INDECOMPOSABLE POSITIVE LINEAR MAPS AND BOUND ENTANGLEMENT

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I,(Vimalesh kumar),Roll no. 2k19/mscmat/15 student of M.Sc(Applied Mathematics),hereby declare that the project Dissertation title "INDECOMPOSABLE POSITIVE LINEAR MAPS AND BOUND ENTANGLEMENT" which is submitted by me to the Department of Applied mathematics,Delhi technological university,Delhi is partial fulfillment of the requirement for the award of the degree of Master of science ,is original and not copied from any other source without proper citation.This work has not previously formed the basis for the award of any Degree,Diploma Associateship,Fellowship or other similar title or recognition.

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CERTIFICATE

This is to certify that the project "INDECOMPOSABLE POSITIVE LINEAR MAPS AND BOUND ENTANGLEMENT " submitted by Vimalesh Kumar is based upon their own work under the supervision of **Mr.ROHIT KUMAR** in the Department of Applied Mathematics,Delhi Technological University (formerly Delhi College of Engineering) that neither the project report nor any part of it has been submitted anywhere else before.

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ABSTRACT

We introduce a family of linear maps from M_2 to M_4 and find,under what conditions the map is ,'Positive and Decomposable','Positive and Indecomposable',and also find 'the constructed indecomposable map detects bound entangled states'.

Contents

1 A Short Introduction to Information Theory

The main goal of this dissertation is the construction of a family of linear map and determine the conditions where the map is Positive Decomposable , positive Indecomposable and the constructed indecomposable map detects bound entangled states.
Throughout this paper $M_{m\times n}(\mathbb{C})$ denotes the space of $m \times n$ matrices with complex entries.We use $M_n(\mathbb{C})$ instead of $M_{n\times n}(\mathbb{C})$ i.e $M_n(\mathbb{C})$ denotes the set of all $n \times n$ complex matrices.I_n denotes the identity matrix of $M_n(\mathbb{C})$ or simply say I and $M_m(M_n)$ denotes the set of all $m \times m$ block matrices with each block in M_n.
For a matrix $A \in M_{m\times n}(\mathbb{C})$, A^T denotes the transpose of A and A^* denotes the transpose of the complex conjugate of matrix A.
The set of all positive matrices of M_n is denoted by M_n^+ and $M_m(M_n)$ can also be denoted by M_{mn} with M_{mn}^+ denotes the set of all positive block matrices.

1.1 A Brief History of Quantum Information Theory

The story of quantum information and quantum computation begin at 20^{th} century.The concept of this theory was developed in the 20^{th} century when classical physics was turned into quantum physics.The quantum mechanics was developed by Schrodinger using wave mechanics and Heisenberg using matrix mechanics.The classical physics theory was predicting idiocy such as ultraviolet catostrophe.In the classical physics at first these problem were brushed aside by adding ad hoc hypothesis,it become a new theory called the theory of quantum mechanics.
The modern theory of quantum mechanics is the creation of resoltent and panic after a quarter century at $1920s$.The structure of DNA,Super conductors,Nuclear fusion in stars,The structure of a atom,The elementary particle of nature and inside the sun are included in quantum mechanics,then we say that quantum mechanics has been a most expensive part of science and has been included most of every things.
Now arise a question,what's in the quantum mechanics or what's the quantum mechanics,any group of rules for the making of physical theories or

the mathematical structure is called quantum mechanics.For example,in the about of quantum electrodynamics is a physical theory,is is characterize with singular purity the interplay of atoms and light.The framework of quantum mechanics is by the built of quantum electrodynamics but it contains specific rule which does not characterize by quantum mechanics.The basic rule of quantum mechanics are quite easy for evaluation but even expert find them counterintuitive.

Albert Einstein is the best reviewer of quantum mechanics.The aim of quantum information and quantum computation is to develop the tools which will be more effective and sharpen in out intuition about quantum mechanics.For example,in the early 1980s ,this is the possible that use of quantum effects to single faster that light.The determination of this problem turn out to blancmange an if only ,it is possible "clone" an unknown quantum state.After in the 1980s the clone has been called superposition state of quantum state,if this clone is possible to a single faster than light using quantum effects.Seeing this situation in the early 1980s, a result come out to in front of the world scientist called "no-cloning theorem" of quantum information and quantum computation.

In 1970s,"The complete control over single quantum system" is the historical development of quantum information and quantum computation.For example,a super quantum mechanical explanation system is in the "superconductor" having a big sample of conducting metal.In this time so many technologies have been developed for controlling single quantum system.For example in 1970s developed the "atom trap system",in this system developed for trapping a single atom.In the 1930s and 1940s was the invention of radio astronomy ,a big discoveries in this time, quasars,pulsars and the galactic core of the Milki way galaxy.In this time the developed the system of finding the ways to lower the temperature of different systems,across the long distance communication a secrete way of communication is the "quantum cryptography" .

In 1750$B.C$ the Babylonians had inventioned a few justly complex algorithmic ideas.In 1936 a paper had published by the great mathematician "Alan Turing".they had developed a programmable computes called "Turing Machine".

1.2 Quantum Information Theory in Future

Quantum information theory is in the running process in this current time.Firstly arise a question ,whats the future of quantum information and quantum computation.How to use the quantum information and quantum computation in the future to develop the science ,technology and humanity? What are the key role of open problems in quantum computation and quantum information.We will build some terribly temporary remarks concerning these over arching queries before moving onto additional elaborate investigation is the challenging subjects for computer scientist ,physicist ,mathematician etc in current time.

I observe that the future of quantum computation and quantum information will be bright and clean.In future we shall do very fast information processing similar to speed of light or faster than light i.e we shall send a information from source to sink by the speed ,faster than light,if it happens ,then in the medical field we can do better treatment with better equipment and found the fastest result from the quantum computation,the journey of universe will be possible and if all will good in future,then we can do time travel.And a one of the most important topic is that quantum security is the highest security level can't possible to hacked it.

In the big level we have learned that any physical theory not just quantum mechanics ,it may be used as the important key of information processing and communication.Quantum computation and quantum information are certainly challenging to physicists,but it is probably a small sensitive what quantum information and quantum computation offers to physics in the future or in the long time.

1.3 Quantum bits and Dirac Notation

The bit is that the basic construct of classical computation and classical information,i.e in classical information 0 and 1 are two states which we called the bits is in the modern day computers.In the other manner,we can introduce a basic unit of quantum computation,this basic unit is called quantum bit in short "qubit" of information in quantum computation,while a qubit has kept a most important speciality called supper position .

Like a bit , a qubit has in one of two states in the case of a cubit,we level the states 0 and 1 by $|0\rangle$ and $|1\rangle$.In quantum theory any object enclosed by the notation $|\rangle$ is called a state,a vector or a ket.

Firstly arise a question,what's the difference between a qubit and a bit? if we talk about a bit,a bit is use in ordinary computer,it is in state 0 or state 1,and if we talk about a qubit ,the state $|0\rangle$ or the state $|1\rangle$ is in a qubit but a qubit can also exist a super states called a super position state.The super position state is the linear combination of the given states $|0\rangle$ and $|1\rangle$ i.e

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1.1}$$

Where α and β are complex number and the state $|\phi\rangle$ called the supper position state,while a qubit can exist in a super position state then if we measured a qubit it is only in the state $|0\rangle$ or state $|1\rangle$. The probability of finding a qubit in state $|0\rangle$ or state $|1\rangle$ is modulus squre of α and β from (1.1). i.e
The probability of finding $|\phi\rangle$ in state $|0\rangle$ is $|\alpha|^2$.And
The probability of finding $|\phi\rangle$ in state $|1\rangle$ is $|\beta|^2$
and the total probability of finding $|\phi\rangle$ in state $|0\rangle$ and $|1\rangle$ is,

$$|\alpha|^2 + |\beta|^2 = 1 \tag{*}$$

Since α and β are complex numbers ,then $|\alpha|^2$ and $|\beta|^2$ can be written as,

$$|\alpha|^2 = \alpha\alpha^* \text{ and } |\beta|^2 = \beta\beta^* \tag{1.2}$$

Where α^* and β^* are the complex conjugate of α and β respectively.

For example,Suppose $|\phi_1\rangle = \frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$ and $|\phi_2\rangle = \frac{i\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ What is the probability of finding $|\phi_1\rangle$ and $|\phi_2\rangle$ in state $|0\rangle$ and $|1\rangle$?
Let us suppose that,$|\phi_1\rangle = \frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$ Now from (1.1)

$$\alpha = \frac{\sqrt{2}}{\sqrt{3}} \text{ and } \beta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow |\alpha^2| = \frac{2}{3} \text{ and } |\beta^2| = \frac{1}{3}$$

And, suppose that $|\phi_2\rangle = \frac{i\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ Now from (1.1)

$$\alpha = \frac{i\sqrt{3}}{2} \text{ and } \beta = \frac{1}{2}$$
$$\Rightarrow |\alpha^2| = \frac{3}{4} \text{ and } |\beta^2| = \frac{1}{4}$$

All the information about a system is stored in a state vector. That is called a ket. Kets belong to Hilbert space. There exits some elements which give real or complex numbers, while taking inner products with the ket vectors.These elements are said to belong to bra space. For every ket $|\psi\rangle$ there exists a unique bra $\langle\psi|$. The scalar product (ϕ, ψ) is denoted by the bra-ket $\langle\phi|\psi\rangle$

$$(\phi, \psi) \rightarrow \langle\phi|\psi\rangle \tag{1.3}$$

The wave functions are basically the projections of ket vectors on position or momentum axis.

$$\langle\overrightarrow{r}, t|\psi\rangle = \psi(\overrightarrow{r}, t) \tag{1.4}$$

In the coordinate representation, the scalar product $\langle\phi|\psi\rangle$ is given by

$$\langle\phi|\psi\rangle = \int \phi^*(\overrightarrow{r}, t)\psi(\overrightarrow{r}, t)d^3r \tag{1.5}$$

1.4 Quantum information theory with example

The name "Quantum information" is used in this world in two different ways in scope of quantum information and quantum computation.The first use of quantum information in a broad catch all for operations that related to information processing in the field of quantum mechanics.This is use in such subjects as quantum teleportation ,quantum computation,the no-cloning theorem etc.

The second use of quantum information is the study of elementary quantum information processing .Quantum algorithm designed does not include typically in elementary quantum information.In the beyond of the scope of elementary are the details of quantum algorithm for the tackling of confusion we will use "quantum information theory" in parallel.The term classical information theory to characterize the classical world .In such a way the tentative presentation of the elementary processes studied by quantum information theory is of the most interested.

The main purpose of quantum information theory is that the introduction of basic structure .This subject is currently under development and its not to clear that how all pieces fit together.Now we introduce a some fundamental goals during work on quantum information theory,

(i)Identify some elementary definitions in quantum information

(ii)Identifying the linear algebra in quantum information theory

(iii)Construction of a family of linear map and find the condition where the map is positive ,decomposable and indecomposable.
The shannon's noisless channel coding theorem and shannon's noisy channel coding theorem are the fundamental results of classical information theory.Now What's the information source? Well,the main problem of the classical and quantum information theory is the information source.
For example,how we can handle this type of problem,what if the source had the state $|0\rangle$ with probability p and the state $\frac{(|0\rangle+|1\rangle}{2}$ with probability $1-p^2$ by the standard classical information theory.it is not possible for us to determine this problem.
The quiet channel coding theorem quantifies what number bits square measure needed to store data being emitted by a supply of knowledge, whereas the shouting channel committal to writing theorem quantifies what quantity information will be faithfully transmitted through a loud communications channel.

2 Introduction to Linear Algebra in Quantum Information Theory

2.1 Linear Algebra

Let V be set of elements (vectors) a_1, a_2,a_3 , $\cdots$ is said to be a vector space V along with an addition and a scalar multiplication on V, if the following properties holds:
(i) Commutative under addition:

$$a_i + a_j = a_j + a_i \text{ for all } a_i, a_j \in V \quad (2.1)$$

(ii) Associative under addition:

$$(a_i + a_j) + a_k = a_i + (a_j + a_k) \text{ for all } a_i, a_j and a_k \in V \quad (2.2)$$

(iii) Associative under multiplication

$$(\alpha\beta)a_i = \alpha(\beta a_i) \text{ for all i and all } \alpha, \beta \in F(scalarfield) \quad (2.3)$$

(iv) Additive identity

$$\exists 0 \in V \text{ such that } a_i + 0 = a_i \text{ for all } a_i \in V \quad (2.4)$$

(v) Additive inverse

$$\text{For every } a_i \in V \text{ ,there exist } a_j \in V \text{ such that } a_i + a_j = 0 \quad (2.5)$$

(vi) Multiplicative identity

$$1a_i = a_i \text{ for all } a_i \in V \quad (2.6)$$

(vii) Distributive properties

$$\text{Left distributive: } \alpha(a_i + a_j) = \alpha a_i + \alpha a_j$$
$$\text{Right distributive: } (\alpha + \beta)a_i = \alpha a_i + \beta a_i \text{ for all } \alpha, \beta \in F \text{ and all } a_i, a_j \in V \quad (2.7)$$

2.1.1 Linear Operators and Pauli Matrices

Generally an operator is mathematical tool that can be apply on a function it gives a another. In this manner the operator Λ is a mathematical rule that transforms a ket ϕ to another ket ψ say:

$$\Lambda|\phi\rangle = |\psi\rangle \quad (2.8)$$

the operator Λ can also act on bra vector.

$$\Lambda\langle\mu| = \langle\nu| \quad (2.9)$$

if the following condition hold then operator Λ is called linear operator,let α and β are complex numbers and the state vectors are $|\phi_1\rangle$ and $|\phi_2\rangle$ then the condition is,

$$\Lambda(\alpha|\phi_1\rangle + \beta|\phi_2\rangle) = \alpha(\Lambda|\phi_1\rangle) + \beta(\Lambda|\phi_2\rangle) \quad (2.10)$$

Using the above equation the Identity operator and the Zero operator are I and 0 respectively,

$$I|\phi\rangle = |\phi\rangle \text{ and } 0|\phi\rangle = 0 \quad (2.11)$$

in quantum computation the pauli operators a set of operators that turn out to fundamental importance.there are four pauli operators including identity operators denoted by I,X,Y and Z.Now we operate the pauli operators on initial state vectors $|0\rangle$ and $|1\rangle$ then,

$$I|0\rangle = |0\rangle \text{ and } I|1\rangle = |1\rangle$$
$$X|0\rangle = |1\rangle \text{ and } X|1\rangle = |0\rangle$$
$$Y|0\rangle = -i|1\rangle \text{ and } Y|1\rangle = i|0\rangle$$
$$Z|0\rangle = |0\rangle \text{ and } Z|1\rangle = -|1\rangle$$

And the pauli matrices are,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2.12}$$

2.1.2 Inner product

Let H_n be the $n - dimensional$ Hilbert space. We will use the conventional bra and ket notations in quantum information theory i.e the ket notation of a vector ψ in H_n is $|\psi\rangle \in H_n$ and bra notation is $\langle\psi| \in H_n$ .The bra notation $\langle\psi|$ is the Hermitian conjugate of ψ i.e ψ^*.
Now the complex inner product between ψ and ϕ in H_n is

$$\langle\psi|\phi\rangle = \psi^*\phi \tag{2.13}$$

Now, $\langle\psi|$ is called normal vector if the inner product of $\langle\psi|$ is equal to 1 i.e

$$\langle\psi|\psi\rangle = 1 \tag{2.14}$$

Now,let us define the matrix form,

$$\rho = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i| \tag{2.15}$$

Where $|\psi_i\rangle$ are the normalised eigenvector of ρ and $\lambda_i \geq 0$ are the eigenvalues. When ρ has trace equal to 1,then the matrix ρ become a density matrix,and if a density matrix ρ has rank 1 then ρ is called a pure state and can be written as

$$\rho = |\psi\rangle\langle\psi| \tag{2.16}$$

Example-1 First you construct the basis for C^4 by using the state vectors $|+\rangle$, $|-\rangle$, and show that the constructed basis are orthonormal.(example from $DavidMcMahonQuantumComputingExplain$)

Let H_1 and H_2 are two Hilbert spaces for quantum bits. Firstly we introduced the basis of $H = H_1 \otimes H_2$.
Since we know that $|0\rangle, |1\rangle$ is the basis of each of qubits. Since all possible products of the basis states for H_1 and H_2 formed the basis of $H = H_1 \otimes H_2$. Now,the basis vectors are

$$|v_1\rangle = |0\rangle \otimes |0\rangle$$
$$|v_2\rangle = |0\rangle \otimes |1\rangle$$
$$|v_3\rangle = |1\rangle \otimes |0\rangle$$
$$|v_4\rangle = |1\rangle \otimes |1\rangle$$

The basis states $|a_i\rangle$ (say) of H_1 and The basis states $|b_i\rangle$ (say) of H_2 , Now the expansion of an arbitrary state vector of H_1 and the expansion of an arbitrary vector from H_2 in terms of the basis states $|a_i\rangle$ (say) and $|b_i\rangle$ (say) of H_1 and H_2 respectively. then,

$$|\phi\rangle = \sum_i \alpha_i |a_i\rangle \text{ and } |\chi\rangle = \sum_i \beta_i |b_i\rangle \tag{2.17}$$

And the vector $|\psi\rangle = |\phi\rangle \otimes |\chi\rangle \in H$ then,

$$|\psi\rangle = \sum_{i,j} \alpha_i \beta_j |a_i\rangle \otimes |b_j\rangle \tag{2.18}$$

Now if the vector $|\psi\rangle = |\phi\rangle \otimes |\chi\rangle$ is a tensor product, then the components of $|\psi\rangle$ are multiplying by the components of the two vectors $|\phi\rangle$ and $|\chi\rangle$ to form the tensor product.
Then basis states of $H = C^4$ are given below (using $|+\rangle$, $|-\rangle$) as the basis for H_1 and H_2. we have

$$|v_1\rangle = |+\rangle \otimes |+\rangle$$
$$|v_2\rangle = |+\rangle \otimes |-\rangle$$
$$|v_3\rangle = |-\rangle \otimes |+\rangle$$
$$|v_4\rangle = |-\rangle \otimes |-\rangle$$

If the basis is orthonormal, then we must have $\langle v_1|v_1\rangle = \langle v_2|v_2\rangle = \langle v_3|v_3\rangle = \langle v_4|v_4\rangle = 1$ and $\langle v_i|v_j\rangle$ where $i \neq j$ is equal to zero. We consider $\langle v_1|v_1\rangle$,$\langle v_2|v_2\rangle$, and $\langle v_1|v_2\rangle$, and

$$\langle v_1|v_1\rangle = (\langle +|\langle +|)(|+\rangle|+\rangle) = \langle +|+\rangle\langle +|+\rangle = (1)(1) = 1$$
$$\langle v_2|v_2\rangle = (\langle +|\langle -|)(|+\rangle|-\rangle) = \langle +|+\rangle\langle -|-\rangle = (1)(1) = 1$$
$$\langle v_1|v_2\rangle = (\langle +|\langle +|)(|+\rangle|-\rangle) = \langle +|+\rangle\langle +|-\rangle = (1)(0) = 0$$
$$\langle v_2|v_1\rangle = (\langle +|\langle -|)(|+\rangle|+\rangle) = \langle +|+\rangle\langle -|+\rangle = (1)(0) = 0$$

2.2 Tensor product of state vectors with example

let $|\phi_1\rangle \in H_1$ and $|\phi_2\rangle \in H_2$ be two state vectors which are belongs to H(Hilbert space).
Now let Λ is an operator that acting on the state vector $|\phi_1\rangle \in H_1$, and let

Γ is an operator that acting on the state vector $|\phi_2\rangle \in H_2$. Now we define the operator $\Lambda \otimes \Gamma$ that acting on the state vectors $|\rangle \in H$ by,

$$(\Lambda \otimes \Gamma)|\phi\rangle = (\Lambda \otimes \Gamma)(|\phi_1\rangle \otimes |\phi_2\rangle) = (\Lambda|\phi_1\rangle) \otimes (\Gamma|\phi_2\rangle) \qquad (2.19)$$

Example-2. Let $|\phi\rangle = |i\rangle \otimes |j\rangle$ and $\Lambda|i\rangle = i|i\rangle$, $\Gamma|j\rangle = j|j\rangle$. find the value of $\Lambda \otimes \Gamma|\phi\rangle$?

Since we know that from equation(2.19) $|\phi\rangle$ can be written as,

$$\Lambda \otimes \Gamma|\phi\rangle = (\Lambda \otimes \Gamma)(|i\rangle \otimes |j\rangle)$$

Then we using (2.19) then the operators:

$$\Lambda \otimes \Gamma|\phi\rangle = (\Lambda \otimes \Gamma)(|i\rangle \otimes |j\rangle) = \Lambda|i\rangle \otimes \Gamma|j\rangle$$

$\Rightarrow$ we use $\Lambda|i\rangle = i|i\rangle$, $\||j\rangle = j|j\rangle$ to write

$$\Lambda|i\rangle \otimes \Gamma|j\rangle = i|i\rangle \otimes j|j\rangle$$

Since we know that $|\phi\rangle \otimes (\alpha|\chi\rangle) = \alpha|\phi\rangle \otimes |\chi\rangle$,where α is an scalar.

$$\Rightarrow i|i\rangle \otimes j|j\rangle = ij(|i\rangle \otimes |j\rangle) = ij|\phi\rangle$$

Hence,

$$\Lambda \otimes \Gamma|\phi\rangle = ij|\phi\rangle$$

2.3 The density operator

firstly we start the density operator at single state vector say $|\phi\rangle$,Now if there exist the orthonormal basis $|u_1\rangle$,$|u_2\rangle$,$\cdots$,$|u_n\rangle$ then the state vector $|\phi\rangle$ can be written as,

$$|\phi\rangle = \alpha_1|u_1\rangle + \alpha_2|u_2\rangle + \cdots + \alpha_n|u_n\rangle \qquad (2.20)$$

Where α_1,α_2,$\cdots$,α_n are the complex numbers.By using the Born rule ,since the probability of state $u_i\rangle$ is $|\alpha_i|^2$.if the system is in the $definitestate$,we say the system is in the $purestate$.The density operator is an average operator and it is describe to statical mixture ,it is denoted by ρ.Now,
The average value of a operator Λ.we write,

$$\langle\Lambda\rangle = \langle\phi|\Lambda|\phi\rangle \qquad (2.21)$$

By using (2.20),

$$\langle\Lambda\rangle = \langle\alpha_1|u_1\rangle + \alpha_2|u_2\rangle + \cdots + \alpha_n|u_n|\Lambda|\alpha_1|u_1\rangle + \alpha_2|u_2\rangle + \cdots + \alpha_n|u_n\rangle$$

$$= (\alpha_1^*|u_1\rangle + \alpha_2^*|u_2\rangle + \cdots + \alpha_n^*|u_n)\Lambda(\alpha_1|u_1\rangle + \alpha_2|u_2\rangle + \cdots + \alpha_n|u_n)$$

$$= \sum_{i,j=1}^n \alpha_i^*\alpha_j\langle u_i|\Lambda|u_j\rangle$$

$$= \sum_{i,j=1}^n \alpha_i^*\alpha_j\Lambda_{ij} \qquad (2.22)$$

Where the coefficients of above expansion,i.e

$$\alpha_k = \langle u_k|\phi\rangle$$

and there complex conjugate $\alpha_k^* = \langle\phi|u_k\rangle$ then we can write

$$\alpha_i^*\alpha_j = \langle\phi|u_i\rangle\langle u_j|\phi\rangle = \langle u_j\rangle|(|\phi\rangle\langle\phi|)u_i\rangle \qquad (2.23)$$

Now let us the density operator is given the above equation (2.23) and it is denoted by $\rho = |\phi\rangle\langle\phi|$.So the average value of an operator Λ with respect to a state vector ϕ can be written as

$$\langle\Lambda\rangle = \sum_{i,j=1}^n \alpha_i^*\alpha_j\Lambda_{ij}$$

$$= \sum_{i,j=1}^n\langle u_j\rangle|(|\phi\rangle\langle\phi|)u_i\rangle\Lambda_{ij}$$

$$= \sum_{i,j=1}^n\langle u_j\rangle|\rho|u_i\rangle\Lambda_{ij} \qquad (2.24)$$

If $\rho = |\phi\rangle\langle\phi| = 1$ then the density operator is in the $Purestate$

3 Decomposable and Indecomposable Map

3.1 Positive Complex Matrix

Let A be a $m \times n$ complex matrix, matrix A is positive if and only if $A = A^*$ i.e A is hermitian with non-negative eigenvalues.

Example-3. Let A be a 2×2 matrix with complex entries i.e

$$A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

Now,

$$A^* = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

$\Rightarrow$ A is hermitian . And now,
The eigenvalues of A are 0 and 2 which is non negative.Hence A is positive matrix with complex entries.

3.2 The Linear Positive Map

Let $\Phi : M_n(\mathbb{C}) \to M_m(\mathbb{C})$ be a linear map, the map Φ is positive if and only if $\Phi(A)$ is positive for all positive matrix A in $M_n(\mathbb{C})$.

A linear map $\Phi : M_n \to M_m$ satisfies the given condition

$$[A_{i,j}]_{i,j=1}^k \geq 0 \Rightarrow [\Phi(A_{i,j})]_{i,j=1}^k \geq 0 \tag{3.1}$$

then Φ is called the $k - positive$ linear map if for $[A_{i,j}]_{i,j=1}^k \in M_{mn}$,

3.2.1 The Completely Positive Map

Let I_p be the identity map on $M_p(\mathbb{C})$.We define the map $I_p \otimes \Phi : M_{pn} \to M_{pm}$ by,

$$I_p \otimes \Phi[(A_{jk})_{1 \leq j,k \leq p}] = [\Phi(A_{jk})]_{1 \leq j,k \leq p} \tag{3.2}$$

$\Rightarrow \Phi$ is completely positive if and only if $I_p \otimes \Phi$ is positive for all positive integer p.

$\Phi : M_n(\mathbb{C}) \to M_m(\mathbb{C})$ be completely co-positive if $A^T o \Phi$ is completely positive.

Example-4 Let us define the linear map $\Phi : M_2(\mathbb{C}) \to M_4(\mathbb{C})$ by,

$$\Phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \tag{3.3}$$

To show that this map is positive.

$$\text{suppose } \Phi(A) = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \text{ where } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then the eigenvalues of $\Phi(A)$ are a and d.
$\Rightarrow \Phi$ is positive.And if a,b,c and d are real number then consider a,b,c and d are all positive.

3.3 Decomposable and Indecomposable Linear Map

If a linear map can be express in the sum of a completely positive linear map and a completely co-positive linear map then the linear map called decomposable map.

From choi[3] result, a positive linear map $\Phi : M_n(\mathbb{C}) \to M_m(\mathbb{C})$ is decomposable if and only if there exist $n \times m$ matrices V_i and U_j such that,

$$\Phi(A) = \textstyle\sum_i V_i^* A V_i + \sum_j U_j^* A^T U_j \tag{3.4}$$

for every $A \in M_n(\mathbb{C})$.
If Φ in not decomposable then called indecomposable.

Example-5. Let us define the linear map $\Phi : M_2(\mathbb{C}) \to M_4(\mathbb{C})$ by,

$$\Phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

To show that this map is decomposable.

$$\text{suppose } \Phi(A) = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \text{ where } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then the eigenvalues of $\Phi(A)$ are a and d.

$\Rightarrow \Phi$ is positive.And if a,b,c and d are real number then consider a,b,c and d are all positive.

By the (3.4), then there exist 2×4 matrices ,

$$V_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, V_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, U_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, U_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$U_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

such that,

$$\Phi(A) =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} +$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} +$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence $\Phi(A)$ is positive and decomposable linear map.

3.4 Partial transpose and Positive partial transpose

Now we tend to introduce what is the definition of the partial transpose(PT) and partial traces. For any $A = [A_{i,j}]_{i,j=1}^{k} \in M_{kn}$, the standard transpose of A is defined by,

$$A^T = \begin{bmatrix} A_{1,1}^T & \cdots & A_{k,1}^T \\ \vdots & \ddots & \vdots \\ A_{1,k}^T & \cdots & A_{k,k}^T \end{bmatrix} \tag{3.5}$$

Now let us define the PT of A by,

$$A^\tau = \begin{bmatrix} A_{1,1} & \cdots & A_{k,1} \\ \vdots & \ddots & \vdots \\ A_{1,k} & \cdots & A_{k,k} \end{bmatrix} \tag{3.6}$$

It is obvious that $A \geq 0$ doesn't essentially $\Rightarrow A^\tau \geq 0$. If each of this A and A^τ are positive semi-definite, then A is called the positive partial transpose. It is to be noted that analyzable maps cannot sight Positive Partial Transpose entangled density operators(ρ). Since we know that a density operator ρ is alleged to the Positive Partial Transpose(PPT) if $(I \otimes T)\rho \geq 0$ wherever T is that the transpose and I is the identity. Therefore, indecomposable maps should sight a minimum of one Positive Partial Transpose entangled density operator.

Now as well, if a linear map satisfies the given condition $Tr[\Phi(X)] = Tr[X]$ for all $X \in M_n$ then a linear amp is called trace preserving. And also if a linear map satisfies the given $\Phi(X)^+ = \Phi(X^+)$ for all $X \in M_n$ then linear map is called hermiticity preserving. Now let us a linear map $\Phi : M_n \to M_n$, and the map $\Phi^+ : M_n \to M_n$ outlined by the relation $\langle \Phi^+(X), Y \rangle = \langle X, \Phi(Y) \rangle$ for any operator $X, Y \in M_n$,then the map Φ is positive iff the map Φ^+ is as well positive.

4 Quantum Entanglement

Firstly the foremost uncommon and interesting aspects of quantum physics is that the truth that elements or systems will be entangled. For the best 2

quantum elements case, Now we prefer to denote the elements X and Y. And if the given elements X and Y are area unit entangled, then the meaning of that the above sentence is the values of sure properties of Quantum element X area unit correlate and with the values of sure properties can assume for Quantum element Y. This properties will become correlated and the 2 quantum elements area unit in a that way relates to space and the position area and size divided head most to the stage horrible action at a margin.
The main part of this idea return a protracted method — Now ltes comeback to the year 1935 when Einstein and 2 colleagues, Podolsky and Rosen (now usually referred to as EPR), kept a paper titled "Can the quantum-mechanical description of reality be considered complete?" in front of the world. In This paper title — written by the quantum challenger Einstein was actually performed to point out that scientific theory is not complete and to form irrelevant sortilege.
A core worth command by EPR and alternative "true litigants" was that the properties of substantial systems have certain values (an purpose entity) whether or not you execute the element or not. otherwise to mention this is often that a given property of a system contains a sharply defined worth before a mensuration is created.

Not all states $|\phi\rangle \in H_A \otimes H_B$ area unit entangled. once 2 systems area unit entangled, the state of every composite system will solely be delineate with relevance the opposite state. If 2 states aren't entangled, we are saying that they're a product state or divisible. If $|\phi\rangle \in H_A$ and $|\psi\rangle \in H_B$ and $|\chi\rangle = |\phi\rangle \otimes |\psi\rangle$, then $|\chi\rangle$ could be a product state. Now form the states in C^4 is that the following: Let

$$|\phi\rangle = \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} \tag{4.1}$$

If $ps = qr$ then this state is separable .
Example-6 Is the state $H \otimes H|00\rangle$ entangled? where H is Hadamard matrix.

Since the Hadamard matrix is given by,

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{4.2}$$

Then,

$$H \otimes H = \frac{1}{\sqrt{2}} \begin{bmatrix} H & H \\ H & -H \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{4.3}$$

Now the representation of state $|00\rangle$,

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4.4}$$

$\Rightarrow ps = (1)(0) = 0 = qr$ so $|00\rangle$ is clearly a product state by (4.1)

Now,

$$H \otimes H|00\rangle = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{4.5}$$

By using (4.1),

$$ps = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \tfrac{1}{4}$$

$$qr = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \tfrac{1}{4}$$

$$\Rightarrow ps = qr$$

$\Rightarrow$ this is also a product state.

5 Construction a Family of Linear Map

5.1 Construction a family of linear map and find the condition where the map positive and decomposable

We construct the linear map $\Phi : M_2(\mathbb{C}) \to M_4(\mathbb{C})$ define by,

$$\Phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & -b & 0 & 0 \\ -c & a & 0 & 0 \\ 0 & 0 & d & b \\ 0 & 0 & c & a \end{bmatrix}$$

To show that this map is positive and decomposable when $b = c = 0$.

suppose $\Phi(A) = \begin{bmatrix} d & -b & 0 & 0 \\ -c & a & 0 & 0 \\ 0 & 0 & d & b \\ 0 & 0 & c & a \end{bmatrix}$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then the eigenvalues of $\Phi(A)$ are a and d when either $b = 0$ or $c = 0$.
$\Rightarrow \Phi$ is positive when either $b = 0$ or $c = 0$.And if a,b,c and d are real number then consider a,b,c and d are all positive.
By the *choi*[3] result,a positive linear map $\Phi : M_2(\mathbb{C}) \to M_4(\mathbb{C})$ is decomposable if and only if there exist 2×4 matrices V_i and U_j such that,

$$\Phi(A) = \textstyle\sum_i V_i^* A V_i + \sum_j U_j^* A^T U_j$$

for every $A \in M_2(\mathbb{C})$.

So, if $b = c = 0$ then there exist 2×4 matrices ,

$V_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
$U_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$,$U_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $U_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,
$U_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

such that,

$\Phi(A) =$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} +$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence $\Phi(A)$ is positive and decomposable when $b = c = 0$.

5.2 Construction a family of linear map and find the condition where the map positive and indecomposable

We construct the linear map $\Phi : M_2(\mathbb{C}) \to M_4(\mathbb{C})$ define by,

$$\Phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} \mu d & -b & 0 & c \\ -b & -a & 0 & 0 \\ 0 & 0 & d & b \\ c & 0 & b & -\mu a \end{bmatrix} \text{ where } 0 < \mu < 1 \tag{5.1}$$

To show that this linear map is positive when $\mu^2 > \frac{b^4 - ac^2 d}{a^2 d^2}$ and indecomposable when $0 < x < 1$ where $x = \frac{1}{1-\mu^2}$

$$\text{Suppose } \Phi(A) = \begin{bmatrix} \mu d & -b & 0 & c \\ -b & -a & 0 & 0 \\ 0 & 0 & d & b \\ c & 0 & b & -\mu a \end{bmatrix} \text{ where } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\Rightarrow$ the determinant of upper left $k \times k$ ($1 \leq k \leq 4$) sub matrices of $\Phi(A)$

are given below,

μd , $\begin{vmatrix} \mu d & -b \\ -b & -a \end{vmatrix} = -\mu a d - b^2$, $\begin{vmatrix} \mu d & -b & 0 \\ -b & -a & 0 \\ 0 & 0 & d \end{vmatrix} = -\mu a d^2 - b^2 d$, and

$$\begin{vmatrix} \mu d & -b & 0 & c \\ -b & -a & 0 & 0 \\ 0 & 0 & d & b \\ c & 0 & b & -\mu a \end{vmatrix} = \mu^2 a^2 d^2 - b^4 - ac^2 d \tag{5.2}$$

So, $\Phi(A) > 0$ when $\mu d > 0$, $-\mu ad - b^2 > 0$, $-\mu ad^2 - b^2 d > 0$ and $\mu^2 a^2 d^2 - b^4 - ac^2 d > 0$, combine there inequality we get $\mu^2 > \frac{b^4 - ac^2 d}{a^2 d^2}$. Hence $\Phi(A) > 0$ when $\mu^2 > \frac{b^4 - ac^2 d}{a^2 d^2}$

Let $x = \frac{1}{1-\mu^2}$ and $y = \frac{\mu}{1-\mu^2}$.If

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} x & 0 & 0 & y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ y & 0 & 0 & x \end{bmatrix} \tag{5.3}$$

And $Q = \begin{bmatrix} I & B \\ B^* & C \end{bmatrix}$ in M_{24} where I in the identity in M_4

then,

$$Q \geq 0 \text{ and } \begin{bmatrix} I & B \\ B^* & C \end{bmatrix} \geq 0 \tag{5.4}$$

We have

$$C - BB^* \geq 0 \text{ and } C - B^*B \geq 0 \tag{5.5}$$

Hence

$$Q = \begin{bmatrix} I & 0 \\ B^* & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & C - B^*B \end{bmatrix} \begin{bmatrix} I & B \\ 0 & I \end{bmatrix} \geq 0 \text{ and}$$

$$\begin{bmatrix} I & B^* \\ B & C \end{bmatrix} = \begin{bmatrix} I & 0 \\ B & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & C - BB^* \end{bmatrix} \begin{bmatrix} I & B^* \\ 0 & I \end{bmatrix} \geq 0 \tag{5.6}$$

Suppose Φ is decomposable then Φ can be written as sum of completely positive and completely co-positive linear map i.e

$$\Phi = \Phi_1 + \Phi_2 \tag{5.7}$$

Where Φ_1 is completely positive linear map and Φ_2 is completely co-positive linear map. Since $[E_ij]_{i,j=1}^2 \in M_{22}^+$ we have

$$[\Phi_1(E_{ij})]_{i,j=1}^2 \geq 0 \text{ and } [\Phi_2(E_{ji})]_{i,j=1}^2 \geq 0 \tag{5.8}$$

Hence

$$\begin{aligned} trace(Q.[\Phi(E_{ij})]_{i,j}) &= trace(Q.[\Phi_1(E_{ij})]_{i,j}) + trace(Q.[\Phi_2(E_{ij})]_{i,j}) \\ = trace(Q.[\Phi_1(E_{ij})]_{i,j}) &+ trace(\begin{bmatrix} I & B^* \\ B & C \end{bmatrix}.[\Phi_2(E_{ji})]_{i,j}) \\ \geq 0 \end{aligned}$$

And now,

$$\Phi(E_{11}) = \Phi\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu \end{bmatrix}$$

$$\Phi(E_{12}) = \Phi\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Phi(E_{21}) = \Phi\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi(E_{22}) = \Phi\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow [\Phi(E_{ij})_{i,j}] = \Phi\left(\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\mu & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & 0 & 0 & y \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & y & 0 & 0 & x \end{bmatrix}$$

$$\Rightarrow [Q.(\Phi(E_{ij})_{i,j}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & 0 & 0 & y \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & y & 0 & 0 & x \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\mu & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\mu & 0 & 0 & 1 & 0 \\ y & 0 & 0 & x & x\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ x & 0 & 0 & y & y\mu & 0 & 0 & 1 \end{bmatrix} \tag{5.9}$$

But

$$trace(Q.[\Phi(E_{ij})_{i,j}]) = x\mu - \mu \tag{5.10}$$

$\Rightarrow$ For $0 < x < 1$ the linear map Φ is indecomposable.
Hence the linear map Φ is indecomposable when $0 < x < 1$.

6 Conclusion

In this report we have introduced a brief history of Quantum information theory.we gave some ideas of Quantum information for future.we have defined Quantum bits and multiple bits.We gave a short introduction of Quantum information theory with example,we introduced to linear algebra in Quantum information theory,we gave a short introduction of linear algebra like about bases and linearly independent vector,linear operator and matrices,the pauli matrices,The inner product of two vectors,tensor product and the brief introduction of Density operator.We have introduced positive complex matrix,linear positive map,completely positive and completely copositive map,decomposable and indecomposable map ,partial transpose and positive partial transpose.We have constructed a new family of map and determine the condition where the map is positive,decomposable and indecomposable.

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