection 3.2, property(1),(2))
4:   if  $leftmostvertex(q) < leftmostvertex(T)$  then
5:     choose a minimum right-end vertex  $w \in V(q)$  adjacent to  $T$ , join it to the leftmost adjacent vertex from  $T$  and  $T$  be the resultant new tree.
6:   if  $leftmostvertex(q) > leftmostvertex(T)$  then
7:     choose a minimum right-end vertex  $w \in V(T)$  adjacent to  $q$ , join it to the leftmost adjacent vertex from  $q$  and  $T$  be the resultant new tree.
8:    $opc = opc / \{q\}$ 
9: return  $T$ .
```

Algorithm 2 contains more than one distinct path components.

Step by step description of formation of connected routing structure (see Fig. 11) using Algorithm 3 with the $opc = \{u_6u_3u_5u_4, u_2u_1\}$ obtained in example 2 on the network given in Fig. 10.

Step 1. A path component P_c is chosen from the opc . Let us take $P_c = u_6u_3u_5u_4$, then $opc = \{u_2u_1\}$ and spanning tree $T = u_6u_3u_5u_4$ is updated.

Step 2. Repeation of steps from 3 to 8 until opc becomes empty.

Step 3. Let us choose another path component q from opc , we take $q = u_2u_1$.

Step 4. We find $leftmostvertex(q) = u_1$, $leftmostvertex(T) = u_3$ and $1 < 3$.

Step 5. Choose a minimum right-end vertex from $V(q)$ adjacent to T which is u_1 and joined to u_3 which is the leftmost adjacent vertex from T to u_1 and routing structure of T gets updated as shown in Fig. 11.

Step 6. This condition is not satisfied for the given opc , so

Step 7. It is skipped.

Step 8. opc is updated again with the removal of q .

Step 9. Return of connected routing structure can be seen in Fig. 11 after the termination of step 2.

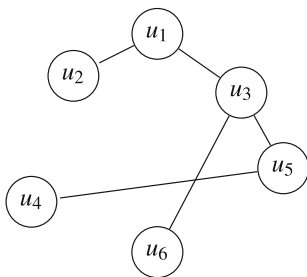


Fig. 11 Formation of connected routing structure with opc obtained in example 2 by Algorithm 3

Theorem 5 Path cover obtained by Algorithm 2 is energy optimal since it forms an optimal routing structure (spanning path/tree) which minimizes the maximum node's power consumption.

Proof Let T be the resultant spanning tree formed with opc , path cover of an interval graph G , obtained by Algorithm 2. Assume that opc is not optimal and opt is an optimal path cover but with same cardinality as opc . Let T' be the optimal spanning tree formed with opt . Consider a link with maximum power consumption interval in T , say l . Then link l does not belong to T' (otherwise opc and T would have been optimal). Also the power consumption intervals of links in T' is less than l 's (Note: comparison of intervals is based on upper bounds since upper bound is the maximum limit of power consumption for each link). This means a connected tree/path can be constructed with using links whose power consumption interval is less than the power consumption interval of link l with the same cardinality of both the path cover. In that case either Algorithm 2 or Algorithm 3 would have terminated before processing l and this leads to contradiction. $\square$

Theorem 6 Algorithm 2 reduces maximum power consumption limit which is the upper bound of the assigned power interval to the rightmost vertex on an arbitrary interval graph, to the minimum value possible, being an upper bound in the resultant graph.

Proof Let's begin by considering the working of Algorithm 2 on Fig. 10 which results to an optimized routing tree given in Fig. 11. Each vertex has power assignment as $u_i = [a_i, b_i]$, $i = 1, \dots, 6$ and $u_6 = [a_6, b_6]$ is joined to $u_3 = [a_3, b_3]$ in the resultant topology (Fig. 11). Upper bound of an interval of u_6 reduces from b_6 to b_3 because of $w_{e_{63}} \in [\max\{a_3, a_6\}, \min\{b_3, b_6\}] = [\max\{a_3, a_6\}, b_3]$ and so $r_{u_6} = [c_{63}, b_3]$ where $c_{63} = \max\{a_3, a_6\}$. Let this reduction is not minimum. Therefore r_{u_6} can be reduced more by lowering the upper bound of its interval. This means u_3 is not the vertex with minimum subscript/index in u_6 's

neighbourhood implies that step(14) of Algorithm 2 is not working correct which is not true. Hence our assumption is wrong. $\square$

Proposition 2 Path cover obtained by Algorithm 2 is energy optimal for its path components form an optimal routing (spanning) tree/path which minimizes the maximum transmission cost over link in worst case scenario of WSNs.

Theorem 7 Algorithm 2 has $\mathcal{O}(n + m)$ time complexity where n is number of nodes and m is number of edges.

Proof Step 1 takes $\mathcal{O}(n)$ time. Step 2-6 can be done in constant time. Steps 8,12 and 14 inside the while loop takes $\mathcal{O}(\text{degree of } u_n)$ time. Step 10 takes $\mathcal{O}(n)$ time and rest of the steps are done in constant time so total time complexity can be $\mathcal{O}(n + m)$. $\square$

Theorem 8 Algorithm 3 has $\mathcal{O}(|V(G)|^2)$ time complexity, where $V(G)$ denotes the number of vertices of a graph G .

Proof Step 1 can be done in constant time. While loop repeats at the most $|V(G)|$ times. It takes $\mathcal{O}(|V(q)|)$ and $\mathcal{O}(|V(T)|)$ time to determine *leftmostvertex* or *rightmostvertex* of q and T respectively. So maximum time complexity of steps 4-7 inside the while loop can be $\mathcal{O}(|V(G)|^2)$. Hence total time complexity is $\mathcal{O}(|V(G)|^2)$. $\square$

Remarks:

1. Algorithm 1 takes $\mathcal{O}(n)$ time in computing an energy optimal path cover in complete interval graph where n is number of nodes.
2. Algorithm 2 takes $\mathcal{O}(|V(G)| + |E(G)|)$ time in computing an energy optimal path cover in an arbitrary interval graph $G(V, E)$.
3. Algorithm 2 is the general algorithm, can apply to complete interval graph also and it gives the same energy optimal path cover/spanning path as Algorithm 1 does.
4. Algorithm 3 is applied after Algorithm 2 only when *opc* contains more than one path component. Time complexity of Algorithm 3 is $\mathcal{O}(|V(G)|^2)$.

6 Evaluation and Comparison

Li et al. in [16] proposed a polynomial time algorithm which finds a maximum internal spanning tree by joining distinct path components of an optimal path cover [3] obtained on an arbitrary interval graph. Optimal path cover problem has been solved in linear time by Arikati and Rangan [3] on a right end ordered interval graph and optimality is taken on the basis of minimum cardinality of the path cover. Path cover, obtained with the algorithm

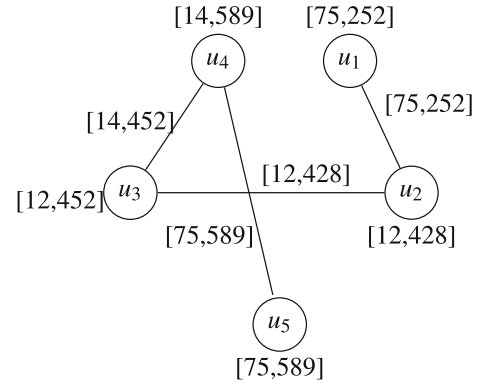


Fig. 12 Spanning path by Algorithm, given in [3], with edge weights as available power interval on each link and nodes weight as assigned power interval

defined in [3] is optimal since it covers all vertices with minimum number of distinct path components, hence minimum cardinality. Path cover obtained by Algorithm 2 doesn't guarantee minimum cardinality in incomplete interval graphs yet it is more energy efficient in order to form connected routing structure (spanning path/tree) by Algorithm 3 in WSN. Path cover obtained on any complete interval graph by newly proposed Algorithms (Algorithms 1 and 2) has minimum cardinality, equal to the cardinality of optimal path cover obtained by Algorithm given in [3], with better optimization of power consumption and load balancing in the resultant network.

Starting with the performance evaluation of Algorithm 1 and Algorithm, mentioned in [3]. Both the Algorithms work to find path cover on complete interval graph. Path cover obtained on complete interval graph contains single path component which spans all vertices, hence path covers are themselves Spanning paths.

Let's consider the example 1 where path cover is obtained by Algorithm 1 gives min-max node power consumption interval is $r_{u_4} = [14, 428]$ and u_5 which is maximally assigned with power interval $[75, 589]$ in the original network [Fig. 6(c)] reduces to $[75, 252]$ in the resultant topology (Fig. 9). Now finding the optimal path cover on the same network [Fig. 6(c)] by the Algorithm in [3], results a spanning path ' $u_1 u_2 u_3 u_4 u_5$ ' (see Fig. 12). Resultant power consumption interval of each node on the obtained spanning path (refer Fig. 12) is as follows : $r_{u_1} = [75, 252]$, $r_{u_2} = [12, 428]$, $r_{u_3} = [12, 452]$, $r_{u_4} = [14, 589]$ and $r_{u_5} = [75, 589]$. Hence min-max node power consumption interval at u_5 is $r_{u_5} = [75, 589]$, higher than lastly computed value using Algorithm 1 at u_4 which is $r_{u_4} = [14, 428]$ (see Fig. 9).

Moving to the worst case and computation of min-max transmission cost and min-total transmission cost over the links in complete interval graphs.

Table 2 Power optimization on a complete interval graph of Fig. 6(c)

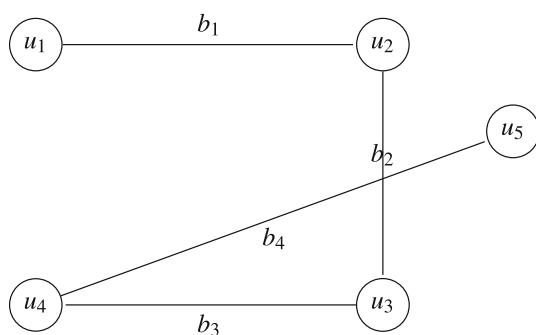
Comparison criteria	Algorithm in [3](refer Fig. 13)	Algorithm 1(refer Fig. 9)
Min-max node's power consumption interval	[75,589]	[75,428]
Min-max transmission cost in worst case	589	428
Min-total transmission cost in worst case	1721	1360
Minimization of Upper limit of Rightmost vertex	no reduction	minimized from 589 to 252

When Algorithm, mentioned in [3], is applied on the network given in Fig. 7, it results to a spanning path ' $u_1u_2u_3u_4u_5$ ' (see Fig. 13).

In the traced path of Fig. 13, min-max transmission cost over the link e_{45} is b_4 and min-total transmission cost is $b_1 + b_2 + b_3 + b_4$ while path traced by Algorithm 1 (refer Fig. 8) results min-max transmission cost is b_2 and min-total transmission cost is $b_1 + b_1 + b_2 + b_2$. Since $b_1 \leq b_2 \leq b_3 \leq b_4 \leq b_5$ hence lesser power consumption over the spanning path obtained by Algorithm 1. Therefore spanning path obtained in Fig. 12 has min-max transmission cost 589 and min-total transmission cost 1721 while Spanning path (Fig. 9) traced by Algorithm 1 has min-max transmission cost 428 and min-total transmission cost 1360. This comparison of power optimization can be seen in Table 2.

Now coming to the performance evaluation of Algorithm 2 and 3 which for convenience written as Algorithm 2and3 and Algorithm mentioned in [16] on arbitrary interval graphs.

Path cover by Algorithm 2 on an arbitrary interval graph given in Fig. 10 results $opc = \{u_6u_3u_5u_4, u_2u_1\}$ having two distinct path components, hence after applying Algorithm 3, a connected routing structure is obtained (refer Fig. 11) gives min-max node power consumption at u_5 whose power consumption interval is upper bounded by b_4 and maximum assigned power interval of the *rightmostvertex* $u_6 = [a_6, b_6]$ reduces to $[\max\{a_3, a_6\}, b_3]$. Now applying the Algorithm of Li et al., mentioned in [16], on the same

**Fig. 13** Spanning path by Algorithm in [3] on the complete network given in Fig. 7

network (Fig. 10). Methodology of the Algorithm in [16] runs with the optimal path cover, obtained in [3] and results to a maximum internal spanning tree ' $u_1u_2u_3u_4u_5u_6$ ' can be seen in Fig. 14. This obtains min-max node power consumption interval at $u_6 = [\max\{a_5, a_6\}, b_5]$ implies that incident link on u_6 has power consumption equal to b_5 which is greater than b_4 .

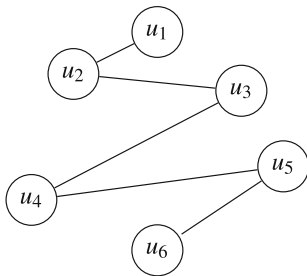
Numerical results by both the algorithms has been evaluated on the same network (Fig. 10) where each node is weighted with power intervals as $u_1 = [50, 105]$, $u_2 = [78, 150]$, $u_3 = [85, 203]$, $u_4 = [161, 230]$, $u_5 = [155, 312]$ and $u_6 = [178, 312]$ and shown in Table 3 (Sect. 7) for $n = 6$.

Trying to apply Li et al. algorithm, given in [16], to the obtained $opc = \{u_6u_3u_5u_4, u_2u_1\}$, problem comes in choosing relevant energy efficient link to u_3 when there are more than one adjacent vertices, in this case there are u_1, u_2 and also a maximum internal spanning tree doesn't form. Algorithm 3 resolves this issue by considering adjacent vertex with minimum right-end since available power interval at e_{13} is less than that at e_{23} . So selectively joining u_3 to u_1 results to have lesser available power consumption interval at the link and also reduces the power consumption interval at node u_2 .

Moreover, performance of Algorithm 2 can be seen on an arbitrary network given in Fig. 15 whose vertices are already right-end ordered according to the assigned power interval. Applying Algorithm 2, single path component spans all the vertices and results to an optimized network can be seen in Fig. 16. Node u_9 which was maximally assigned with [115, 140] in original network (refer Fig. 15) minimized to [115, 125] and so min-max node power consumption interval becomes [115, 125] in the resultant topology (refer Fig. 16). Also nodes u_8 and u_7 which were assigned with power intervals [94, 140] and [95, 138] reduces to [95, 112] and [94, 125] respectively and upper limit of power consumption of rightmost vertex minimized from 140 to 125. Fig. 17 shows the spanning path obtained by applying Algorithm in [16] on the same network (Fig. 15) whose min-max node power consumption interval [115, 140] is more than the lastly computed [115, 125].

Table 3 Summarized simulation results

Number of nodes	Number of edges	Min-max node power consumption interval		Min-max transmission cost in worst case	
		Algorithm 2and3	Algorithm in [16]	Algorithm 2and3	Algorithm in [16]
6	9	[155,230]	[178,312]	230	312
9	21	[115,125]	[115,140]	125	140
10	34	[6,82]	[41,87]	82	87
13	38	[72,162]	[121,236]	162	236
15	49	[9,246]	[93,277]	246	277
18	100	[47,190]	[84,263]	190	263
20	120	[10,366]	[319,394]	366	394
23	149	[82,379]	[347,412]	379	412
25	174	[149,399]	[120,455]	399	455
28	202	[24,404]	[165,446]	404	446
30	268	[15,485]	[93,560]	485	560
33	265	[75,601]	[417,633]	601	633
35	380	[170,611]	[163,688]	611	688
37	322	[337,680]	[481,717]	680	717
40	376	[370,709]	[275,771]	709	771
42	448	[76,781]	[97,825]	781	825
45	479	[351,823]	[367,894]	823	894
48	608	[568,919]	[641,940]	919	940
50	632	[713,872]	[713,974]	872	974

**Fig. 14** Maximum internal spanning tree

Performance comparison of both the algorithms including the worst case can be seen in Table 3 for $n = 9$.

7 Simulation results on arbitrary interval graphs

We run the simulation considering large number of WSNs whose underlying graphs are interval graphs in Spyder (Python 3.7). Input of arbitrary right-end ordered interval graph in term of nodes, edges and data of right end ordered intervals are randomly generated and given to the combined python code of Algorithm 2 and 3 to evaluate the

combined performance of Algorithm 2 and 3 (for sake of convenience it is written as Algorithm 2and3 in this section). Simulation is done with using Algorithm 2and3 since Algorithm 2 is the generalisation of Algorithm 1 and applicable on both complete as well as incomplete interval graphs. When obtained path cover, by Algorithm 2, contains more than one distinct path components then Algorithm 3 results them to a connected routing topology. Therefore python code of Algorithm 2and3 is used for the simulation which takes the input of any arbitrary interval graph in term of number of nodes, number of edges, list of all edges and list of intervals according to right-end ordering of nodes.

The aim of taking input of arbitrary interval graphs is fulfilling the evaluation requirement on complete as well as incomplete interval graphs.

Results by Algorithm 2and3 on min-max node power consumption interval and min-max transmission cost in worst case for each input data have been recorded and plotted against the results obtained by Algorithm, mentioned in [16] on the same data. Fig. 18 shows the plotting of min-max node power consumption interval on the power line (i.e. nodes transmit power level ranging from 0 to 1000) with the variation of number of nodes from 5 to 50 consisting of both the complete as well as incomplete interval graphs with different edge sets. The range covered

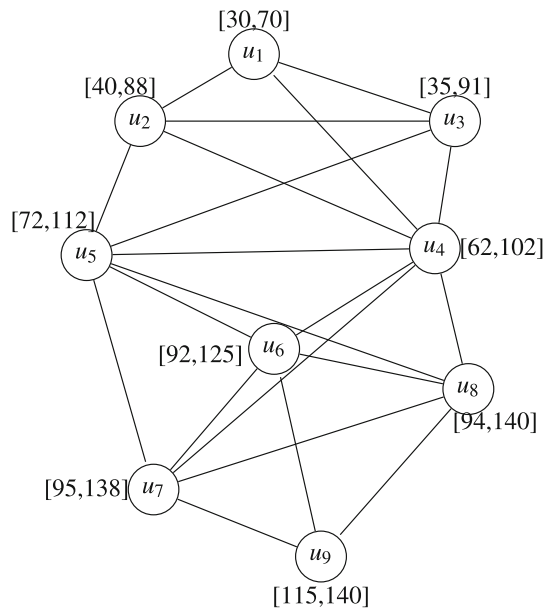


Fig. 15 Arbitrary right-end ordered interval graph with nodes weight as power interval

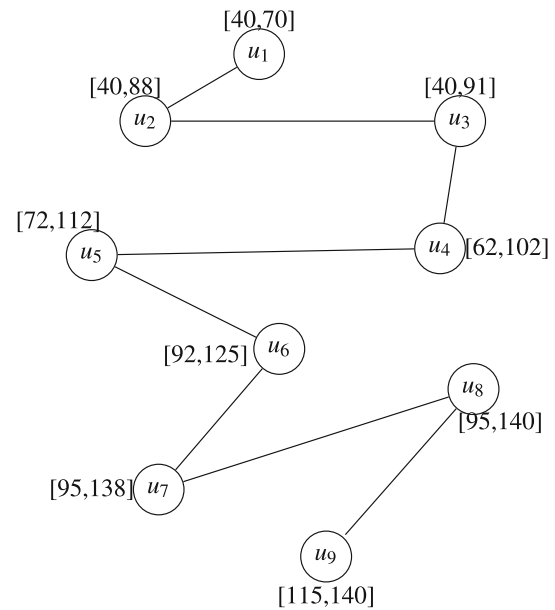


Fig. 17 Resultant Spanning path after applying Algorithm in [16] on the network given in Fig. 15

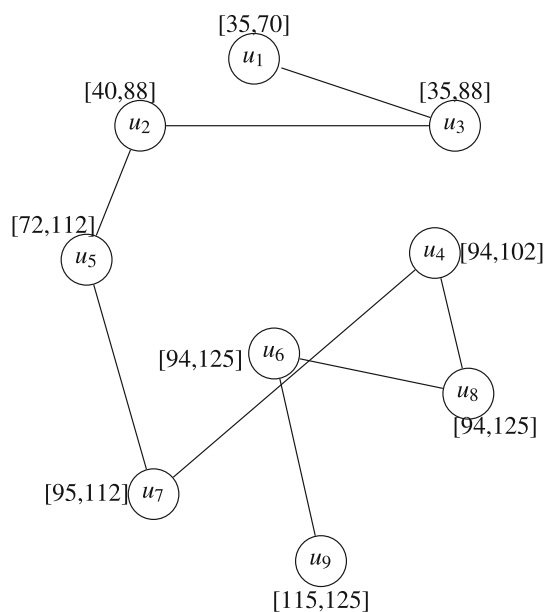


Fig. 16 Path cover by Algorithm 2 with reduced power intervals on the network given in Fig. 15

by 'blue color' shows the maximum node power consumption interval obtained by Algorithm 2and3 and range with 'red color' shows the maximum node power consumption interval obtained by Algorithm, mention in [16]. It has been observed that blue line terminates before the red line everytime parallel to power line, this implies that routing topology obtained by Algorithm 2and3 results minimum of maximum node power consumption interval on interval graphs everytime. Fig. 19 shows the range of

power conserved by Algorithm 2and3 with red color, extracted from Fig. 18.

Simulation results of maximum transmission cost on links in worst case has been plotted in Fig. 20. Blue dots whose tracing is shown by blue curve depicting the performance of Algorithm 2and3 and red dots whose tracing is shown by red curve depicting the performance of Algorithm, mentioned in [16]. It can be seen in Fig. 20 that blue curve falls below the red curve for everycount of nodes which implies that max transmission cost on links in worst case is minimized by Algorithm 2and3 more than the Algorithm in [16].

Summarized simulation results can be seen in Table 3, depicting the difference in power consumption with the input of randomly generated right end ordered interval graph whose nodes and edges are mentioned in the table.

Hence the goal of energy saving in topology control of wireless sensor networks whose underlying graphs are interval graphs, could be achieved when path cover is obtained by Algorithm 1 or 2 and distinct path components are joined by Algorithm 3.

Remarks

1. No routing solution exists in literature on heterogeneous WSNs where nodes are explicitly assigned with power intervals to optimize power consumption subject to the connectivity constraint. Proposed methods are the first approach in this respect.
2. There exists no another path cover with cardinality less than or equal to opc , obtained by Algorithm 2, which can be more energy efficient in general and worst case

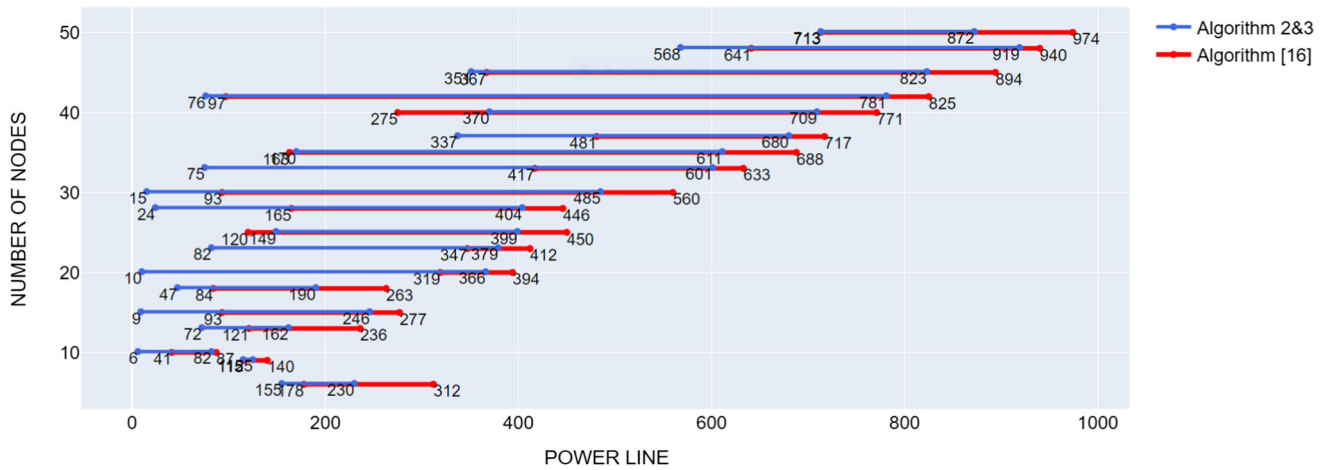


Fig. 18 Evaluation of maximum node power consumption interval: blue color shows the performance of Algorithm 2and3 and red color shows the performance of Algorithm, given in [16] (Color figure online)

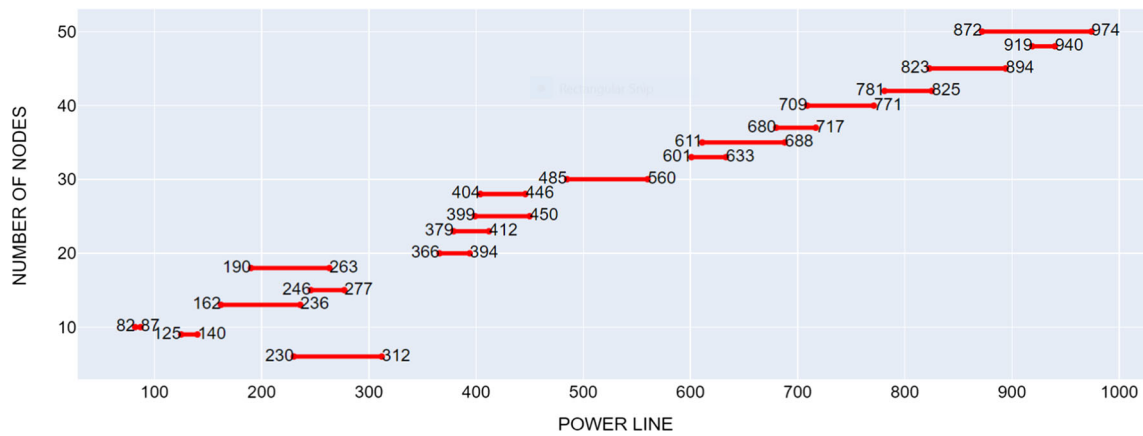


Fig. 19 Range of Power conserved by Algorithm 2and3

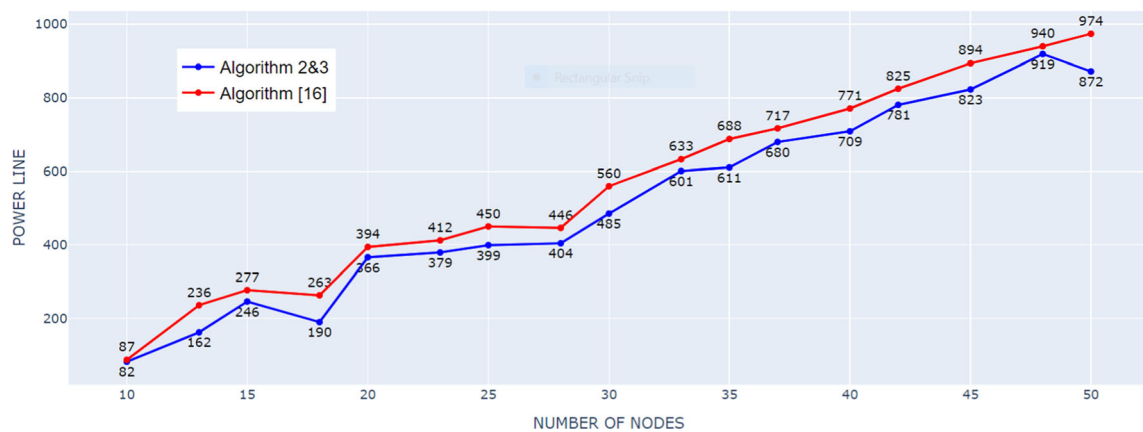


Fig. 20 Evaluation of maximum transmission cost in worst case: blue color shows the performance of Algorithm 2and3 and red color shows the performance of Algorithm, given in [16] (Color figure online)

too. Thus, it is seen that Algorithm 2/Algorithm 1 gives energy optimal path cover/spanning path on an interval graph.

8 Conclusion and summary

This paper is about a persisting problem of power consumption and network lifetime in wireless sensor networks where nodes are allowed to vary their transmission range over a uniquely assigned power interval in accordance to make required/necessary links at a particular instant of time. Several works have been done aiming to reduce energy consumption by reducing nodes transmission range by proposing Transmission Power Control (TPC) approaches, explicitly designed interference model in topology control of WSN, topology control algorithms and routing protocols [7, 9, 23, 27, 28], most of the works are dealing with static transmission power/range of nodes. This paper is different in view of modelling WSN as a ‘connected bidirectional interval graph’, assigning transmission range in term of power interval to each node and minimization of maximum power consumption in a connected topology by newly proposed polynomial time algorithms, based on interval graph theory.

Assigned power interval of each node is ranging from minimum energy to maximum energy necessarily required for successful transmission of data packets bidirectionally. Hence every node is assigned with an appropriate power interval in which node can vary its transmission range accordingly, forming an interval graph and consequently, a right-end ordered interval graph layout is formed by nodes. Every wireless sensor network whose underlying graph is complete, has an interval graph layout. The problem is to find energy efficient routing structure (spanning path/tree) with connectivity among the nodes i.e. a structure which can be a path/tree, covering all vertices of an interval graph with low power consumption with successful routing among all nodes. Considering this, an idea of energy optimal path cover on an interval graph comes into picture and so two algorithms are proposed aiming the same for non-mobile wireless sensors. Algorithm 1 is used to find energy optimal path cover in complete bidirectional interval graph only and since network is complete, a single path covers all vertices and gives an energy optimal spanning path. This means path cover in complete interval graph is itself a connected routing structure which minimizes maximum node’s power consumption interval and optimizes the worst case situation also by minimizing maximum transmission cost over the link and total transmission cost over the resultant network. Algorithm 2 is the generalisation of Algorithm 1 which computes an energy optimal path cover on an arbitrary interval graph. Path cover computation on an arbitrary interval graph may

result more than one distinct path components so Algorithm 3 is designed to join distinct path components to have an energy efficient connected routing structure which is applied after Algorithm 2. If there is only one path component in the path cover obtained by Algorithm 2 then that path component is itself an energy optimal routing structure (spanning path). The resultant routing structure by Algorithm 2 and 3 optimally minimizes maximum node power consumption interval and reduces upper power consumption limit of rightmost vertex to the minimum value possible causing load balancing in the network. All the proposed Algorithms have polynomial time complexity. Algorithm 1 has linear time complexity i.e. $O(n)$, Algorithm 2 has $O(n + m)$ time complexity and Algorithm (3) has $O(n^2)$ time complexity where n is the number of nodes and m is number of links. Although Algorithm 2 does not guarantee optimal min-total transmission cost in the worst case for incomplete interval graph. A comparison has been shown using evaluation and simulation on the basis of power consumption between the performance of Algorithm, given in [16] and the combined performance of Algorithm 2 and 3 on arbitrary interval graphs, reveals that routing topology formed with path cover, obtained by Algorithm 2 is more energy efficient and that’s why path cover obtained in this paper is called energy optimal path cover.

Thus the proposed algorithms work when connected topology formed by wireless sensors has interval graph layout and vertices are represented by right-end ordering. In future, more general algorithms independent of the restriction of right-end ordering will be discovered. Moreover, the proposed solution is centralized in nature. So we intend to explore some distributed solutions for the same problem with the addition of more results based on power optimization and load balancing. A static network is considered in this paper. Our next work will incorporate nodes mobility as well.

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Optimization of energy and delay on interval data based graph model of wireless sensor networks

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Abstract

Emerging applications of wireless sensor networks (WSNs) in various domain of real-life require establishment of such routing topology for wireless sensors which can balance energy consumption with delay minimization. Despite a plethora research on energy and delay optimization in this field, work on developing delay minimum and energy efficient connected routing topology on heterogenous bi-directional WSNs where each node is assigned with a power interval, remain untouched. Considering this problem, we have introduced ‘Interval data based graph model (IDGM)’ and ‘Sorted-interval data based graph model (S-IDGM)’ of WSNs to explicitly deal with nodes’ power interval and proposed ‘Energy and Delay Optimization (EDO)’ algorithm to optimize S-IDGM such that the maximum topology delay, total topology delay and maximum node’s power interval become minimum in polynomial time complexity. A new function is formulated to estimate topology delay based on link distance and link interference after showing dependency analysis between link distance and link interference on large number of WSNs towards achieving optimal solutions. Extensive simulation work, graphical and statistical *t*-test analysis have been carried out to show the performance of EDO algorithm in minimizing topology delay and nodes’ power consumption, better than the existing algorithms from similar grounds. *t*-test analysis shows that the proposed EDO algorithm achieves optimal energy saving of nodes at 5% level of significance along with optimal minimization of max and total topology delay at 2% level of significance on S-IDGMs.

Keywords Wireless sensor networks (WSNs) · Algorithm · Energy-Delay Tradeoff · Minimum spanning tree (MST) · Path cover

1 Introduction

Wireless sensor network (WSN) is the most evolutionary technology now-a-days because of the facilitation of low-cost multi-functional wireless sensors that are small in size and communicate untethered using radio signals, contributing to large number of applications in various fields involving tasks like environmental monitoring, health and well-ness monitoring, power monitoring, inventory and location monitoring, military surveillance, tracking objects, animals, humans and vehicles etc [3, 27, 44]. Each sensor has capability of sensing, processing and transmitting radio signals by organizing a multi-hop routing topology among

themselves [4, 32, 38]. Since sensors are battery powered and once their energy gets drained, they will no longer contribute to any of the task in the network. Depletion of their power is largely depending on transmission than sensing and processing [30]. Therefore in order to conserve nodes power, authors in history develop and engage many strategies like minimization of transmission energy by following shortest path, transmission power adaptation, data gathering or clustering, duty-cycle concept, construction of low interference topology and etc [9, 11, 12, 21, 28, 41].

1.1 Energy-delay tradeoff

It has been observed that routing topologies which aim to minimize nodes’ power consumption or focus on saving nodes’ power adversely affect delivery of data packets [10, 20]. This means that route with minimum energy

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consumption causes longer delay in transmitting data packets between the nodes. This is because nodes use smaller transmission range to communicate in order to save their energy and prefer to have hop-by-hop transmission [25, 29]. Delay increases with number of intermediate hops existing between source to destination nodes because data is sensed, processed and transmitted as many time as number of intermediate hops and this contributes processing, propagation, transmission and queuing delay [24, 31]. Therefore delay minimization costs maximization of energy consumption since it requires longer transmission range of every source node to directly transmit data packets to its destination node. Work alone to minimize end-to-end routing delay shortens the network lifetime and work alone to minimize network energy consumption promotes routing delay. This issue in WSNs is called Energy-Delay Tradeoff [1, 5, 8, 22, 42] and has been a prime concern from past many years to resolve. Many authors work towards this Energy-Delay Tradeoff by formulating weighted function of some energy consumption causing factors as well as delay causing factors and use value of their formulated function to achieve desired routing goals. Some authors attempt to minimize delay factor more if the goal is to meet packet delivery deadline and some prioritize low transmission cost path in order to save nodes energy with some delay constraint [5, 13, 16, 36, 43, 45].

1.2 Research gaps

Existing approaches on Energy-Delay Tradeoff result delay constraint energy aware path between source to destination node based on hop counts by ignoring rest of the topology. Also work on minimization of nodes power consumption with delay optimization at topology level is so rare. Existing attempts do not visualise nodes power consumption explicitly while minimizing delay [15, 43] whether nodes are having variable transmission range or not. Some authors estimate queuing, transmission and propagation delay [18, 36] with the ignorance of transmission failure or retransmission and collision of data packets due to interference factor in large and dense networks. This retransmission and packets collision contribute to more delay in data delivery along with nodes' power consumption [8, 17, 31, 34]. Selvi et al. in [36] estimate end-to-end delay including propagation delay, processing delay, transmission delay and queuing delay, from source S to destination D using Eq. (1).

$$Delay_{end-to-end}(S, D) = \sum_{i,j} \left(\left(\frac{1}{\alpha - \beta} \right) + \frac{1}{\phi} + \frac{d(i,j)}{\mu} \right) \quad (1)$$

where α, β are constant, ϕ is bandwidth link, μ is the propagation speed and $d(i, j)$ is the distance between intermediate nodes, called hop length or link distance. Here authors didn't consider retransmission or interference factor in estimating end-to-end delay.

Moreover solution to Energy-Delay Tradeoff on bi-directional connected heterogeneous WSNs where nodes are allowed to vary their transmission range and are assigned with appropriate power interval is still unexplored on topology level.

1.3 Motivation

These existing gaps motivate us to bring out a methodology to obtain delay minimum energy efficient connected routing topology considering interference factor which simultaneously work to minimize nodes' power consumption and topology delay on bidirectional connected heterogeneous WSNs where each sensor node has been allowed to vary its transmission range within a power interval. To estimate delay, hop counts won't be considered since this work is dealt with bi-directional connected topology and each node has to present to maintain the topology connectivity therefore hop length (or link distance) between every communicating node is the first factor to estimate delay between every communicating sensor nodes as Eq. (1) implies that $Delay_{end-to-end}(S, D) \propto \sum_{i,j} (d(i, j))$ where ϕ and μ are assumed to be same for each sensor nodes in the network.

As delivery time between any two nodes increases with distance but opting a shortest distance doesn't imply fast data delivery between any two nodes until the path carries low interference in a dense network. In view of this, the new link-level interference model, introduced by Sun et al. in [39] has been engaged to mitigate interference in the network which in turn reduces topology delay as well as nodes' power consumption, is the new metric and second factor to estimate topology delay in the proposed work.

Briefing the points of motivation and improvements:

- (1) Proposed methodology attempts to minimize overall topology delay with power consumption while the existing work focuses to minimize end-to-end delay between sender and receiver nodes only.
- (2) Proposed work shows minimization of nodes power consumption explicitly in the form of power interval while no delay minimization work from literature shows reduction of nodes power consumption explicitly.
- (3) Our work on developing energy and delay minimum topology on those WSNs whose nodes are assigned with power interval, considers interference factor

along with distance factor to estimate topology delay by engaging the link interference model introduced in [39] while most of the existing work ignores this retransmission or delay causing factor.

1.4 Contribution

The manifold contributions of the proposed work are as follows:

- (1) Introduction of ‘Interval data based graph model (IDGM)’ and ‘Sorted-interval data based graph model (S-IDGM)’ for type of bi-directional connected heterogeneous WSNs where each node is weighted with a power interval so that nodes can vary their transmission range in the power interval according to communication requirement.
- (2) A new weighted ‘Delay estimation (DE) function’ based on distance and interference factors has been formulated after showing dependency analysis between link distance (hop length) and link interference towards optimality.
- (3) A polynomial time ‘Energy and Delay Optimization (EDO) algorithm’ has been proposed to obtain an energy and delay optimized connected routing topology on S-IDGMs using DE function. EDO algorithm works under an optimal delay constraint in an energy efficient way and so it minimizes node power consumption and topology delay simultaneously.
- (4) Energy efficiency of EDO algorithm is shown in term of percentage of energy saving by reducing the upper bound of maximally assigned power intervals along with minimization of topology delay.
- (5) Extensive simulation work, graphical and statistical *t*-test analysis have been carried out to show the performance and comparison of EDO algorithm with delay minimum spanning tree, algorithms described in [23] and [19] on S-IDGMs in term of maximum and total topology delay, maximum power consumption interval and percentage of energy saving.

1.5 Organization of paper

The rest of the paper is organized in the following section: Sect. 2 surveys the related work, Sect. 3 introduces important terminologies, details of opted and proposed models. Section 4 presents the proposed problem with linear programming, pseudo-code and explanation of proposed algorithm. Performance evaluation and comparison are shown in Sect. 5. Section 6 concludes the paper.

2 Related work

There has been lot of research work carried out during past many years to optimize power consumption and routing delay as well as to have an optimal solution to the Energy-Delay Tradeoff in WSNs. Authors worked with different strategies and introduced many methodologies to either optimize transmission energy or routing delay or both with the involvement of different estimation metrics. Here, we have mentioned some of them targeting the same goal with different approaches. Some may involve different models of WSNs.

Authors in [40] proposed an approach to minimize energy consumption where packets need to be delivered within a deadline. The approach involves optimal scheduling for packet transmission by varying power level of nodes and transmission time so that the packets must be delivered within a given time frame. In this, delay causing factors while routing has not been considered explicitly. In [2], an energy-aware multi-hop centralized routing scheme can be seen where static as well as mobile gateway nodes have been engaged to handle packet delivery tasks to minimize energy consumption and end-to-end transmission delay. Authors used weighted fair queuing (WFQ) packet scheduling methodology to acknowledge traffic at each intermediate node and a node with least load had been considered for data forwarding from source to destination node. However, WFQ mechanism might be complicated and costly for resource constraint sensor nodes. Authors in [30] come-up with an algorithm to find lowest energy consuming path following a delay bound. In this, path delay has been estimated undertaking medium access layer contention. Authors in [15] introduced energy efficient routing scheme with delay bound using Bellman-Ford algorithm, path length and hop count concept. Approaches involved in this paper limits the number of iterations and hop counts while determining the delay constraint routing path. A real-time delay attack detection and isolation scheme has been proposed in [33] where methodology is based on machine learning mechanism along with a route-handoff mechanism. A concentric circular bands (CCBs) based data forwarding scheme from source to sink node optimizing transmission energy or routing delay or both has been proposed in [5]. In this, authors considered dense and uniformly distributed homogenous sensor network to estimate energy consumption and delay. An energy optimizing technique has been proposed in [14] where data collection in large and dense WSNs are accomplished using projection-based compressive data gathering which aids to conserve nodes’ energy and extend network life-time. Authors in [37] proposed an algorithm for detecting cluster head based on trust value among neighbor nodes

based on the Received Signal Strength Index (RSSI) to reduce the network energy drainage and enhance the network lifetime. Authors in [8] introduced delay concerned routing protocol by believing that delay can be minimized by reducing intermediate hop counts. Using realistic unreliable link models with hop counts, hop length and transmission power factors, authors in [45] obtained lower bound of energy-delay tradeoff in Rayleigh fast fading and Rayleigh block fading channels. Authors in [16] used multiple mobile base stations to minimize latency and engaged minimum Wiener index spanning tree (MWST) as a routing topology. In this, number of hop counts had been considered to estimate latency. Although adopting multiple mobile base stations make the network costly. Further, a ‘delay-constrained energy multi-hop (DCEM)’ algorithm, proposed in [18], provides a solution of TED (Tradeoff between Energy and Delay) optimization problem in real-time cluster-based routing by adjusting the distance and residual energy factor. Authors used queuing theory to find queuing delay to estimate end-to-end routing delay. However, authors ignored the unreliability factor in queuing delay estimation, also it is defined for homogenous WSN only. Authors in [17] estimated delay considering link reliability as well as unreliability factors by involving access delay and collision delay. Collision delay had been taken equivalent to the number of interfering nodes at a time of transmission lying inside the transmission disc of other nodes. Working of the algorithm is very local. Authors in [43] dealt with the Energy-Delay Tradeoff considering unreliable links in both hop-by-hop routing and end-to-end routing of data packets. With the minimization of delay based on transmission and queuing delay, authors provided an optimal power assignment to each links involved in transmission. Optimal power assignment to the nodes in the topology had been left in this work. Authors in [10] proposed MDET algorithm to construct minimum delay and energy-efficient tree for data flooding in low-duty-cycle WSNs. This algorithm finds shortest path using expected transmission count and improves the energy efficiency in a local manner. Authors involved sleep-wakeup delay of duty-cycle with packet transmission delay of each hop under unreliability situation to estimate end-to-end delay. Minimization of delay and energy consumption by engaging optimal duty-cycle can be also be seen in [42]. Authors believed to take minimum intermediate hops in dense WSNs undergoing duty-cycle. Authors in [36] proposed DCEMRA algorithm to minimize energy consumption in cluster-based routing without increasing end-to-end delay. In this, nodes’ remaining energy was computed using distance and data delivery time. DFS algorithm was used to have low cost multiple paths and one of the best path was chosen based on beacon messages. Authors formulated a function to estimate end-end delay comprises of

transmission delay, propagation delay, queuing delay and processing delay which majorly depends on distance between the intermediate nodes. However, the algorithm works on homogenous WSNs.

Major research in this domain has been done to optimize end-to-end routing delay between sender to receiver node and transmission energy cost of data packets travelling the same route. No work has explicitly shown reduction of nodes’ power consumption with delay minimization, neither at transmission level nor at topology construction level.

Different from previously related work, we first attempt to obtain energy and delay optimized connected topology for a heterogeneous bidirectional WSN where power assignment of each node has been explicitly considered throughout the methodology. In this paper, we work with those WSNs whose nodes have been assigned with power intervals so that each node can adjust its transmission range according to the communication requirement. Furthermore, two factors that are distance between the nodes [36] and link interference model, introduced in [39], have been considered to estimate topology delay to obtain delay minimized and energy efficient reliable routing topology on an interval data based graph model of WSNs.

3 Proposed work

Our work comprises of previously defined definitions, formulas, models, newly proposed models, new functions and algorithms. Moving ahead step-by-step with proper description of every important terminology, models, functions and algorithms.

3.1 Preliminaries

Here we introduce some important terminologies, concepts and abbreviations which have been a part of the proposed work.

- (1) *Graphical layout of WSNs* A bidirectional WSN can be represented as a connected undirected graph, denoted as $G(V, E)$ where V denotes the set of all sensor nodes and E denotes the set of all transmission links.
- (2) *Transmission cost TC* TC is the cost of energy required by a node u to transmit data packets to another node v and is equivalent to $c_1 + c_2 d^\alpha$, where $d = ||uv||$, α ($2 \leq \alpha \leq 4$) is the path loss gradient depending on transmission environment, c_1 and c_2 are some constant depending on electronic characteristics of wireless devices [26]. It has been denoted as $TC(u, v)$ when the transmission

is from u to v and $TC(v, u)$ when the transmission is from v to u . In bi-directional WSNs, $TC(u, v) = TC(v, u) = \max\{TC(u, v), TC(v, u)\}$, can be taken symmetrically [28] and written as $TC(uv)$, uv depicts the bi-directional link. Panda and Shetty in [28] compute transmission cost of a bi-directional link uv as $TC(uv) = ||uv||^2$, an approximate estimation.

- (3) *Nodes power assignment* In WSNs, a power function $P: V \rightarrow R^+$ has been defined to assign sufficiently high transmit power to each sensor node u so that it can directly communicate its farthest node v i.e. $P(u) = \max_{uv \in E} \{TC(uv)\}$ [35].
- (4) *Power interval PI* When the power assignment to each node has been done in the form of interval so that nodes can vary their transmission range from minimum to maximum power level depending upon the transmission need. It is denoted as $PI(u) = [P_{min}(u), P_{max}(u)]$ for any arbitrary node u . $P_{min}(u) = \min_{uv \in E} \{TC(uv)\}$ and $P_{max}(u) = \max_{uv \in E} \{TC(uv)\}$ [19]. Let's see an illustration of PI assignment on a network, redrawn from [28], using the theory introduced in [19]. The weight on each edge of Fig. 1 is the square of the Euclidean distance between end nodes i.e. $||ab||^2$ for two adjacent nodes a and b , taken as a transmission cost of bi-directional link ab [28]. Now consider node 'a' of Fig. 1, mentioning all the links incident on 'a' with Euclidean distance i.e. $||ab||^2 = 252, ||ac||^2 = 14, ||ad||^2 = 589, ||ae||^2 = 34$. Minimum and maximum weight among all the links incident on 'a' is 14 and 589 respectively. Therefore $PI(a) = [14, 589]$ is the range of power assigned to node a i.e. $P_a \leftarrow PI(a)$ and likewise $PI(b) =$

$[75, 252], PI(c) = [12, 428], PI(d) = [75, 589]$ and $PI(e) = [12, 452]$ are computed power intervals corresponding to nodes b, c, d , and e respectively.

$$P_a = [14, 589],$$

$$P_b = [75, 252],$$

Hence $P_c = [12, 428]$, and $P_e = [12, 452]$ are

$$P_d = [75, 589]$$

assigned power intervals in Fig. 1. Moreover, interval graph theory results a power interval to each link which is the intersection of power intervals of its end nodes as for example available power interval on a bi-directional link ab is $PI(a) \cap PI(b)$ and denoted by PI_{ab} . Table 1 shows the available power interval on each link of the WSN given in Fig. 1 [19].

- (5) *Maximally assigned power node* A node having power interval with maximum upper bound i.e. $\max PI$, is called maximally assigned power node. A maximally assigned power node retains a communication link with maximum transmission cost as in Fig. 1 nodes a and d retain a communication link of max transmission cost 589 and so their power intervals have maximum upper bound. Therefore a and d are called maximally assigned power nodes.
- (6) *Minimally assigned power node* A node having power interval with minimum upper bound i.e. $\min PI$, is called minimally assigned power node as in Fig. 1 node b is minimally assigned power node.

3.2 Interval data based graph model (IDGM) and Sorted-interval data based graph model (S-IDGM) of WSNs

In this section, we introduce IDGM and S-IDGM of a bidirectional WSN where each node is carrying a weight in interval form. The S-IDGM has been used in our work to compare the nodes' power intervals by just seeing their labels which consequently reduces the algorithmic complexity. Not every WSN can form an interval graph layout even having interval weight on each node [7]. Therefore the type of WSNs whose underlying graph models show the interval weight on each node, do not require to satisfy interval graph properties, are called 'interval data based

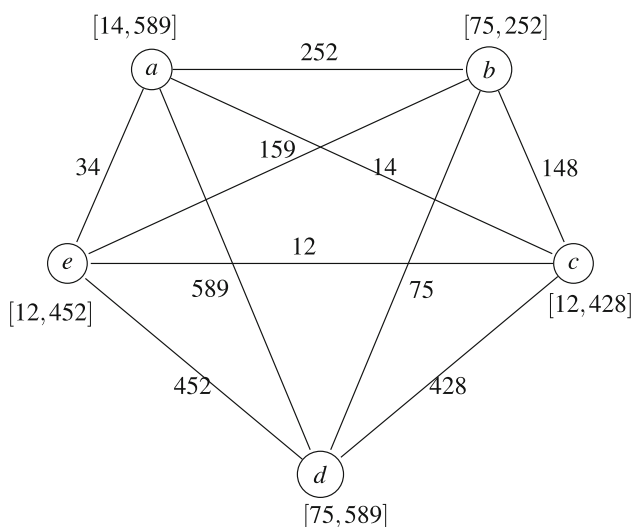


Fig. 1 Weighted complete graph with power intervals

Table 1 Available power interval on each link of Fig. 1

$PI_{bc} \in [75, 252]$	$PI_{ac} \in [14, 428]$
$PI_{be} \in [75, 252]$	$PI_{dc} \in [75, 428]$
$PI_{bd} \in [75, 252]$	$PI_{ae} \in [14, 452]$
$PI_{ba} \in [75, 252]$	$PI_{de} \in [75, 452]$
$PI_{ce} \in [12, 428]$	$PI_{ad} \in [75, 589]$

graph model (IDGM)' of WSNs. Figure 2 is one of an illustration of IDGM.

Every interval graph is an IDGM but converse doesn't hold always because in an interval graph, two nodes are adjacent if and only if their power intervals intersect [6]. In Fig. 2, one can see power intervals of non-adjacent nodes are also intersecting hence this can't be an interval graph so it is called an IDGM of WSN.

Sorted-interval data based graph model of WSN (S-IDGM) Sorting power intervals of an IDGM in an increasing order of their upper bounds results a S-IDGM. Figure 4 shows S-IDGM of Fig. 2.

An intersection model, described in [6] has been used for sorting the power intervals because of its low complexity. Even though the intersection model, given in [6] has been described for interval graphs, it can work for IDGM also but in one way only, not viceversa i.e. one can have an intersection model of an IDGM for a sorting purpose but can't form an IDGM from an intersection model if the IDGM is not an interval graph. Figure 3 shows the intersection model of Fig. 2.

Sorting and relabelling the nodes as $u_1, u_2, \dots, u_n$ following increasing order of power intervals is the very first

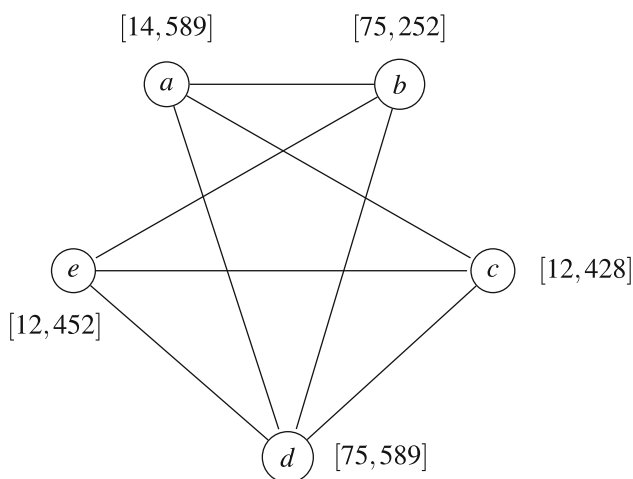


Fig. 2 Interval data based graph model (IDGM) of bidirectional WSN

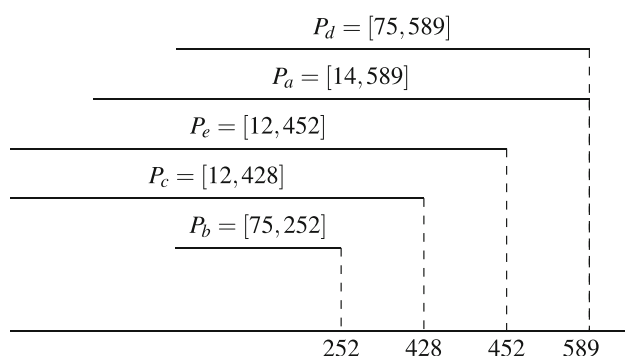


Fig. 3 Intersection model of IDGM, given in Fig. 2

requirement of our methodology. A node with higher subscript is the node with higher power interval. Figure 4 which is a S-IDGM of Fig. 2, visualising that power intervals are increasing with nodes' subscript and vice-versa. Although there are two cases, which have been observed during sorting; first is two or more nodes can have same upper bound of power consumption as seen in Fig. 2, therefore labelling order depends on the lower bound of their power intervals i.e. node with lesser lower bound of power consumption has been labelled with lesser subscript and the node with higher power consuming lower bound has been labelled with higher subscript, as seen in Fig. 4. Secondly, if two or more nodes have same power intervals as shown in Fig. 5a, labelling of the same power nodes can take any consecutive order of the running subscript since all the resultant S-IDGM come out to be isomorphic, see Fig. 5b and c representing the isomorphic models of Fig. 5a. The second requirement of our methodology is to rename minimally assigned power node with leftmost vertex (LMV) and maximally assigned power node with rightmost vertex (RMV) of the obtained S-IDGM. Therefore it can be said that a vertex whose subscript is the minimum number is called leftmost vertex (LMV) and a vertex whose subscript is the maximum number among all vertices' subscript of a S-IDGM is called rightmost vertex (RMV). Hence LMV is the minimally assigned power node, having *min PI* and RMV is the maximally assigned power node, having *max PI* [19]. Let $G(V, E)$ be a S-IDGM, then

$$LMV(G(V, E)) = \{v_j | j = \min_{v_i \in V} (i)\}$$

$$RMV(G(V, E)) = \{v_k | k = \max_{v_i \in V} (i)\}$$

3.3 Link-level interference model

Now introducing Link-level interference model which has been proposed by Sun et al. in 2015 [39] to compute link interference in bidirectional WSNs. For a given graph

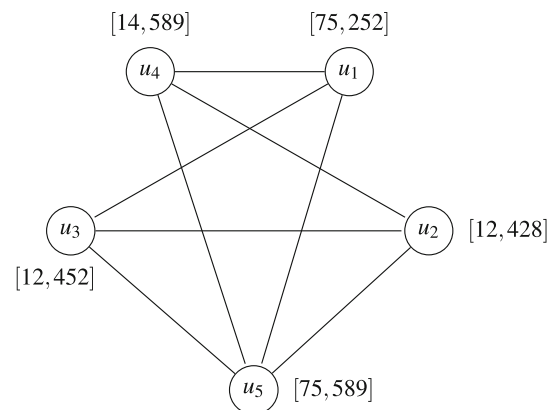


Fig. 4 S-IDGM of IDGM, given in Fig. 2

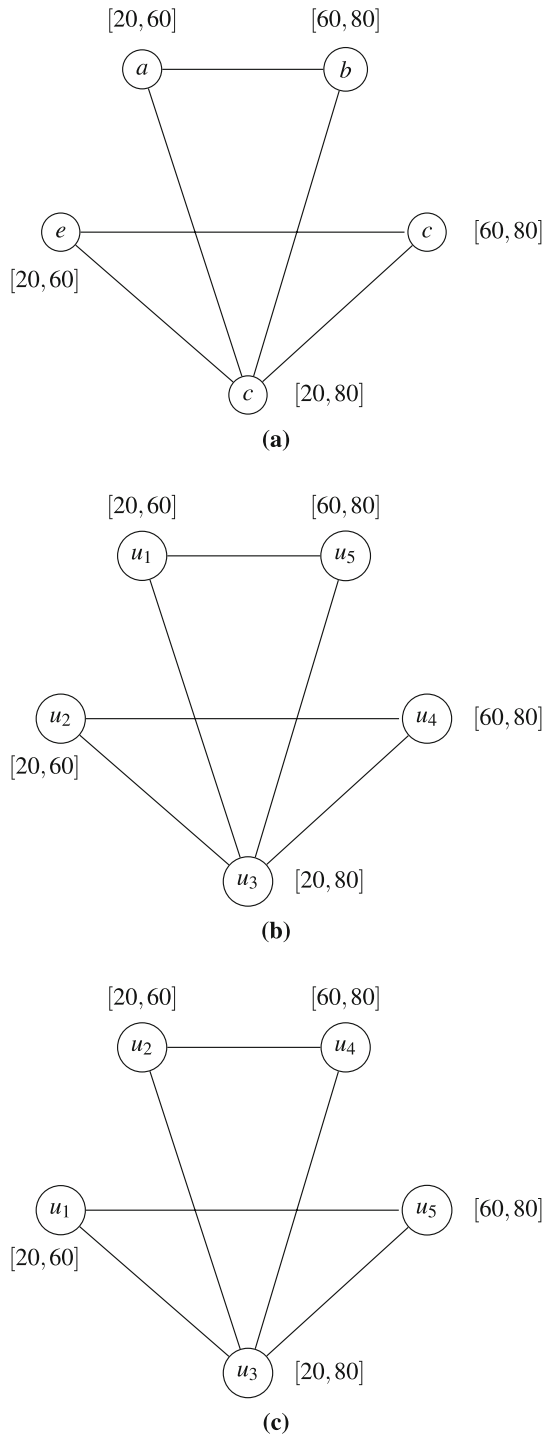


Fig. 5 **a** An IDGM where nodes are having same power intervals **b** First S-IDGM of Fig. 5(a) **c** Second S-IDGM of Fig. 5(a)

$G(V, E)$, interference on any bidirectional link uv is denoted by I_{uv} and is equal to the number of distinct interfering links if any of their end node resides inside the disc of node u or v , is called Link-level interference model and is given by Eq. (2).

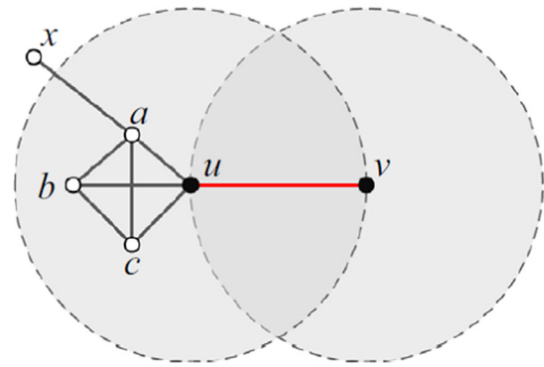


Fig. 6 Disc topology of link uv

$$I_{uv} = |\{xy \mid xy \in E(G), x \text{ or } y \text{ or both} \in \text{Disc}(u, d(u, v)) \text{ or } \text{Disc}(v, d(v, u))\}| \quad (2)$$

where xy is another bidirectional link, $d(u, v)$ is an Euclidean distance between u and v , $\text{Disc}(u, d(u, v))$ is the transmission disc whose centre is u and radius is $d(u, v)$. Similarly $\text{Disc}(v, d(v, u))$ is the transmission disc whose centre is v and radius is $d(v, u)$, where $d(v, u) = d(u, v)$ [41].

An illustration of computing link-level interference model using Eq. (2) are shown on network, given in Fig. 6, taken from [39]. The link-level interference of link uv using Eq. (2) come out to be 7. This link-level interference model outperforms the link interference model described in [28] which counts the number of nodes existing inside the transmission disc of u or v . According to the model described in [28], the link interference of link uv , given in Fig. 6, come out 3 based on nodes count. Hence link-level interference model, proposed by Sun et al. [39] provides more significant results.

3.4 Dependency analysis of link distance and link-level interference model towards optimality

It has been claimed in many research papers that link distance and its interference are so much depending on each other such as more the distance between the nodes more will be the interference between them [17, 26, 41]. Sun et al. in his paper [39] says that link interference increases with link distance and shows the effect of link distance on link-level interference model with positive correlation.

This motivates us to analyze the effect on interference when the approach is related to obtain distance minimum spanning tree as well as the effect on distance when the approach is related to obtain interference minimum spanning tree. For this, first we put the weight on each link is its

Table 2 Total link distance and total link interference calculation on 20 distinct networks

Sr. no	WSNs		Euclidean distance-based MST		Link-level interference model-based MST	
	Nodes	Edges	Total distance	Total interference	Total distance	Total interference
1	9	27	4.044	38	4.154	34
2	11	30	3.54	44	3.63	43
3	13	38	3.68	60	3.88	55
4	15	49	3.73	97	4.17	88
5	18	100	5.394	96	5.65	87
6	20	120	3.68	124	3.84	117
7	23	129	4.152	144	4.36	137
8	25	141	4.106	180	4.425	172
9	28	202	6.177	165	6.459	161
10	30	268	5.731	174	5.754	173
11	32	291	5.29	240	5.485	218
12	33	265	4.91	245	5.17	243
13	35	307	5.507	254	5.753	239
14	37	304	5.284	310	5.468	283
15	39	433	7.88	251	8.25	230
16	41	461	6.49	248	6.65	239
17	43	436	6.034	353	6.28	300
18	45	479	7.53	341	7.75	334
19	48	608	7.27	363	7.56	326
20	50	608	6.92	394	7.35	377

distance, compute the MST using Prim's algorithm and observe the reduction in total distance as well as total interference. Then, we put weight on each link is its interference using Eq. (2), compute the MST using Prim's algorithm and observe the reduction in total interference as well as total distance. This process has been done on 20 distinct networks and resultant data values have been tabulated in Table 2.

Some concluding points have been made after observing the tabulated data

- Resultant data in Table 2 reveals that there is always a difference between value of total interference obtained in interference-based MST and distance-based MST i.e. optimal interference reduction is achieved in interference-based MST only, not in distance-based MST, same things happen with link distance. This implies that one cannot optimally reduce both the distance and interference factor by considering the single factor; interference is optimally reduced in interference-based MST and distance is optimally reduced in distance-based MST. Therefore to minimize both the factors

simultaneously, a weighted function of link distance and link interference needs to formulate. (Discussion of weighted function is carried forward to next section.)

- Link-level interference is more depending on network density than link distance. More dense the network is, more will be data collision and so more retransmission.
- In dense network, minimization of interference factor when the hop length are not much longer, reduces total distance to some optimal extend as well but this doesn't happen in sparse network. This implies that in dense network, interference factor has more dominance over distance factor.

3.5 Delay estimation (DE) function

After analysing the results obtained in previous section, a conclusion is made to construct a new function which optimally balances the minimization of link interference with hop length (link distance) when any algorithm is applied after it. Since both the hop length and link-level interference are used metrics to estimate delay between

every communicating nodes, therefore this new function is known as ‘Delay estimation (DE) function’ and is given by Eq. (3).

This DE function assigns weight on each edge which is a combination of link-level interference and link distance, using Eq. (3). Since $\alpha = 1$ of DE function results interference-based MST providing optimal total interference value and $\alpha = 0$ results distance-based MST providing optimal total link distance value. Exclusion of both these alpha values from Eq. (3), the DE function assigns weight to each link with the combination of link distance and link interference which results a weighted MST whose total interference and total hop length values lie close to their respective optimal values (better than when $\alpha = 0$ and $\alpha = 1$).

Moreover, for maintaining the generality, $\alpha = 0.5$ is taken to assign the weight on each link in the network. Therefore Eq. (3) reduces to Eq. (4).

$$DE(uv) = \alpha * I_{uv} + (1 - \alpha) * d(u, v) \quad (3)$$

where $\alpha \in (0, 1)$ and uv is any bidirectional link.

$$DE(uv) = I_{uv} + d(u, v) \quad (4)$$

Role of DE function

- DE function assigns weight to each edge using Eq. (4) which is nothing but the required delay estimation of each link in the network.
- DE function is used to obtain optimal delay constraint Δ for the complete execution of proposed algorithm.
- DE function estimates total and maximum topology delay (TD) in the resultant topology using Eq. (4).

4 Problem definition and proposed algorithm

To find an energy and delay optimized connected routing topology on S-IDGM $G(V, E)$ of WSN, following is the formulated linear programming problem (LPP) demonstrating the required optimization problem.

$$\min \max_{uv \in E} DE(uv) = \min \max_{uv \in E} \{I_{uv} + d(u, v)\} \quad (5)$$

$$\min \max_{u \in V} PI(u) = \min \max_{u \in V} [P_{min}(u), P_{max}(u)] \quad (6)$$

subject to

$$\sum_{uv \in E} uv = |V| - 1 \quad (7)$$

$$DE(uv) \leq \Delta \quad (8)$$

$$P_{min}(u) \leq PowerConsumption(u) \leq P_{max}(u) \quad (9)$$

$$\max\{P_{min}(u), P_{min}(v)\} \leq TC(uv) \leq \min\{P_{max}(u), P_{max}(v)\} \quad (10)$$

$$u, v > 0 \quad \forall u, v \in V \quad (11)$$

where uv 's are bidirectional transmission links, Δ is an optimal delay constraint i.e. an optimal max delay bound obtained using delay minimum spanning tree by the proposed algorithm during its execution. Equations (6) and (7) depict the desired optimization goals that are minimization of maximum topology delay and minimization of maximum node's power interval respectively. Equations (8)–(11) depict the required constraints attached with the given optimization problem. Equation (8) gives a bound on number of links i.e. total number of links in the resultant topology should be equal to one less than the number of nodes, Eq. (9) directs to keep only those link whose delay estimation (DE) must be less than or equal to the obtained delay constraint, Eq. (10) allows each node to spend its energy within its assigned power interval, Eq. (11) allows variation of transmission cost of each link within the given bounds (which is actually an available power interval on each link) as transmission cost is directly depending on the power used by a link during transmission and Eq. (11) requires each node to present in the resultant topology.

Energy and Delay Optimization (EDO) algorithm

To the defined optimization problem, a polynomial time algorithm has been proposed, named as ‘Energy and Delay Optimization (EDO)’ algorithm. The proposed EDO algorithm undergoes three main phases. The very first phase (step 1 and 2) provides an optimal delay constraint i.e. Δ based on DE function and DE MST, second phase (from step 3 to 17) computes the delay constraint energy efficient path cover on S-IDGM of WSN and third phase (step 18 onwards) constructs energy and delay optimized connected routing topology of S-IDGM if the obtained path cover contains more than one path component. Methodology of the proposed EDO algorithm is the motivation of the algorithms proposed in [19] and [23].

Algorithm 1 Energy and Delay Optimization (EDO) algorithm**Input :** A S-IDGM $G(V, E)$ where $|V| = n$ and $|E| = m$ of a bidirectional WSN.**Output :** A delay-constraint and energy-efficient connected routing topology which results Eq. (5) and (6) of the given LPP following all the attached constraints.

```

1: Compute  $DE(uv)$  of each link  $uv$  of  $G(V, E)$  using Eq. (4).
2: Obtain MST of the delay weighted graph  $G(V, E)$  using Prim's algorithm and optimal delay constraint  $\Delta = \max_{uv \in MST} \{DE(uv)\}$ .
3: Define path cover,  $PC = \phi$  initially
4: Let  $UncoverV = \{v_1, v_2, \dots, v_n\}$ 
5: Start  $num = 1$  and  $Cnum = v_n$   $\triangleright$  ( $num$  shows the number of path components;  $Cnum$  shows the running path component)
6:  $UncoverV = UncoverV / v_n$ 
7: while  $UncoverV \neq \phi$  do
8:   Compute  $S = \{v_k \mid v_k \in UncoverV, DE(v_k v_n) \leq \Delta \text{ and } k > n\}$  and  $T = \{v_k \mid v_k \in UncoverV, DE(v_k v_n) \leq \Delta \text{ and } k < n\}$ 
9:   if  $S$  and  $T$  of  $v_n$  is empty then
10:      $[v = RMV(UncoverV); num = num + 1; Cnum = v]$ 
11:   if  $S \neq \phi$  then
12:      $[v = RMV(S); Cnum = Cnum \cup v]$   $\triangleright$  ( $\cup$  shows the joining of vertices)
13:   else
14:      $[v = LMV(T); Cnum = Cnum \cup v]$ 
15:    $v_n = v$ 
16:    $uncoverV = uncoverV / \{v_n\}$ 
17: return  $PC = \{C_1, C_2, \dots, Cnum\}$ 
18: if  $|PC| = 1$  then
19:    $C_1$  is the required solution
20: else
21:   Choose a component  $C_i$  from  $PC$  and  $PC \leftarrow PC / \{C_i\}$ 
22: while  $PC \neq \phi$  do
23:   Pick another component  $C_j \in PC$  such that  $C_j$  and  $C_i$  are intersecting
24:   Compute  $nghC_j = \{v_i \mid v_i \in C_i, \text{providing the direct connectivity to } C_j \text{ under } \Delta\}$  and  $nghC_i = \{v_j \mid v_j \in C_j, \text{providing the direct connectivity to } C_i \text{ under } \Delta\}$ 
25:   if  $LMV(nghC_i) < LMV(nghC_j)$  then
26:      $C_i = C_i \cup C_j$  will be the new path with the joining of  $LMV(nghC_i)$  to the leftmost adjacent vertex from  $nghC_j$ .
27:   else
28:      $C_i = C_i \cup C_j$  will be the new path with the joining of  $LMV(nghC_j)$  to the leftmost adjacent vertex from  $nghC_i$ 
29:    $PC = PC / \{C_j\}$ 
30:  $C_i$  is the required delay constraint energy efficient connected routing topology.
31: For every link  $uv \in C_i$ , update  $DE(uv)$  using Eq. (4) and compute  $\min \max_{uv \in C_i} DE(uv), \min \sum_{uv \in C_i} DE(uv)$ .
32: For each node  $v \in C_i$ , update  $PI(v) = [\min\{PI_{vu}, \forall vu \in C_i\}, \max\{PI_{vu}, \forall vu \in C_i\}]$  and compute  $\min \max_{v \in C_i} PI(v)$ .

```

Working illustration with step-by-step explanation of EDO algorithm consider a S-IDGM of an arbitrary WSN given in Fig. 7 where each node has been weighted with a power interval which results to power interval on each edge as shown in Fig. 7, $PI(v_1) = [50, 105]$ and $PI(v_2) = [78, 150]$, then the edge between v_1 and v_2 has $PI_{v_1 v_2} = PI(v_1) \cap PI(v_2) = [78, 105]$. Likewise each edge has been resulted with a power interval due to its end nodes and so these power intervals are called available power intervals. Weight other than the available power interval on each edge in Fig. 7 is a delay estimation.

Step 1 Computes delay estimation on each edge of graph, given in Fig. 7, using Eq. (4) and so $DE(v_1 v_2) = 2.617$, $DE(v_1 v_3) = 8.923$, $DE(v_2 v_3) = 8.923$, $DE(v_3 v_4) = 7.76$, $DE(v_3 v_5) = 7.76$, $DE(v_3 v_6) = 8.99$, $DE(v_4 v_5) = 5.78$, $DE(v_4 v_6) = 5.52$.

Step 2 DE-MST is obtained using Prim's algorithm of Fig. 7 with edges $\{v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_5, v_4 v_6\}$ and results $\Delta = \max\{6.803, 8.697, 7.584, 7.717, 7.876\} = 8.697$.

Step 3 Path cover is defined as $PC = \phi$ initially depicts that no vertex is covered by any path component of PC .

Step 4 $UncoverV$ is the set of unvisited vertices which are not covered by PC . So in this illustration $UncoverV = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ since no vertex is covered by PC .

Step 5 $num = 1$ shows that PC starts to obtain its first path component C_1 as PC is like $\{C_1, C_2, \dots, Cnum\}$. C_1 visits maximally assigned vertex first i.e. $RMV(UncoverV)$, therefore $C_1 = v_6$.

Step 6 $UncoverV$ updates to $UncoverV = \{v_1, v_2, v_3, v_4, v_5\}$ as v_6 is covered by C_1 .

Step 7 Initialization of while loop brings execution of steps from 8 to 16 until $UncoverV$ becomes empty.

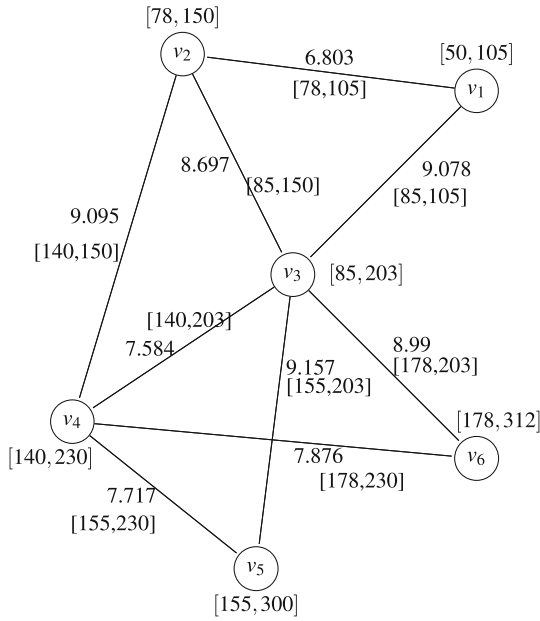


Fig. 7 A S-IDGM of an arbitrary WSN where weights on each edge is the estimated delay of each link and available power interval on each link

Step 8 S and T has been computed for each node present in the original graph. For $v_n = v_6$, $S(v_6) = \phi$ since S comprises only those unvisited neighbour nodes of v_6 whose subscript is greater than 6 with link delay ≤ 8.697 . Unlikely $T(v_6) = \{v_4\}$ comprises of those unvisited neighbour nodes of v_6 whose subscript is less than 6 with link delay ≤ 8.923 .

Step 9 Checks the emptiness of S and T .

Step 10 If incase both S and T are empty then C_1 gets terminated with the initialization of C_2 which again starts with RMV of $UncoverV$ and jumps to step 15.

Step 11 Checks the non-emptiness of set S if the condition of step 9 is not satisfied. If S comes out non-empty then

Step 12 C_1 extends itself by adjoining RMV from S .

Step 13 Checking of T 's emptiness if the condition of step 11 is not approved like in the given illustration where S is empty and T is non-empty then

Step 14 C_1 extends itself by adjoining LMV from T so C_1 becomes $C_1 = v_6v_4$.

Step 15 Now v_4 becomes v_n i.e. $v_n = v_4$.

Step 16 Therefore $UncoverV = \{v_1, v_2, v_3, v_5\}$. (Since $UncoverV \neq \phi$ execution of while loop causes computation of S and T for v_4 , results $S = \{v_5\}$ and $T = \{v_3\}$ and $C_1 = v_6v_4v_5$. Now $v_n = v_5$, $UncoverV = \{v_1, v_2, v_3\}$, $S(v_5) = \phi$ and $T(v_5) = \phi$ therefore step 9 and 10 terminate C_1 with the initiation of $C_2 = RMV(UncoverV)$ i.e. $C_2 = v_3$. Now $v_n = v_3$, $UncoverV = \{v_1, v_2\}$, $S(v_3) = \phi$ and $T(v_3) = \{v_2\}$ so C_2 extends to v_3v_2 and likewise $v_n =$

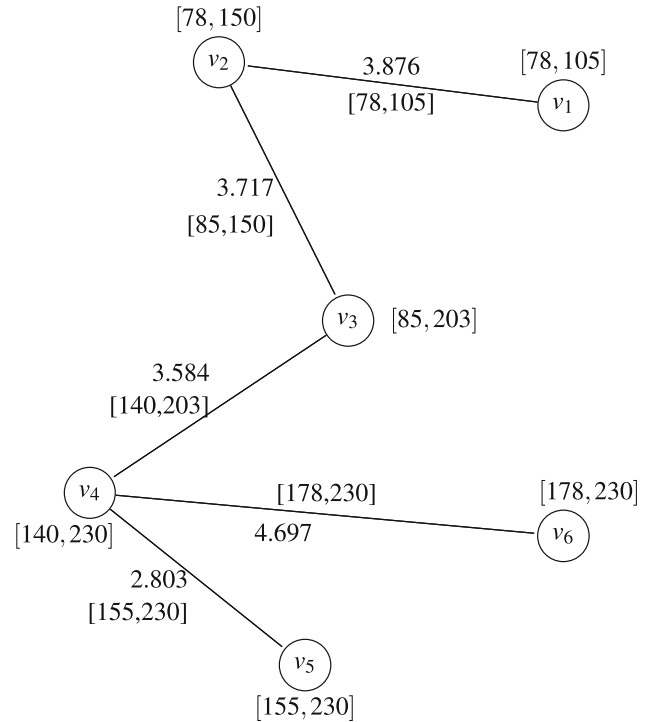


Fig. 8 Connected topology by EDO algorithm of S-IDGM of Fig. 7

v_2 results $C_2 = \{v_3v_2v_1\}$. Finally $UncoverV$ becomes empty and while loop gets terminated.)

Step 17 Return $PC = \{C_1, C_2\}$ i.e. $PC = \{v_6v_4v_5, v_3v_2v_1\}$.

Step 18 Checks the cardinality of PC , if $|PC| = 1$ then

Step 19 C_1 is the required solution.

Step 20 In the given illustration we follow the else part as $|PC| = 2$.

Step 21 Let $C_i = v_6v_4v_5$ and update $PC = \{v_3v_2v_1\}$.

Step 22 Initialization of while loop since $PC \neq \phi$.

Step 23 Let $C_j = v_3v_2v_1$, also C_i and C_j are intersecting in the original graph, given in Fig. 7.

Step 24 $nghC_i$ and $nghC_j$ shows the the direct neighbourhood of path component C_i and C_j respectively under the delay constraint. Therefore $nghC_j = \{v_4\}$ and $nghC_i = \{v_3\}$ for the given illustration.

Step 25 $LMV(nghC_i)$ is v_3 , $LMV(nghC_j)$ is v_4 and $v_3 < v_4$, this satisfies the condition that $LMV(nghC_i) < LMV(nghC_j)$.

Step 26 Therefore C_i is joined to C_j by opting the connection from v_3 to v_4 , see in Fig. 8.

Step 27 If incase condition of step 25 doesn't satisfy then

Step 28 C_i is joined to C_j by making connection of $LMV(nghC_j)$ to its leftmost adjacent vertex from $nghC_i$.

Step 29 PC becomes empty for the given illustration, therefore end of while loop.

Step 30 Resultant C_i , given in Fig. 8 is the optimized connected routing topology of Fig. 7. *(Fig. 8 shows the updated weights on nodes as well as on edges.)

Step 31 Computation of link delay of the optimized topology using Eq. (4). Delay weights are updated in Fig. 8 which results $\min \max_{uv \in C_i} DE(uv) = 4.697$ and $\min \sum_{uv \in C_i} DE(uv) = 18.677$.

Step 32 For each node of the optimized topology, let's find the updated power assignment to each node; $P(v_1) = [78, 105]$,

$$P(v_2) = [\min\{78, 85\}, \max\{105, 150\}] = [78, 150],$$

$$P(v_3) = [\min\{85, 140\}, \max\{150, 203\}] = [85, 203],$$

$$P(v_4) = [\min\{140, 155, 178\}, \max\{203, 230\}] = [140, 230],$$

$P(v_5) = [155, 230]$ and $P(v_6) = [178, 230]$. Therefore $\min \max PI = [140, 230]$ at node v_4 .

Remarks

- (1) Max TD in the original graph, given in Fig. 7 is 9.157 and max node PI is $[178, 312]$. EDO algorithm reduces max TD from 9.157 to 4.697 and max node PI from $[178, 312]$ to $[140, 230]$ (refer Fig. 8). Also

total TD in the original graph, given in Fig. 7 is 74.997 reduces to 18.677 in the optimized topology of Fig. 8.

- (2) Minimization of max node PI leads to the minimization of max power consumption as 320 is the maximum power consumption of atleast one link in the original graph and hence it reduces to 230 in the optimized topology (refer Figs. 7 and 8). This leads to network energy saving. Therefore % of energy saving by EDO algorithm when max node PI reduces from $[178, 312]$ to $[140, 230]$ is obtained by doing $\frac{(312-230)}{312} \times 100 = 26\%$.

Theorem 1 Δ is an optimal delay constraint obtained by EDO algorithm.

Proof Δ is the maximum edge weight on the obtained MST where the edge weight is the delay estimation of each link (computed using Eq. (4)). Assume that Δ is not an optimal constraint this means that there exists another spanning tree say Tr whose maximum edge weight must be less than Δ . But this is not possible as Tr cannot be another MST giving different solution of the same graph. Hence Δ is an optimal delay constraint. $\square$

Table 3 Data obtained on maximum topology delay and total topology delay after simulation

Number of nodes	Number of edges	Maximum topology delay				Total topology delay			
		DE MST	EDO algorithm	Algorithm in [19]	Algorithm in [23]	DE MST	EDO algorithm	Algorithm in [19]	Algorithm in [23]
6	9	4.923	4.923	4.997	4.923	15.345	15.606	18.081	15.606
9	27	6.931	6.931	7.970	7.89	38.154	42.898	56.615	48.6
11	30	7.445	8.445	9.722	9.577	45.595	52.083	81.405	55.646
13	38	9.566	11.566	11.793	11.614	58.875	87.303	95.438	92.79
15	49	11.336	11.512	13.6	13.6	95.002	116.324	145.06	136.54
18	100	11.7	11.7	16.923	16.915	91.626	126.336	253.425	176.82
20	120	17.479	18.479	18.602	18.586	121.793	246.42	294.52	214.1
23	129	17.557	17.557	21.59	20.653	142.275	265.92	340.22	228.676
25	141	17.614	17.614	23.424	19.542	178.243	281.27	378.288	303.522
28	202	14.3	16.468	27.046	27	170.4	302.278	503.12	419.67
30	268	11.373	20.44	28.84	28.614	177.744	232.017	600.5	456.34
32	291	13.282	23.35	30.856	29.714	226.422	305.87	761.024	674.07
35	307	17.685	18.31	33.712	33.67	244.72	364.86	740	634.65
37	304	26.786	28.786	35.667	33.582	288.433	672.15	856.106	698.47
39	433	12.363	23.853	38.1	38	237.133	406.834	1041.672	668.4
41	461	10.305	16.367	40	38.654	246.535	355.252	1065.673	774.827
43	436	20.513	20.513	41.819	37.58	306.282	461.506	1192.872	796.134
45	479	14.3	20.368	42.758	42.854	340.684	412.144	1279.416	943.376
48	608	15.178	22.446	46.847	45.817	333.56	666.5	1483	1100
50	608	16.27	18.22	49	48.757	384.235	486	1679.355	1324.142

Theorem 2 EDO algorithm results energy and delay optimized connected routing topology which minimizes maximum delay as well as node's power interval simultaneously on S-IDGM of a WSN.

Proof We proceed the proof by considering two possible cases. Case (i) Maximum power consumption and maximum delay exist on distinct links: Let v_i be any node having *max PI* on the obtained topology T by EDO algorithm. This implies that there exists atleast one link of max power consumption incident on v_i , let l be that link. Let $l' \in T$ be another link, distinct from l , having maximum delay. Let us assume that T is not an energy and delay optimal topology obtained by EDO algorithm and T' is an optimal one where the cardinality of path cover obtained for T' must not be greater than the cardinality of path cover obtained for T 's construction. Hence links l and l' which are having max power consumption and max delay

respectively must not belong to T' (otherwise T would have been optimal). This means that a connected routing topology can be constructed using links whose power consumption and delay estimation are less than l and l' respectively. In that case EDO algorithm would have terminated before processing l and l' and this leads to contradiction. Hence T is the optimal topology.

Case (ii) Maximum power consumption and maximum delay lie on a same link:

Let l be the link of max power consumption and max delay on the topology T obtained by EDO algorithm, incident on node v_i having *max PI*. The proof carries forward by assuming the contradicting situation in the same way as we did in case(i) which finally results that T is the energy and delay optimal topology, obtained by EDO algorithm. $\square$

Theorem 3 Time complexity of EDO algorithm is $\mathcal{O}(n^2)$ where n is number of nodes.

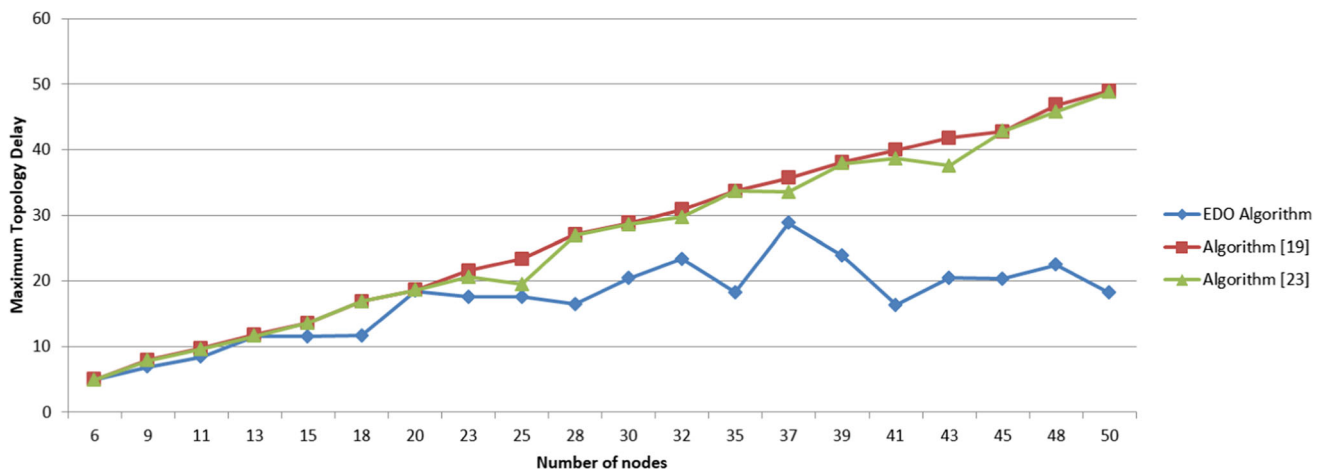


Fig. 9 Max Topology Delay with network density

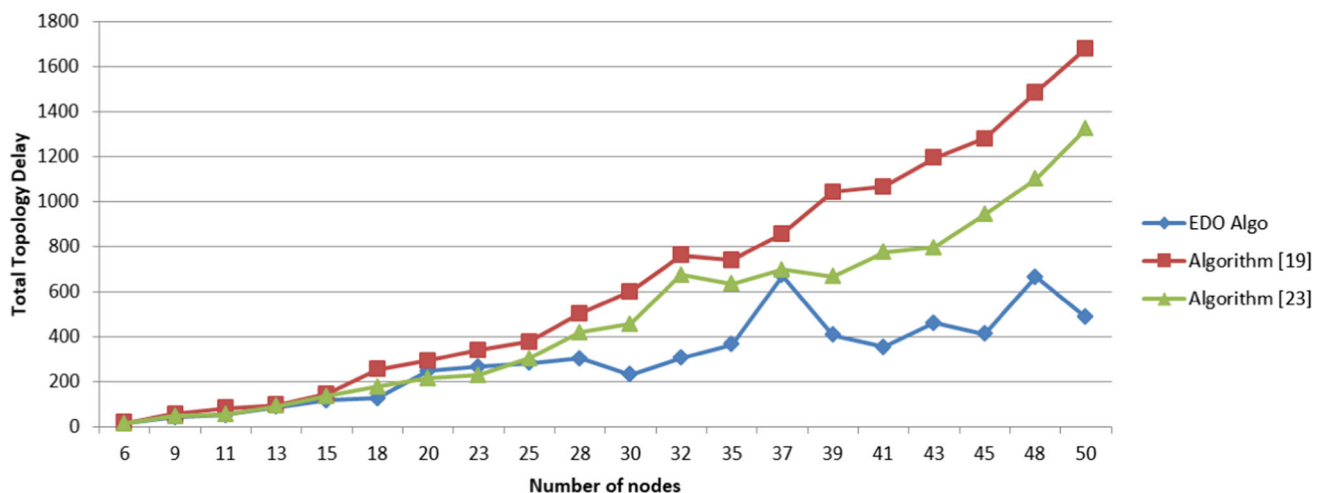


Fig. 10 Total Topology Delay with network density

Proof Step 1 takes $\mathcal{O}(n^2)$ to compute weighted graph. Step 2 also takes $\mathcal{O}(n^2)$ to obtain MST using Prim's algorithm. Step 4 takes $\mathcal{O}(n)$ and steps 3,5,6 are done in constant time. Step 7 initiates the first while loop and executes the steps from 8 to 16 atmost $\mathcal{O}(n)$ time. Step 8 of finding S and T takes $\mathcal{O}(\text{degree}(v_k))$ and maximum complexity from steps 9 to 14 can be $\mathcal{O}(n)$ since finding LMV and RMV of any component can take $\mathcal{O}(n)$. Steps 15 and 16 take constant time and here's the end of first while loop. Now step 18 is done in $\mathcal{O}(n)$ time and steps 19 to 21 take constant time. Step 22 initiates the second while loop and executes the steps from 23 to 29 atmost $\mathcal{O}(n)$. Steps 23 and 24 can take $\mathcal{O}(n^2)$ to find intersection and adjacency of two distinct components. Step 25 takes $\mathcal{O}(|V(C_i)|) + \mathcal{O}(|V(C_j)|)$ where $|V(C_i)|$ and $|V(C_j)|$ depict the number of nodes in components C_i and C_j respectively. Step 26 takes $\mathcal{O}(|V(C_j)|)$ and step 28 takes $\mathcal{O}(|V(C_i)|)$. Final steps 31 and 32 can be done in $\mathcal{O}(n^2)$. Therefore the maximum time complexity of EDO algorithm is $\mathcal{O}(n^2)$. $\square$

5 Performance evaluation

Let us now evaluate the performance of EDO algorithm in term of topology delay minimization and energy efficiency by computing max node's power interval and percentage of energy saving and show the comparison with predefined algorithms, given in [23] and [19] on interval data based graph models of WSNs. Algorithm, given in [23], constructs a spanning tree of minimum number of pendant vertices by joining disjoint path component of path cover, obtained by Arikati and Rangan in [6] on an interval graph. The path cover obtained in [6] is called optimal in term of minimum cardinality (minimum number of path components) of the obtained path cover. Algorithm, given in [19] provides energy optimal path cover and energy efficient connected routing topology which minimizes max node's power interval and max link's transmission cost of worst case scenario. This routing topology [19] is more energy efficient than the spanning tree obtained in [23] on interval graphs.

Table 4 Data obtained on maximum node's power interval and % of energy saving after simulation

Number of nodes	Number of edges	Max node power interval				Percentage of energy saving			
		DE MST	EDO algorithm	Algorithm in [19]	Algorithm in [23]	DE MST	EDO algorithm	Algorithm in [19]	Algorithm in [23]
6	9	[155,312]	[155,230]	[155,230]	[161,312]	0	26	26	0
9	27	[16,133]	[42,116]	[16,116]	[22,116]	4	16	16	16
11	30	[81,169]	[103,169]	[55,131]	[103,193]	20.3	20.3	38	9
13	38	[123,176]	[72,162]	[72,162]	[121,236]	31	36	36	7
15	49	[68,227]	[9,189]	[9,246]	[93,277]	23	35.5	16	5.5
18	100	[87,226]	[87,207]	[47,190]	[84, 263]	21	28	34	8
20	120	[197,369]	[10,366]	[10,366]	[319,394]	8	8.5	8.5	2
23	129	[86,371]	[99,349]	[99,349]	[272,349]	7	12.5	12.5	12.5
25	141	[226,478]	[273,457]	[273,478]	[226,481]	1	5	1	0
28	202	[165,408]	[165,375]	[24,404]	[165,446]	9.5	16.9	10.4	1
30	268	[332,492]	[12,477]	[15,485]	[93,560]	18	20.5	19.2	6.7
32	291	[308,569]	[350,543]	[41,534]	[308,581]	6.9	11.13	12.6	5
35	307	[516,663]	[479,663]	[455,663]	[516,679]	5	5	5	2.6
37	304	[44,646]	[328,612]	[544,619]	[359,694]	9.3	14	13	2.5
39	433	[486,725]	[412,671]	[412,671]	[156,771]	6.2	13.2	13.2	0.3
41	461	[287,746]	[282,696]	[287,696]	[270,806]	8.13	14.3	14.3	0.71
43	436	[546,825]	[548,807]	[396,803]	[548,825]	4	6.1	6.5	4
45	479	[205,875]	[205,875]	[351,823]	[367,894]	2.2	2.2	8	0.1
48	608	[641,927]	[16,803]	[568,919]	[641,940]	3	16	4	1.7
50	608	[513,942]	[443,883]	[163,870]	[604,959]	5.5	11.4	12.7	4

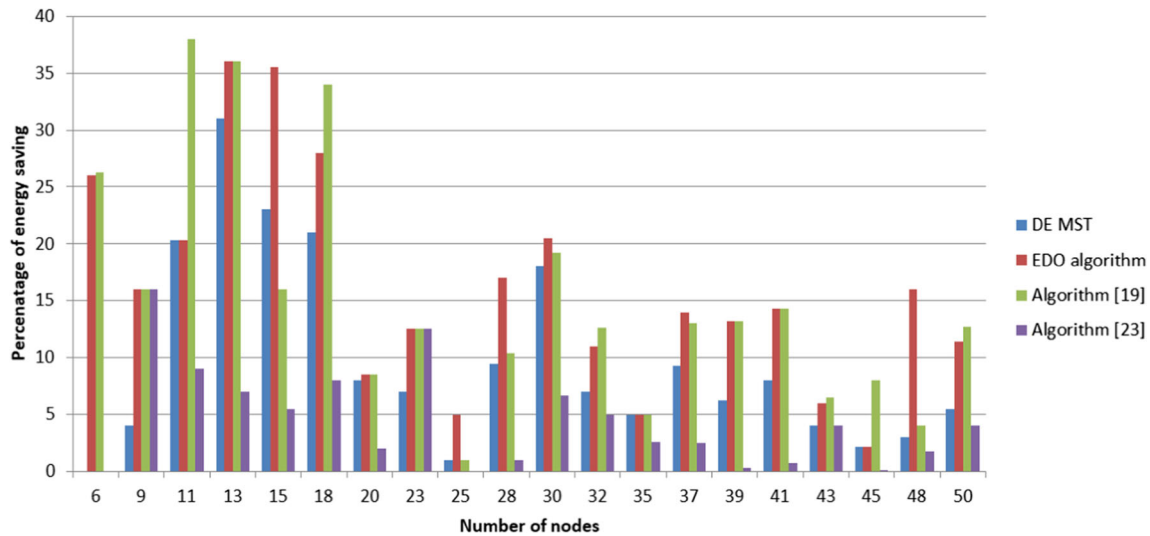


Fig. 11 Percentage of energy saving

Here we will show that EDO algorithm outperforms both the algorithms on S-IDGMs of WSNs in simultaneous minimization of topology delay and nodes' power consumption interval using simulation results, graphical plotting and statistical *t*-test tool. Using *t*-test, we show that topology formed by EDO algorithm has no significant difference in optimizing power consumption to save network energy, comparing with the energy efficient topology obtained by algorithm [19].

In addition, comparison of EDO algorithm with delay minimum spanning tree where the weight on each edge is computed using Eq. (4) and MST is obtained using Prim's algorithm, for convenience we call it DE MST, is also shown. No doubt, this DE MST is the most optimal in minimizing max topology delay and total topology delay but through our evaluation we show that DE MST is not as much energy efficient as the topology formed by EDO algorithm.

5.1 Simulation environment

We run the simulation on large number of WSNs whose underlying graphs are interval graphs in Spyder (Python 3.7) to show the required comparison. Input of arbitrary interval graphs are randomly generated in term of number of nodes, number of edges, list of edges and list of all intervals in increasing order of their upper bounds. The data of each interval graph has been given to the python code of EDO algorithm to evaluate a connected routing topology and compared it with the topology formed by using algorithms given in [19, 23] and DE MST.

Simulation parameter α of EDO algorithm The variable α when approaching to 1 gives more weightage to link level interference and α approaching to 0 gives more weightage

to link distance by Eq. (3). After observing the relationship between link distance and link interference on distinct WSNs towards optimality in Sect. 3.4, α is fixed to a constant 0.5 in simulation work of EDO algorithm in order to consider both the metrics of delay estimation at equal weightage.

5.2 Metric 1: Topology delay minimization

The obtained data after simulation on maximum topology delay and total topology delay of the routing topology by each algorithm has been noted down in Table 3. It has been seen from the data given in Table 3 and their corresponding plotting in Fig. 9 that EDO algorithm performs better in minimizing maximum topology delay than Algorithm [19] and Algorithm [23]. Moreover, data plotting in Fig. 10 depicts that EDO algorithm performs better in minimizing total topology delay than Algorithm [19] in each case. Comparing with Algorithm [23], EDO algorithm minimizes more total topology delay in dense networks. Further, we use two-tailed *t*-testing to show that EDO algorithm optimizes topology delay at 2% level of significance, comparing with DE MST which has been optimal in minimizing topology delay.

***t*-testing on the data given in Table 3:** *t*-test compares mean of data obtained by two different methodologies where results which are obtained on samples, are believed to exist on large number of networks (or we call it population of S-IDGMs).

To examine the performance of EDO algorithm with respect to DE MST through 'two tailed *t*-test', two types of hypothesis have to make. The first hypothesis is called null hypothesis comprises of a statement which we want to claim/proof and the other one is called alternate hypothesis

whose statement is against the claim. Presently our claim for a null hypothesis is that the topology delay minimization by EDO algorithm is as optimal as DE MST. For this to test, null hypothesis becomes ‘mean of data by EDO algorithm is equal to the mean of data by DE MST’ and alternate hypothesis contradicts the null hypothesis and becomes ‘mean of data by EDO algorithm is not equal to the mean of data by DE MST’.

Mean of max TD by DE MST is 13.844 and mean of total TD by DE MST is 187.175 on the networks, given in Table 3. Let μ_0 be the mean of max TD and μ_1 be the mean of total TD by EDO algorithm on population of S-IDGMs.

Null hypothesis H_0 : $\mu_0 = 13.844$ and $\mu_1 = 187.175$.

Alternate hypothesis H_a : $\mu_0 \neq 13.844$ and $\mu_1 \neq 187.175$.

Now computing t -values for both the max TD and total TD using formula $t_0 = \frac{\bar{x}_0 - \mu_0}{SD_0/\sqrt{n}}$ and $t_1 = \frac{\bar{x}_1 - \mu_1}{SD_1/\sqrt{n}}$ respectively where n is number of sample networks, $\bar{x}_0$ is mean of max TD , SD_0 is standard deviation of max TD by EDO algorithm on the networks, given in Table 3. Likewise $\bar{x}_1$ is mean of total TD and SD_1 is standard deviation of total TD by EDO algorithm on the networks, given in Table 3.

Therefore $n = 20$, $\bar{x}_0 = 16.89$, $\bar{x}_1 = 294.978$, $SD_0 = 6.978$ and $SD_1 = 190.688$. Substituting these values in the formulas (given above) result $t_0 = 2.23395$ and $t_1 = 2.52827$. Hence the null hypothesis is accepted for both μ_0 and μ_1 with 2% level of significance. It can be said that topology delay minimization by EDO algorithm is as optimal as DE MST at 2% level of significance.

Remark t -values of Algorithm [19] and Algorithm [23] for max TD come-out to be 4.36 and 4.20 respectively, are significantly different from optimal value at 2% level of significance. Also t -values of Algorithm [19] and Algorithm [23] for total TD come out to be 4.0 and 3.536, are again significantly different from optimal value at 2% level of significance. Therefore overall performance of EDO algorithm in optimizing TD is much better than Algorithm [19] and Algorithm [23] on interval graph models of WSNs.

5.3 Metric 2: Energy efficient topology

The obtained data after simulation on max node’s power interval and percentage of energy saving on the topology by each algorithm has been noted down in Table 4. It has been remarked already that percentage of energy saving is directly depending on max node’s power interval; more is the upper bound of PI lesser is the energy saving. Figure 11 which shows the percentage of energy saving by each respective algorithm, also depicts that more energy saving by any algorithm implies lesser maximum power consumption and this can be seen in Table 4 as well.

Data in Table 4 and plotting in Fig. 11 are explicitly showing that EDO algorithm performs better in minimizing max node’s power consumption than DE MST and algorithm, given in [23] in each case and so EDO algorithm conserves more energy than DE MST and algorithm, given in [23] on interval data based graph models of WSNs.

However, it has been observed that in some networks EDO algorithm conserves more energy than algorithm, given in [19] while in other networks EDO algorithm conserves equal and lesser energy than Algorithm [19]. This is because algorithm, proposed in [19] has been an energy optimal in providing connected routing topology on interval graphs without considering delay factors and EDO algorithm which works to obtain an energy and delay minimized connected routing topology, considers DE function depending on link interference and distance factor which helps in minimizing topology delay and power consumption simultaneously. Since tradeoff between energy consumption and delay minimization brings fluctuation in energy saving while minimizing topology delay. Therefore to evaluate the performance of EDO algorithm in compare to the Algorithm, given in [19], two-tailed t – test has been used on the obtained data, given in Table 4 to show that topologies formed by EDO algorithm and algorithm, given in [19] have no significant difference in saving nodes’ energy.

t-testing on the data given in Table 4: Considering the topology formed by Algorithm [19] is energy optimal and claiming that topology formed by EDO algorithm is also energy optimal while optimizing topology delay. So null hypothesis becomes ‘Mean of percentage of energy saving by EDO algorithm is equal to the mean of percentage of energy saving by Algorithm [19]’ and alternate hypothesis contradicts the null hypothesis and becomes ‘Mean of percentage of energy saving by EDO algorithm is not equal to the mean of percentage of energy saving by Algorithm [19]’.

Mean of percentage of energy saving by Algorithm [19] is 15.36 on the networks, given in Table 4 and let μ_0 is the mean of percentage of energy saving by EDO algorithm on population of S-IDGM.

Let the null hypothesis H_0 : $\mu_0 = 15.36$ and alternate hypothesis H_a : $\mu_0 \neq 15.36$.

Mean of percentage of energy saving by EDO algorithm on the networks, given in Table 4 is 15.92 i.e. $\bar{x} = 15.92$ and standard deviation is $SD = 9.53026$. Computing t -value using formula $t = \frac{\bar{x} - \mu_0}{SD/\sqrt{n}}$ where n is the number of sample networks, in this case $n = 20$. Substituting the values in the given formula results $t = 0.2628$. Hence the null hypothesis is accepted with 5% level of significance. This shows that there is no significant difference in minimizing maximum nodes’ power interval and percentage of

energy saving between EDO algorithm and Algorithm [19]. So it can be said that EDO algorithm results energy optimal topology while minimizing topology delay on S-IDGMs of WSNs.

Remark t -values of DE MST and Algorithm [23] for average percentage of energy saving come out to be -3.0381 and -11.12 respectively, are significantly different from optimal value at 5% level of significance. Therefore overall performance of EDO algorithm in optimizing max power consumption is much better than DE MST and Algorithm [23] on interval graph models of WSNs.

6 Conclusion

In this work, we have proposed an energy and delay optimizing polynomial time algorithm, named as EDO algorithm, to construct an energy and delay optimized connected routing topology on heterogenous bi-directional connected WSNs where nodes are allowed to vary their transmission range within their assigned power interval. A sorted interval data based graph model (S-IDGM) is introduced to show nodes power interval explicitly and proposed EDO algorithm runs on S-IDGM to result an optimized topology which minimizes max topology delay, total topology delay and max nodes' power interval. Minimization of max nodes' power interval consequently results efficient energy saving on S-IDGMs. Additionally, dependency analysis between link distance and link interference has been shown which helps to formulate a new weighted 'Delay estimation (DE)' function towards the optimality of EDO algorithm. Extensive simulation work, graphical and t -test analysis have been carried out to compare the performance of EDO algorithm with delay minimum spanning tree (DE MST) and algorithms [19, 23] on S-IDGMs of WSNs. The proposed EDO algorithm outperforms algorithm [19] and algorithm [23] in minimizing max and total topology delay while conserving more energy than algorithm [23] and DE MST. The t -test analysis has shown that EDO algorithm results an energy efficient topology on S-IDGM at 5% level of significance, comparing with algorithm [19] and delay minimization at 2% level of significance, comparing with DE MST.

In future, work on energy and delay optimization can be extended to dynamic WSNs where topologies are required to be more adaptable and stable with respect to node's movement. Moreover we look forward to develop new interference and delay models under different possible factors like network density, speed of node's movement and data collision in this direction.

Data Availability Authors can confirm that all relevant data are included in the article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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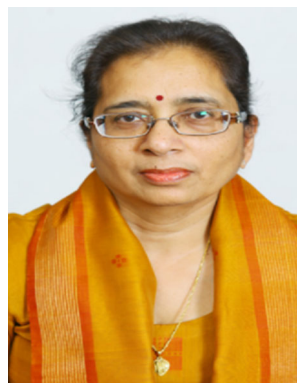


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Dual-interference minimization routing techniques in wireless sensor networks

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Abstract

Interference minimization whether on topology or routing level has been a prominent concern in wireless sensor networks (WSNs). Existing graph-based approaches focus on minimizing whole topology and network interference. Work on having an interference minimum routing path on bidirectional connected graph models of WSNs is still left to explore under different interference models. So, this paper focuses on establishing an efficient routing path from the source to the destination node of minimum total link interference and maximum receiver node interference by optimally balancing between both the interference levels on any bidirectional connected graph model of WSN. We have proposed a dual-interference model based optimization (DIMO) routing algorithm by engaging ant colony optimization technique with two distinct interference models simultaneously that are link SUM interference model and receiver interference model. Moreover, a new interference model named as Disc-SUM model has also been developed whose implementation with Dijkstra's algorithm results in a dual-interference minimized routing path in quadratic time complexity. Simulation work has been carried out to validate the DIMO algorithm and the Disc-SUM model for implementing a routing path between the source to the destination node with optimally reduced total link interference, maximum receiver node interference, total transmission cost, and hop counts.

Keywords Wireless sensor networks (WSNs) · Topology · Graph algorithms · Ant colony optimization (ACO) · Time complexity · Interference models

1 Introduction

The trend of WSN has been growing very rapidly with time as it has got a wide range of applications in various fields like power monitoring, inventory location monitoring, military surveillance, hospital services, tracking objects, animals, humans, and vehicles, etc. These applications are functioned by numerous battery-powered, light weight and omnidirectional wireless sensors when they are deployed in any target area [1]. These sensor nodes sense information

from any target area and pass on the sensed data to other nodes till the collective information in the form of signals reached to a destination node [2]. This leads to frequent data transmission from the source node to the destination node via relay nodes and so, multiple signal receptions to receiver nodes cause lots of data collision and retransmission in the network. The collision of data packets leads to transmission hindrance, and retransmission which is called interference in WSNs. Interference contributes to more power consumption of each sensor node in WSNs where sensors are power constraint and non-rechargeable [3, 4]. In history, many researchers have attempted their best to get rid of network interference by developing many distinct interference models to measure hindrances/obstacles existing on links and nodes during data transmission in WSNs. These interference models have been further engaged to construct the interference-reduced topology of sensors through some interference minimization techniques, and algorithms [5–10].

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Existing interference minimization algorithms primarily focus on developing a topology of minimum interference by connecting all sensor nodes yet this does not bring lesser interference at both the link and node level when transmission happens from the source to the destination node on the connected topology. Also existing interference minimization algorithms for route determination between the source to the destination node work on a single interference model at a time and achieve optimality either at link or node level. This motivates us to bring new routing techniques for determining an efficient path between the source to the destination node to achieve a simultaneous reduction of maximum receiver node interference and total link interference during transmission on bidirectional connected graph models of WSNs.

Briefing the paper contributions

- (1) For the very first time, ACO technique has been introduced with two distinct interference models to result in dual-interference reduced routing path from the source to the destination node. The technique is named as DIMO algorithm.
- (2) DIMO algorithm simultaneously minimizes max receiver node interference and total link interference on bidirectional routing paths from the source to the destination node.
- (3) A new link interference model named as Disc-SUM model has been introduced to compute interference on each bidirectional link based on transmission discs interruption for the very first time.
- (4) Another routing algorithm has been proposed with an implementation of the Disc-SUM model with the Dijkstra's algorithm, named as DSD algorithm which not only minimizes total link interference but also minimizes max receiver node interference in quadratic time complexity.
- (5) Proposed work validation has been shown via simulation, graphical, and statistical *t*-test analysis on the ground of total link interference, max receiver node interference, transmission cost, and hop counts minimization.

The outline of the paper is as follows, Sect. 2 provides a brief on related work. The proposed work is explained in Sect. 3 with preliminaries. Section 3.1 provides the purpose and the problem statement, Sect. 3.2 provides the methodology of the proposed DIMO routing algorithm. Description of the Disc-SUM model and the DSD algorithm are given in Sects. 3.3 and 3.4 respectively. Simulation and evaluation are given in Sects. 4 and 5 concludes the paper.

2 Related work

As interference in WSNs has been a major issue in energy consumption and delay in data transmission, various attempts have been made to develop techniques for reducing interference in the history of WSNs. In [11], authors have come up with a routing technique that avoids a route of heavy interference and collisions by getting information on each next node about its busyness using a backpressure routing algorithm. The protocol is localized in nature yet it is good in reducing network delay. An approach to mitigate co-channel interference for distributed homogenous WSNs has been developed in [12]. The authors have focussed to reduce data collision between the sensor nodes at the topology level by using Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) techniques. In [13], an interference-free communication has been developed by choosing appropriate time slots for data transmission between the nodes using a signal-to-interference-plus-noise ratio (SINR) model. The authors have shown the effectiveness of the work on the tradeoff between power consumption and schedule length. An interference cancellation-based data aggregation technique has been introduced in [14] to reduce aggregation delay, and energy consumption simultaneously under a physical interference model. The work has been defined for directed graph models of WSNs.

In [15], authors have introduced cellular clustering architecture to reduce interference for underwater wireless data transmission. The approach is based on inter-cell time division multiple access (TDMA) scheduling, and intra-cell hierarchical routing. Another TDMA-based approach has been developed by authors in [16] to reduce radio frequency interference. Authors in [17] have put forth an interference and distortion minimization approach for harvesting WSNs where a constraint of resource allocation has been taken into account. The problem has been modeled as a convex optimization problem and modified Newton's method has been applied to result an approximate solution.

In [18], authors have proposed energy-efficient algorithms to minimize total and maximum network interference under the sender interference model subject to a connectivity constraint of directed graph models. The approach contributes to the topology control of wireless sensors. Then authors in [6] have proposed MAX and SUM models to measure the interference of WSNs and interference-minimizing algorithms under these models. Also, algorithms under receiver centric interference model have been tested which provide a 2-approximate solution. An algorithm to minimize maximum topology interference under receiver centric interference model has also been

proposed in [19]. A minimum interference multipath routing protocol has been introduced in [20] which establishes multiple paths between sensor nodes to sink nodes to enhance data transmission via minimum interference routes. This strategy outperforms in terms of load balancing, throughput, and latency. Furthermore, a new link interference model has been introduced in [7] to minimize topology-based interference, and power consumption. Optimal transmission range assignment based attempt to minimize maximum and total receiver node interference of the topology has been introduced by authors in [8]. Authors in [9] have come up with a new scheduled-based routing technique to mitigate interference for software-defined WSNs. Moreover, authors in [2] have introduced a new metric to measure interference level among multiple paths on undirected graph models of WSNs and proposed a routing technique to establish multiple paths of minimum interference to reach the destination node under the same metric. In [21], authors have developed a new technique to minimize maximum sender interference of a connected network. The technique consists of adding some new nodes to the topology so that the interference can be reduced while ensuring the connectivity factor. Authors in [22] have provided two more interference minimization algorithms based on power assignment to each node from the discrete set of two power levels. Each node has been assigned with one of the two given power levels such that maximum node interference gets minimized while maintaining the network connectivity. Furthermore, a probabilistic interference model has been developed by authors in [10] to measure receiver interference of the network and a centralized algorithm to construct an interference minimum spanning tree.

The existing relevant techniques minimize interference by targeting the whole connected topology of sensors. Also, existing graph-based interference minimization models and algorithms are single objective. Work on minimizing total link interference along with node interference on a routing path between the source to the destination node is still left to explore with different interference models. Henceforth in this paper, we bring an efficient connected routing path between the source node to the destination node by simultaneously minimizing total link interference and maximum receiver node interference on bidirectionally connected graph models of WSNs. This dual-interference minimization on any bidirectional connected graph model has been achieved by introducing the ACO technique with the link SUM interference model and receiver interference model, given in [6, 8, 22]. Moreover, a new link interference model has been proposed whose implementation with the Dijkstra's algorithm achieves a simultaneous reduction in total link interference and maximum receiver node interference in quadratic time. The

proposed interference model with the Dijkstra's algorithm shows an efficient reduction in hop counts and the total transmission cost of the path between source to destination node.

3 Proposed work

This paper deals with dual-interference minimization i.e. simultaneous minimization of total link interference and maximum receiver node interference of a routing path between the source to the destination node on bidirectional connected graph models of WSNs. We have accomplished this via two distinct ways:

- (1) Introduction of ACO technique with two existing link and node interference models.
- (2) Proposal of a new interference model and engaging it with Dijkstra's algorithm.

Starting with some preliminaries used in the proposed work.

- (1) *Graph model of WSNs* $G(V, E)$ WSN can be represented as a bidirectional connected graph $G(V, E)$ where each sensor is taken as vertex (node) forming a vertex set V and communication link between sensors is taken as an edge in the plane forming an edge set E .
- (2) *Transmission range* When a sensor node say u has been assigned with a power level $r_u = \max_{uv \in E} ||uv||^2$ then u can sense and communicate directly till r_u distance so, r_u is said to be a transmission range of node u .
- (3) *Transmission disc* $D(u, r_u)$ of node u A circular disc $D(u, r_u)$ is assumed to exist around u where node u is a centre and r_u is a radius. Every other node which comes inside the disc is known as a direct neighbor of node u and can communicate directly with u or vice versa.
- (4) *Receiver interference (I_R) model* This model gives a measurement of interference that exists on each receiver node v by counting the transmission discs of other nodes to which v belongs. Mathematically it is given by Eq. (1).

$$I_R(v) = \{u \in V \setminus \{v\} | v \in D(u, r_u)\} \quad (1)$$

where $D(u, r_u)$ is a transmission disc of another node u with transmission range r_u [8]. A topological illustration of receiver interference is given in Fig. 1, redrawn from [22]. Also, a measure of receiver interference on each node of the illustrated topology is shown in Table 1.

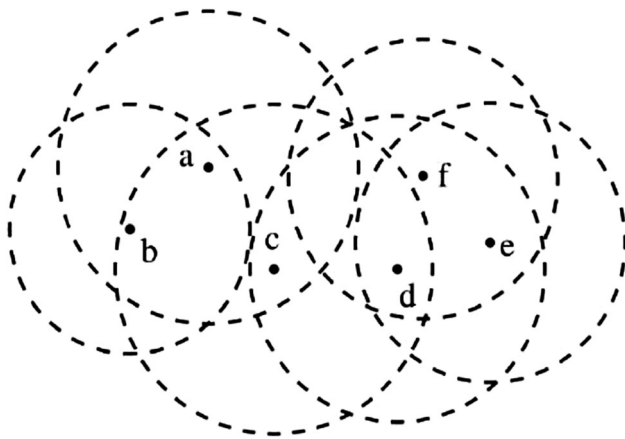


Fig. 1 A topological illustration of receiver interference

Table 1 Measure of receiver interference of topology given in Fig. 1

Node	a	b	c	d	e	f
$I_R(v)$	2	2	2	3	2	2

- (5) *Link-SUM interference (I_{LS}) model* This model is introduced by Panda and Shetty in [6] to compute interference existing on each bidirectional link uv by counting the number of nodes other than u and v belonging to the transmission disc of u and v . Mathematically it is given by Eq. (2).

$$I_{LS}(uv) = |\{w \mid w \in D(u, r_{uv}) \text{ or } D(v, r_{vu})\}| \quad (2)$$

A topological illustration of link interference is shown in Fig. 2, redrawn from [6]. I_{LS} model computes interference on bidirectional link uv given in Fig. 2 and so, $I_{LS}(uv) = 11$.

- (6) *Dijkstra's algorithm* It is a polynomial time algorithm used to find a shortest path between the source to the destination node on any directional/bidirectional graph where each edge of the graph has been weighted with the Euclidean distance (or any positive weight).
- (7) *ACO technique* ACO is an abbreviation of 'ant colony optimization'. This technique has been introduced based on the foraging behavior of ants seeking a shortest path between their colony and the source food. When an ant moves in search of food, it leaves a pheromone (a kind of liquid marker) along the path which evaporates over time. Other ants start following the same path by smelling the pheromone

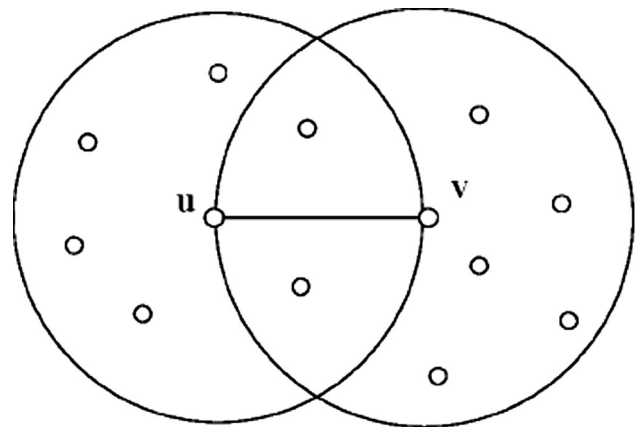


Fig. 2 A topological illustration of a link interference under I_{LS} model

concentration. This way, paths which have been traveled more frequently would have more pheromone quantity. Due to the involvement of pheromone evaporation, shorter routes would have more pheromone quantity as compared to longer routes. Hence after many rounds, numerous ants converge to a single path which would be considered a near-optimal solution [23, 24].

ACO technique has been used for a long time to solve route optimization problems like minimization of transmission cost, minimization of transmission delay, minimization of nodes power consumption, maximization of throughput, and reliability factor in WSNs. To deal with single or multi-objective optimization problems, ACO technique has been modified by introducing new factors to its probability and pheromone function. These factors can be distance, cost, power of nodes, nodes residual energy, nodes power consumption, node degree, etc. [25–27].

In this paper, well-known Dijkstra's and ACO techniques have been engaged on bidirectional connected graph models of WSNs where each edge/link has been weighted with interference instead of distance.

3.1 Problem definition

On any wireless sensor network whose graphical layout is a bidirectional connected graph, a transmission route from the source to the destination node of minimum interference at both the link and node level is required to find. So, the problem is stated as follows.

Dual-interference minimization routing problem (DIMR problem) Find an efficient routing path from the source to the destination node such that the resultant path has a minimum of maximum receiver node interference with a

simultaneous reduction in total link interference on bidirectional connected graph models of WSNs.

To deal with the DIMR problem, two new approaches with a new interference model have been proposed whose discussion are followed in the subsequent sections.

3.2 Dual-interference model based optimization (DIMO) technique

The first proposed technique is named as DIMO routing algorithm, given by Algorithm 1, introduces ACO with a new probability and pheromone update function under two distinct interference models that are I_R and I_{LS} models. The probability function is given by Eq. (3).

$$P_{ij}^k(t) = \frac{\tau_{ij}^\alpha(t) \times (\frac{1}{I_R(j)})^\beta}{\sum_{il \in E(G)} \tau_{il}^\alpha(t) \times (\frac{1}{I_R(l)})^\beta} \quad (3)$$

where $P_{ij}^k(t)$ calculates the probability for k th ant to pick its next move from i th to j th node for data transmission between the source to the destination node at iteration t . τ_{ij} represents the pheromone quantity on edge from i th to j th node based on link interference. $I_R(j)$ computes the receiver interference at each unvisited j th node using Eq. (1). Parameters α and β are engaged to regulate decisions based on link interference and receiver node interference respectively by varying their value between 0 and 1.

Algorithm 1 DIMO algorithm

Input: Bidirectional connected graph $G(V, E)$ with $|V| = n$, source node $v_s = v_1$ and destination node $v_d = v_n$
Output: DIM-routing path from v_s to v_d i.e. $DIMP(v_s \rightarrow v_d)$

- 1: Define a set of unvisited nodes as $UncoverV = V \setminus \{v_1\}$
- 2: Initialise ACO by setting number of ants = m , iteration = IN and $\tau_e(t) = 1 \forall e \in E$
- 3: **for** $t \in [1, IN]$ **do**
- 4: Set α, β and ρ in $[0, 1]$
- 5: **for** $k \in [1, m]$ **do**
- 6: Evaluate $P_{v_s v_j}^k(t)$ for each $v_s v_j \in E$ and $v_j \in UncoverV$ using Eq. (3)
- 7: Select v_j as next node whose $P_{v_s v_j}^k(t)$ is maximum
- 8: **if** $v_j \neq v_d$ **then**
- 9: $v_s \leftarrow v_j$ and go to step 6
- 10: Return $Path(v_s \rightarrow v_d)$
- 11: Compute $\max\{I_R(v)\} \forall v \in Path(v_s \rightarrow v_d)$ using Eq. (1) and $\sum\{I_{LS}(e)\} \forall e \in Path(v_s \rightarrow v_d)$ using Eq. (2)
- 12: **if** $\max\{I_R(v)\}$ and $\sum\{I_{LS}(e)\}$ do not satisfy the required criteria of the DIMR problem **then**
- 13: Back to step 4
- 14: **if** $\max\{I_R(v)\}$ and $\sum\{I_{LS}(e)\}$ satisfy the required criteria of the DIMR problem **then**
- 15: $DIMP(t) = Path(v_s \rightarrow v_d)$
- 16: Evaluate pheromone update on $DIMP(t)$ for next iteration using Eq. (4) and Eq. (5)

Each ant selects a next node of higher probability using Eq. (3) to transmit its data on a bidirectional connected graph model $G(V, E)$ as a node with higher probability implies lesser interference on transmission between the sensors.

The significance of using t lies with pheromone update on paths followed by m ants in each iteration where a total number of iterations is denoted by IN in the algorithm.

The new definition of pheromone update function based on link interference model I_{LS} is given by Eqs. (4) and (5).

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k \quad (4)$$

$$\Delta\tau_{ij}^k = \frac{1}{I_{LS}(ij)} \quad (5)$$

where $\rho \in [0, 1]$ is engaged to regulate the evaporation rate of previously deposited pheromone in each iteration. $I_{LS}(ij)$ computes link interference on each bidirectional link ij using Eqs. (2). $\sum_{k=1}^m \Delta\tau_{ij}^k$ is the total value of pheromone deposit on an edge ij traveled by m number of ants in the current iteration. If some edge is traveled by 5 ants in a single iteration then the pheromone deposit on the edge is summed up to 5 times based on the Eqs. (5). It is revealed from Eqs. (4) and (5) that the update of the pheromone in the next iteration depends on the pheromone quantity deposited till previous iterations. So initially when $t = 1$, the pheromone on each edge is assumed to be the same constant or 1.

Step-by-step explanation of the DIMO algorithm Step 1 defines a set of unvisited nodes as $UncoverV$ which are not yet traveled or visited and initially all vertices except source node are unvisited. Step 2 initialises ACO technique

with total number of desirable ants m , iterations IN and pheromone value 1 on all edges e . Step 3 initiates a for loop which iterates IN times and step 4 can choose any value of α, β, ρ in interval $[0, 1]$ during the execution of the algorithm. Step 5 starts the travel for m number of ants one-by-

one from source v_s to destination v_d . Steps 6-15 result a routing path optimal to the DIMR problem when m ants complete their rounds from source-destination-source using Eqs. (3), (4) and (5) at some α, β, ρ after comparing the various solutions to the DIMR problem at distinct values of α, β, ρ in each iteration $t \in [1, IN]$. Step 16 updates the pheromone quantity in each iteration for better and fast convergence of ants to a single solution. Hence, a resultant routing path with balanced minimization of max receiver interference and total link interference at IN^{th} iteration is considered as optimal.

Theorem 1 *The time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$ where IN is number of iterations, m is number of ants and n is total number of nodes.*

Proof Step 1 and 2 can be done in constant time. Step 3 repeats at the most IN times. Step 4 takes constant time and step 5 runs atleast m times. The total time complexity from steps 6 to 10 is $\mathcal{O}(n^2)$. Steps 11 can be done in $\mathcal{O}(n)$ time. Steps 12-15 take constant time and step 16 takes $\mathcal{O}(n)$. Hence total time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$. $\square$

3.3 Proposed disc-SUM interference model

DIMO algorithm may reach to higher time complexity due to presence of the IN and m variables. Hence to deal with the DIMR problem with lesser time complexity, a new interference model named as Disc-SUM model has been proposed. The proposed model computes link interference based on a number of transmission discs intersecting or covering the link on a routing topology of WSN. Mathematically Disc-SUM model is given by Eq. (6) where $D(w, r_w)$ is a transmission disc of node w with transmission range r_w .

$$\text{Disc-SUM}(I_{uv}) = \{w \in V \setminus \{u, v\} \mid uv \in D(w, r_w) \text{ or } u/v \in D(w, r_w)\} \quad (6)$$

A topological illustration of the Disc-SUM model on bidirectional link uv is shown in Fig. 3 where the transmission disc of node x is covering the link uv and the transmission disc of nodes y, w, z are intersecting the link uv . All the interrupting transmission discs are shown using blue color.

Remark 1 Interference computation by the Disc-SUM model on the bidirectional link uv given in Fig. 3 is found to be 4 whereas the SUM model introduced by Panda and Shetty in [6] computes interference quantity on uv is 1 only. It is because the model, given in [6] measures link interference by counting the number of nodes inside the transmission disc of u and v or both, and in the proposed model nodes outside the transmission disc of uv are also

taken into account based on transmission discs interruption of the nodes to the link.

Remark 2 To the best of our knowledge, this is the first time that transmission disc interruption based link interference is taken into account and results on minimization of maximum receiver node interference along with total link interference, intermediate hop counts and total transmission cost are shown in the subsequent section.

3.4 Disc-SUM Dijkstra's algorithm

Introducing Dijkstra's algorithm on bidirectional connected graph models of WSNs where each edge is weighted with link interference measured using the proposed Disc-SUM model. The approach of finding a dual-interference minimum routing path from the source to the destination node using the Dijkstra's algorithm with the Disc-SUM model is called Disc-SUM Dijkstra's (DSD) algorithm.

Algorithm 2 Disc-SUM Dijkstra's (DSD) algorithm

Input: Bidirectional connected graph $G(V, E)$ with $|V| = n$, source node $v_s = v_1$ and destination node $v_d = v_n$.
Output: DIM-routing path from v_s to v_d .
 1: Compute $\text{Disc-SUM}(I_e) \forall e \in E$ using Eq. (6).
 2: Initiate Dijkstra's algorithm.
 3: Return $\text{Path}(v_s \rightarrow v_d)$
 4: Compute $\max\{I_R(v)\} \forall v \in \text{Path}(v_s \rightarrow v_d)$ and $\sum\{\text{Disc-SUM}(I_e)\} \forall e \in \text{Path}(v_s \rightarrow v_d)$.

Proposition 1 *DSD algorithm has $\mathcal{O}(n^2)$ time complexity where n is a number of nodes.*

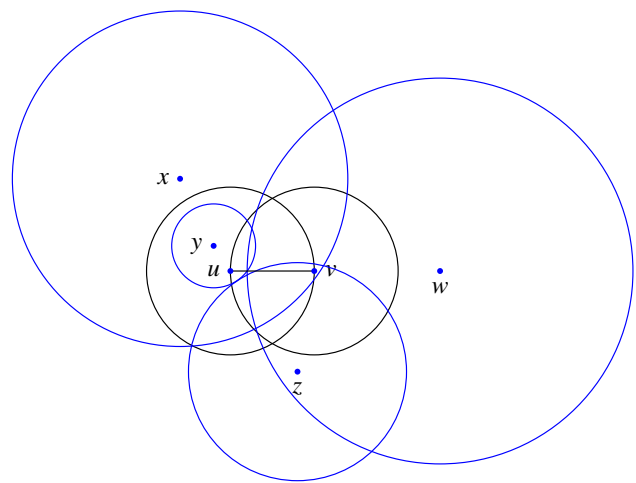


Fig. 3 Topology of Disc-SUM interference model on bidirectional link uv

Theorem 2 *DSD algorithm minimizes total link interference of a routing path from the source to the destination node.*

Proof Dijkstra's algorithm has been optimal in finding a path from the source to the destination node of minimum total weight and when weight is link interference by the Disc-SUM model then the DSD algorithm results in an optimal path of minimum total link interference. $\square$

Remark 3 DSD algorithm minimizes max receiver node interference of a routing path from the source to the destination node which is shown in the subsequent section through simulation.

4 Simulation and comparison

To evaluate the performance of the proposed DIMO and DSD algorithms in terms of simultaneous minimization of max receiver node interference and total link interference on a routing path from the source to the destination node, simulation work on hypothetical bidirectional connected graph models of WSNs has been carried out. On each bidirectional connected graph model $G(V, E)$ of n number of nodes, it is assumed that $v_1 \in V$ is a source node and $v_n \in V$ is a destination node under each simulation.

We have opted Python language for simulations and analyzed the resultant data using graphical plotting and statistical t -test analysis in IBM SPSS.

4.1 Performance evaluation of the DIMO algorithm

To evaluate the performance of the DIMO algorithm, we have compared the data obtained by DIMO algorithm with the Dijkstra's algorithm, MISBT coverage algorithm, given in [6] and MRI-TPL algorithm, given in [22]. Benchmarks of performance evaluation are max receiver node interference i.e. $\max I_R$ and total link interference i.e. $\sum I_{LS}$ of a routing path traced from the source to the destination node on bidirectional connected graph models of WSNs.

The significance of comparing our simulation results with Dijkstra's algorithm is lying in the fact that Dijkstra's algorithm always provides an optimal solution by minimizing total weights of the path traced. Further, comparison of DIMO algorithm with MISBT coverage algorithm [6] and MRI-TPL algorithm [22] is lying in the fact that both the algorithms have been developed to result an interference optimal topology under I_{LS} and I_R model respectively. According to our benchmark, we have focused on a transmission route from the source to the destination node on the topologies obtained by the MISBT and MRI-TPL algorithms for comparison.

Table 2 shows the simulation results on the DIMR problem by the DIMO algorithm, Dijkstra's algorithm, MISBT coverage algorithm, and MRI-TPL algorithm on various graph models of WSNs. It can be observed from the data given in Table 2 that the DIMO algorithm results in better dual-minimization by better balancing between max receiver node interference and total link interference in comparison to Dijkstra's, MRI-TPL [22] and MISBT coverage algorithm [6]. Dijkstra's algorithm works on a single objective i.e. optimal minimization of total link interference, does not control the max receiver node interference on the transmission route. Also, MISBT and MRI-TPL algorithms perform well in minimizing total link interference, not max receiver node interference.

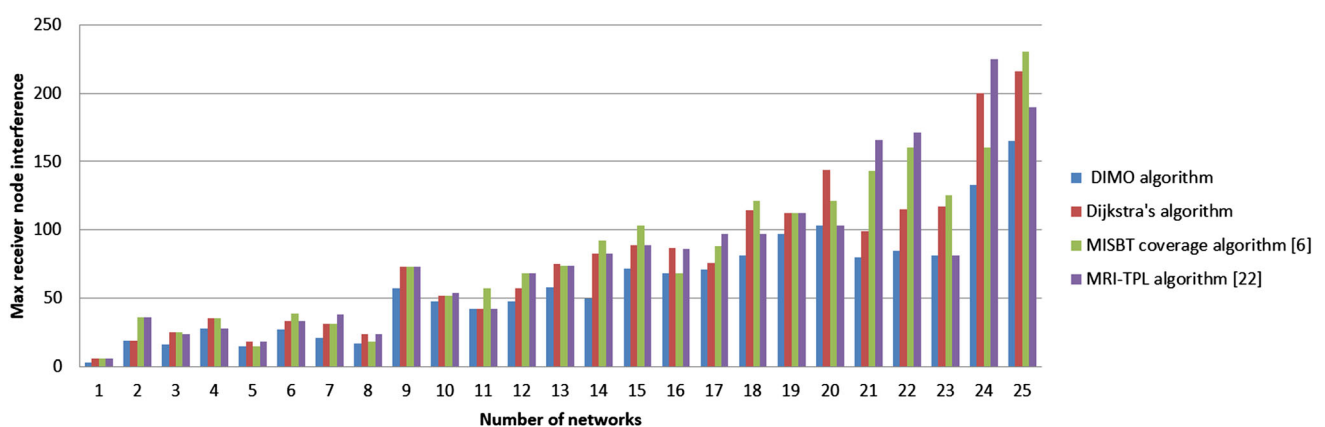
Figure 4 clearly shows that the DIMO algorithm is far better in minimizing max receiver node interference on routing path from the source to the destination node. Though minimization of total link interference by the DIMO algorithm is not as much effective as by the Dijkstra's algorithm yet it is better fulfilling our goal of the DIMR problem as seen from Fig. 5. Plots of Fig. 5 have been derived by subtracting the resultant data, given in Table 2, of the DIMO algorithm from the Dijkstra's algorithm. The difference on positive axis depicts that the DIMO algorithm outperforms the Dijkstra's by how much and difference on negative axis depicts that the DIMO is lacking behind by how much in compare to the Dijkstra's. Therefore it is clearly understood from Fig. 5 that difference of minimized max receiver node interference by the DIMO from the Dijkstra's is much larger than the difference of minimized total link interference by the Dijkstra's algorithm from the DIMO. It can be said that the routing path by the Dijkstra algorithm has a poor balance between the minimization of total link interference and max receiver node interference. Only focus on minimizing total link interference results in higher receiver node interference on the routing path as seen from Table 2, Figs. 4, and 5. Hence DIMO algorithm is more optimal in dealing with the DIMR problem than the Dijkstra's algorithm.

Likewise, Figs. 6 and 7 depict the difference of resultant data (given in Table 2) of the DIMO algorithm from the MISBT coverage algorithm [6] and the MRI-TPL algorithm [22] respectively. Plots of the figures clearly depict that the DIMO algorithm is not only better in minimizing max receiver node interference but also the total link interference of a routing path from the source to the destination node on various networks in comparison of the MISBT, and MRI-TPL algorithms.

Remark 4 The proposed DIMO algorithm outperforms Dijkstra's algorithm, MISBT coverage algorithm [6], and MRI-TPL algorithm [22] in dealing with the DIMR problem.

Table 2 Data obtained on max receiver node interference under I_R model and total link interference under I_{LS} model via simulation

WSNs (nodes, links)	DIMO algorithm			Dijkstra's algorithm		MISBT algorithm, given in [6]		MRI-TPL algorithm, given in [22]	
	α, β, ρ	$\max I_R$	$\sum I_{LS}$	$\max I_R$	$\sum I_{LS}$	$\max I_R$	$\sum I_{LS}$	$\max I_R$	$\sum I_{LS}$
(7,10)	.5,.6,.3	3	4	6	3	6	3	6	3
(8,23)	.9,.4,.2	19	3	19	3	36	7	36	8
(9,22)	.5,.6,.2	16	7	25	4	25	10	24	8
(10,28)	.7,.5,.2	28	6	35	3	35	7	28	6
(11,31)	.6,.4,.2	15	10	18	9	15	10	18	10
(12,40)	.6,.4,.2	27	8	33	5	39	12	33	10
(13,51)	.5,.6,.4	21	9	31	6	31	6	38	9
(14,36)	.5,.3,.3	17	12	24	9	18	12	24	11
(15,58)	.5,.4,.3	57	8	73	5	73	6	73	12
(16,77)	.5,.4,.3	48	11	52	7	52	13	54	15
(17,64)	.6,.4,.3	42	17	42	15	57	28	42	22
(17,94)	.6,.3,.2	48	6	57	5	68	24	68	25
(18,100)	.6,.3,.2	58	17	75	9	74	10	74	10
(19,100)	.7,.4,.2	50	10	83	9	92	12	83	10
(20,124)	.6,.5,.2	72	14	89	12	103	16	89	11
(21,114)	.6,.4,.2	68	6	87	5	68	6	86	8
(22,114)	.5,.2,.2	71	25	76	19	88	26	97	24
(23,149)	.8,.4,.2	81	18	114	17	121	31	97	26
(24,167)	.6,.5,.2	97	8	112	5	112	7	112	7
(25,158)	.8,.4,.4	103	26	144	22	121	42	103	37
(26,150)	.6,.5,.3	80	29	99	21	143	34	166	29
(27,186)	.7,.4,.1	85	23	115	18	160	28	171	44
(28,171)	.8,.3,.1	81	22	117	19	125	23	81	24
(29,207)	.8,.4,.3	133	20	200	17	160	18	225	36
(30,238)	.7,.6,.3	165	24	216	18	230	38	190	41

**Fig. 4** Comparison of max receiver node interference

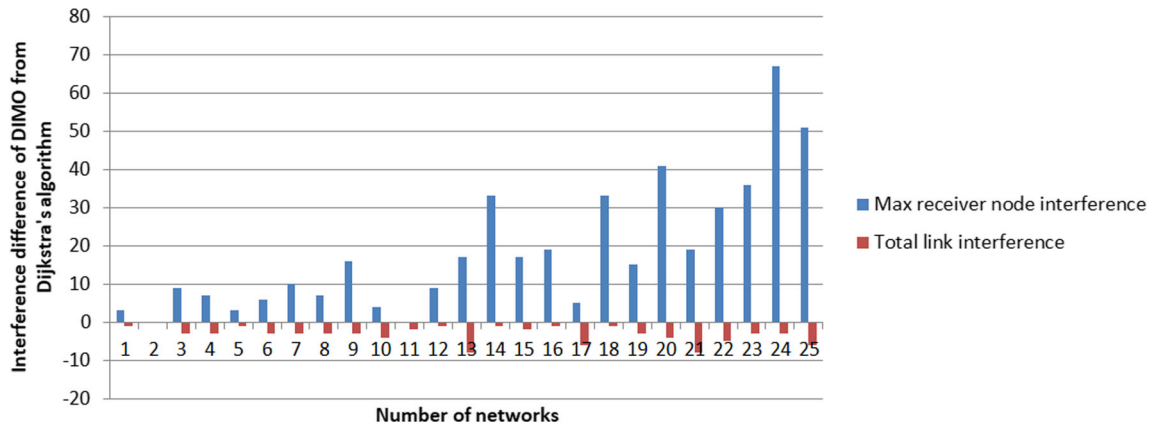


Fig. 5 Difference of reduced max receiver node interference and total link interference of DIMO from Dijkstra's algorithm

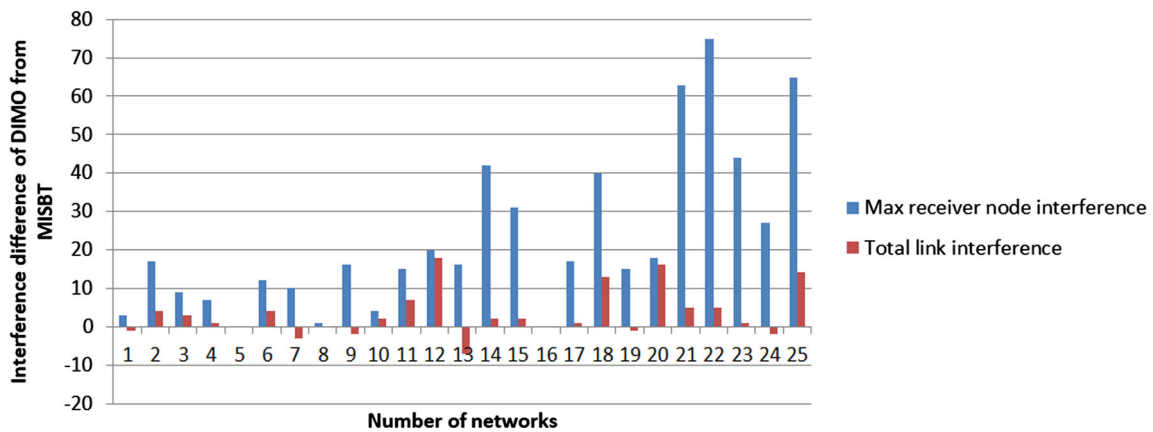


Fig. 6 Difference of reduced max receiver node interference and total link interference of DIMO from MISBT coverage algorithm [6]

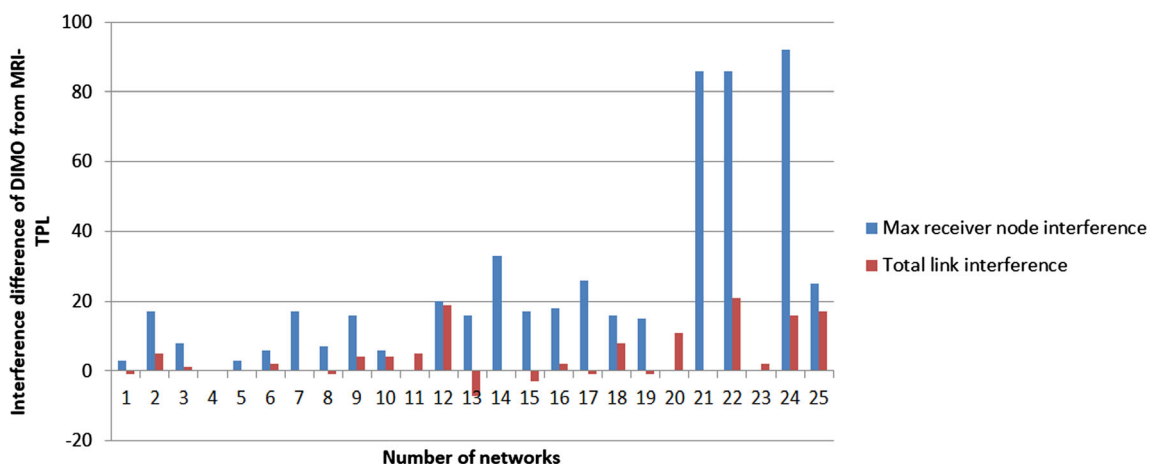


Fig. 7 Difference of reduced max receiver node interference and total link interference of DIMO from MRI-TPL algorithm [22]

4.2 Performance evaluation of the DSD algorithm

The proposed DSD algorithm comprises of the Disc-SUM model with the Dijkstra's algorithm. As Dijkstra's algorithm results in an optimal path of minimum total edge weight so it can be claimed that the total link interference on a routing path from the source to the destination node obtained using the DSD algorithm is optimal as well. Furthermore, minimization of the total link interference on the routing paths are shown by evaluating total transmission cost as link interference reduction directly affects the link's transmission cost [3]. For any bidirectional link uv of Euclidean distance ' d ', link transmission cost has been evaluated as $\|d^2\|$ [6]. The total transmission cost on each routing path by the DSD algorithm has been evaluated and compared with the DIMO algorithm. Likewise, optimality of the max receiver node interference on each routing path by the DSD algorithm has been compared with the DIMO algorithm as DIMO algorithm has been declared as optimal in dealing with the DIMR problem in the previous section.

Table 3 shows the evaluated total transmission cost as well as max receiver node interference on the resultant routing path using DSD and DIMO algorithms. It is observed from Table 3 that in some cases DSD algorithm performs better than DIMO algorithm and vice versa. Therefore, for better analysis of resultant data by the DSD algorithm and the DIMO algorithm on total transmission cost and max receiver node interference, paired sample ' t -test' statistical tool has been used in IBM-SPSS software.

To examine the validation of the DSD algorithm through the ' t -test', two types of hypothesis have to make. The first hypothesis is called the null hypothesis comprises a statement that we want to claim and the other one is called the alternate hypothesis whose statement is against the claim. Presently our claim for a null hypothesis is that the dual-interference minimization by the DSD algorithm is as optimal as the DIMO algorithm. For this to test, the null hypothesis becomes 'mean of data by the DSD algorithm is equal to the mean of data by the DIMO algorithm' and the alternate hypothesis becomes opposite to the null hypothesis i.e. 'mean of data by the DSD algorithm is not equal to

Table 3 Resultant data on total transmission cost and max receiver node interference on routing paths via simulation

WSNs (Nodes, links)	Total transmission cost		max receiver interference	
	DSD algorithm	DIMO algorithm	DSD algorithm	DIMO algorithm
(7, 10)	165	180	3	3
(8, 23)	6	6	19	19
(9, 22)	14	28.53	16	16
(10, 28)	34	30.42	26	28
(11, 31)	211.8	211.8	15	15
(12, 40)	87.5	87.5	27	27
(13, 51)	100	100	21	21
(14, 36)	128.33	102	17	17
(15, 58)	22.45	22.45	57	57
(16, 77)	102	102	48	48
(17, 64)	226	94.8	42	42
(17, 94)	79.6	79.6	48	48
(18, 100)	128.3	128.3	58	58
(19, 100)	51.7	51.7	50	50
(20, 124)	101	98	31	72
(21, 114)	15.5	15.5	68	68
(22, 114)	171	171	71	71
(23, 149)	121.4	121.4	81	81
(24, 167)	19.73	19.73	97	97
(25, 158)	141.73	141.73	103	103
(26, 150)	100.33	144	105	80
(27, 186)	86.53	86.53	85	85
(28, 171)	94.77	94.77	81	81
(29, 207)	76.53	76.53	133	133
(30, 238)	103.64	79.3	165	165

the mean of data by the DIMO algorithm'. Figures 8 and 9 show the paired sample t -test results on total transmission cost and max receiver node interference respectively, obtained in the IBM SPSS. In Fig. 8, resultant p -value in column Sig. (2-tailed) is .436, shows the significance level greater than .05 and so, the null hypothesis is accepted with a 5% level of significance. Hence total transmission cost by the DSD algorithm is as optimal as the DIMO algorithm. Likewise in Fig. 9, resultant p -value in column Sig. (2-tailed) is .716, greater than .05 implies that the null hypothesis is accepted with a 5% level of significance. Hence average max receiver node interference by the DSD algorithm is as optimal as the DIMO algorithm.

Moreover, the effect of interference minimization by the DSD algorithm has been shown on hop counts in Fig. 10. Hop counts are the number of intermediate nodes on a routing path from the source to the destination node, and more the hop counts more is a delay in data delivery. Figure 10 reveals that the routing path by the DSD

algorithm has lesser hop counts than the routing path by the DIMO algorithm in many cases. Hence interference minimization by the DSD algorithm favors delay reduction as well.

Remark 5 DSD algorithm is as effective as DIMO algorithm in minimizing max receiver node interference and total link interference of a routing path from the source to the destination node on bidirectional connected graph models of WSNs.

Remark 6 The significance of using the DSD algorithm for the DIMR problem instead of the DIMO algorithm is lying on the ground of time complexity as the total time complexity of the DSD algorithm is lesser than the DIMO algorithm in dealing with the DIMR problem.

Remark 7 The total time complexity of the DIMO algorithm is $\mathcal{O}(IN \times m \times n^2)$ where IN is number of iterations, m is number of ants and n is number of nodes of a WSN whereas the total time complexity of the DSD algorithm is $\mathcal{O}(n^2)$.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	DSD - DIMO	4.61000	29.10673	5.82135	-7.40467	16.62467	.792	24	.436

Fig. 8 Paired sample t -test of DSD and DIMO algorithms on total transmission cost

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	DSD - DIMO	-.7200	9.7831	1.9566	-4.7583	3.3183	-.368	24	.716

Fig. 9 Paired sample t -test of DSD and DIMO algorithms on max receiver node interference

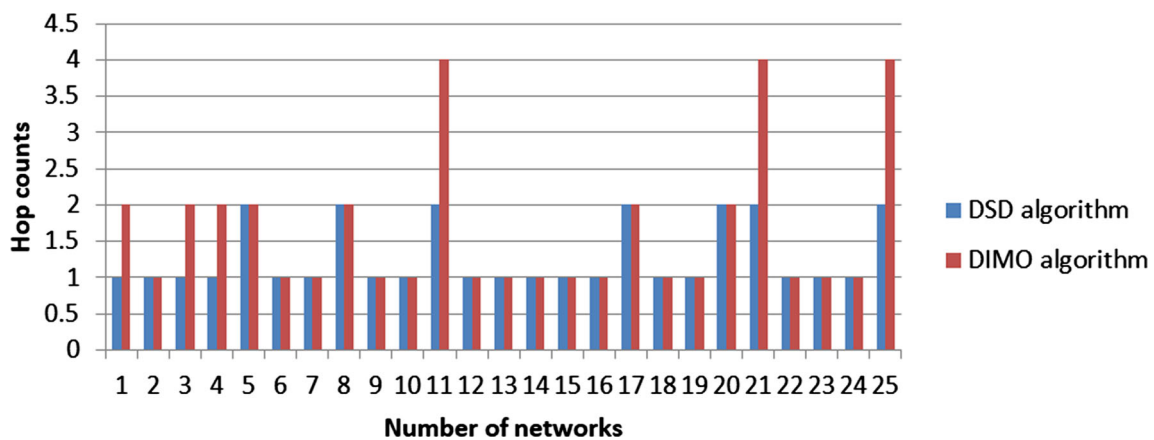


Fig. 10 Comparison of hop counts from the source to the destination node

Remark 8 The effect of interference minimization by the DSD algorithm is seen on reduction in hop counts on a routing path from the source to the destination node. DSD algorithm has lesser hop counts than DIMO algorithm and so, it can be said that DSD algorithm favors delay reduction along with transmission cost on a routing path from the source to the destination node on bidirectional connected graph models of WSNs.

5 Conclusion

In this paper, two new dual-interference minimization routing approaches have been proposed which simultaneously minimize total link interference and max node interference during data transmission from the source to the destination node on bidirectional connected graph models of WSNs. The first approach, named as DIMO algorithm engages ACO technique with two distinct interference models via probability and pheromone update function. This is the very first time that ACO has been introduced with interference models to minimize dual-interference in WSNs. The second approach is based on the newly proposed interference model named as Disc-SUM model which provides new weights to each edge as link interference and use of Dijkstra's algorithm on the weighted graph results in a dual-interference minimum routing path from the source to the destination node in quadratic time complexity. The validation of the proposed techniques and model has been done by carrying out simulations. Data obtained via simulations has been analysed and compared against the algorithms from similar grounds. Analysis on minimization of max receiver node interference, total link interference, total transmission cost, and hop counts have been accomplished graphically and statistical *t*-test tool in IBM SPSS.

The proposed techniques are better in minimizing dual-interference on the routing paths by optimally balancing both interference levels during data transmission. In future, dual-interference minimization would be extended to dynamic routing paths under different interference models or by developing some new suitable models for dynamic WSNs.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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