FOURIER TRANSFORM AND ITS RECENT ADVANCEMENT

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We, Oshika Singh and Divya, who are currently pursuing a Master of Science in Applied Mathematics with Roll Number 2K22/MSCMAT/28 and 2K22/MSCMAT/11 respectively, hereby declare that the project dissertation submitted by us to the Department of Applied Mathematics at Delhi Technological University to fulfill the requirement for the award of the degree of Master of Science in Applied Mathematics, is original and has not been copied from any source. Furthermore, this work has not been previously used as the basis for conferring a degree, diploma, associate's degree, fellowship,
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CERTIFICATE

We hereby bear witness that the Project Dissertation submitted by Oshika Singh, Roll Number 2K22/MSCMAT/28 and Divya, Roll Number 2K22/MSCMAT/11 of the Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Applied Mathematics, is a record of the project work completed by the student under my supervision. To the best of my knowledge, neither a portion nor the entirety of this work has ever been submitted to this university or any other institution for a degree or diploma.

Place: Delhi

Date: 06 June 2024

Mr. Jamkhongam Touthang

Supervisor

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We would like to end by thanking God Almighty for seeing to it that no unforeseen problems occurred while we were working.

ABSTRACT

Fourier analysis and signal processing are vital aspects in the various scientific fields of mathematics, engineering and physics, among others. This abstract provides an extensive overview of the essential aspects, approached and applications of Fourier analysis and signal processing. Fourier analysis is a process invented by the renowned scientist Joseph Fourier, revolving around the decomposition of complicated signals into simple units known as Fourier series or transforms.

The central theme to Fourier's discovery is expressing a signal as a sum of sinusoidal functions, and this representation facilitates the analysis and the synthesis of signals, including time and frequency domains. Signal processing, on the other hand, entails the manipulation, improvement, and extraction of significant content from the signals.

This includes a variety of approaches and algorithms that intend to optimize, clean, or evaluate signals for numerous uses. The connection between Fourier analysis and signal processing can be seen in different fields, including speech and picture processing, data compression, pattern recognition, and wireless communication.

Therefore, this method may deliver the ability of Fourier analysis to effectively extract important components from complicated signals. It can help advance performance in various realms where the presented summarized perspective. This means that Fourier analysis and signal processing themselves form a critical but insufficiently explored aspect in which further discoveries and enhancements are possible. I anticipate that more comprehensive study will reveal new possibilities, allowing me and other scholars and professionals to be more creative.

CONTENTS

Chapter 1

INTRODUCTION

Any function, continuous or discontinuous, can be expanded into a sequence of sines, (see Joseph Fourier § The Analytic Theory of Heat) according to Fourier's 1822 assertion. Others improved and refined that significant work, which laid the groundwork for the different implementations of the Fourier transform that have been in use ever since.

Fourier analysis and the Fourier transform have a special status in the annals of mathematics and signal processing, having revolutionized the areas of analysis and understanding of signals. From their initial inception by the French mathematician Jean Baptiste Joseph Fourie, these related concepts have become indispensable tools in many scientific and engineering disciplines. Our main objective in this introduction is to provide a comprehensive overview of Fourier analysis and the Fourier transform, pointing out the central concepts, the points of similarity and difference, and broad applications of each.

The essence of the Fourier analysis process is that complicated signals are resolved into small, manageable components called sinusoidal functions. The resolution provides insightful information into the original structure and nature of the signal in both temporal and frequency domains. Such an analysis method has numerous applications in diverse fields, including data processing, image processing, signal processing, and communications. The process is versatile and powerful.

The Fourier Transform is a method applied to mathematics that trans- fers a signal from its original domain, usually time, to the frequency do- main. It is a natural extension of Fourier analysis. It plays a significant role in linking a signal's spectrum representation to that in time. In so doing, it offers a deeper understanding of the properties and composition of the signal. With these two concepts being very similar and coming from the same roots, the terms Fourier Transform and

Fourier Analysis are often used interchangeably, even though the former can be considered an extension of the latter. In summary, the related concepts of Fourier Analysis and the Fourier Transform have had a huge impact on the fields of mathematics, engineering, and science. As we try to explore these two concepts in depth, we realize that there is a wide range of applications that include data compression, pattern detection, wireless communication systems, voice, and image processing. In the creation of new technologies and the advancement of scientific— depend critically on the understanding and the expertise in these methods.

Signals

An information, that varies with time, space or some other independent variable is called a signal. Signals are patterns that carry data in a wide variety of disciplines, such as telecommunications, electronics, and data processing. Some examples of such patterns are electrical voltage variations, audio waves, or fluctuations in the intensity of light. The messages, data, or instructions, such data carry, whether analog or digital are applied to a wide variety of uses such as communication, measurement, control systems, among others. Understanding signals, therefore, becomes essential for interpretation and application of data for many technical and scientific applications.

Basic Signals -

Most of the naturally occurring signals have arbitrary amplitude configurations. Fundamental signals with well-defined characteristics, such as pulses, steps, ramps, sinusoidal, and exponential signals are used to analyse. Furthermore, systems are implemented using either hardware or software that alters signals or collects data from them. Their response to these cues identifies them too. The underlying bandwidth or duration of the signal is unlimited. They achieve ideal precision to a close approximation because of practical considerations. Each of these variants of Fourier analysis utilizes the characteristics of the signal. Therefore, it is necessary to study both continuous and discrete types of signals.

Chapter 2

FOURIER SERIES AND TRANSFORM

Joseph Fourier was a French mathematician and scientist credited with the foundation of both the Fourier Transform and Fourier Series. He was born in Auxerre, France, and educated in mathematics at the École Normale Supérieure in Paris. He later became a mathematics professor at the École Polytechnique, instructing famous students such as Claude-Louis Navier and Siméon Poisson.
During the early 19th century, Fourier worked on a problem involving heat conduction, a field of major interest for the development of the theories of thermodynamics. He tried to find a mathematical solution that could describe in detail how heat diffuses in a solid body, taking into account the temperature distribution in the body.

2.1 Background

In 1807, Fourier published "Mémoire sur la propagation de la chaleur", in which he introduced The concept of representing a periodic function as a combination of sine and cosine functions. Consequently, technique of decomposing a function into an unending series of trigonometric functions became recognized as the Fourier Series.

During the mid 19th century, mathematicians such as Augustin-Louis Cauchy and Bernhard Riemann extended the concept Fourier Series to encompass functions that do not exhibit periodic behavior. This advancement resulted in the establishment of the Fourier Transform as it is recognized today. Fourier Transform is a mathematical procedure that breaks down function into an infinite series of sine and cosine functions across all possible frequencies.

2.2 Fourier Series

In the context of Fourier series representation, a continuous periodic signal, indicated as p(t), with a period T and cyclic frequency f_0 = 1/T, can be expressed as the summation of a constant term and sinusoidal components with frequencies f_0. These sinusoids, known as the fundamental frequencies, contribute to the overall representation of the signal, and
$\{2f_0, 3f_0, ..., \infty\}$
referred to as the harmonic frequencies. The kth harmonic of a sinusoid along a fundamental frequency of f_0 is a sinusoid with a frequency of kf_0. The frequencies that are connected or associated with each other, expressed in radians, will be
$\{\omega_0 = 2\pi f, 2\omega_0 = 2\pi(2f_0), 3\omega_0 = 2\pi(3f_0), ..., \infty\}$
Then, using sinusoids, x(t) is expressed as,

$$\begin{aligned} p(t) =& P_s(0) + P_s(1)\cos(\omega_0 t + \phi_1) \\ &+ P_s(2)\cos(2\omega_0 t + \phi_2) + \cdots + P_s(\infty)\cos(\infty\omega_0 t + \phi_\infty) \\ =& P_s(0) + \sum_{k=1}^{\infty} P_s(k)\cos(k\omega_0 t + \phi_k), \quad \omega_0 = \frac{2\pi}{T} \end{aligned}$$

The frequencies of the sinusoids and x(t) are specified in the given equation. The Fourier analysis problem involves determining the amplitudes and phases of sinusoids in order to satisfy the equation with the least squares error. While it is not possible for any physical instrument to generate an infinite number of harmonics, theoretically, the frequency range of sinusoids is unlimited. However, in practice, only a finite number of harmonics are used.

The equation can be expressed in an equivalent manner by utilizing trigonometric identities to represent it in terms of sine and cosine waveforms.

$$p(t) = P_c(0) + \sum_{k=1}^{\infty} \left(P_c(k)\cos(k\omega_0 t) + P_s(k)\sin(k\omega_0 t)\right), \quad \omega_0 = \frac{2\pi}{T}$$

Euler's formula can also be used to write this equation in terms of complex exponentials with an exponent that is only imaginary.

$$p(t) = \sum_{k=-\infty}^{\infty} P_{fs}(k) e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

Fourier Analysis has two types of classification -
In first type we have the time domain classification.

Table 1

Time-domain classification

	Periodic	Aperiodic
Continuous	FS	FT
Discrete	DFT	DTFT

In second type we have frequency domain classification.

Table 2

Frequency-domain classification

	Periodic	Aperiodic
Continuous	DTFT	FT
Discrete	DFT	FS

Fourier Series as a Limiting case of DFT

DFT is a computational procedure used in Fourier analysis and signal processing to calculate discrete-time Fourier transform (DTFT) for a finite-duration signal.The continuous-time Fourier transform (DTFT) depicts a signal as a summation of complex sinusoids at different frequencies.

The duration of the signal being analyzed by the Discrete Fourier Transform (DFT) directly affects the level of accuracy with which the DFT approximates Discrete-Time Fourier Transform (DTFT).

Here DFT converges to DTFT in limit as the signal's length becomes closer to infinity. The DTFT is comparable to the continuous-time Fourier transform (CTFT) for periodic signals.

For periodic signals, we can think of the Fourier series as a limiting instance of the DFT. An infinite sum of complex sinusoids at various frequencies, with coefficients that are calculated through integration or other techniques, is how the periodic signal is represented by Fourier series. Which is the same as taking the limit as the signal's length approaches infinity, which is effectively the same as computing the DTFT of an infinitely long periodic signal using DFT.

The orthogonality attribute of sinusoids can be utilized to construct Fourier Series (FS) in a manner analogous to that of Discrete Fourier Transform (DFT). In this section, we obtain Fourier Series (FS) by considering Discrete Fourier Transform (DFT) when the time-domain sequence's sampling interval approaches zero. For ease of usage, we choose the center-zero format. Let $P(k), N \leq k \leq N$ be DFT of the sequence $p(n), N \leq n \leq N$. Next, $p(n)$'s Fourier representation is provided as

$$p(n) = \frac{1}{2N+1} \sum_{k=-N}^{N} P(k) e^{j\frac{2\pi}{(2N+1)}nk}, \quad n = 0, \pm 1, \pm 2, \ldots, \pm N$$

where

$$P(k) = \sum_{m=-N}^{N} p(m) e^{-j\frac{2\pi}{(2N+1)}mk}$$

Replacing $P(k)$, we get

$$p(n) = \frac{1}{2N+1} \sum_{k=-N}^{N} \left(\sum_{m=-N}^{N} p(m) e^{-j\frac{2\pi}{(2N+1)}mk} \right) e^{j\frac{2\pi}{(2N+1)}nk}$$

Assume that a periodic signal of period is sampled to produce the $2N+1$ samples. T s, During the time period which has a sampling interval of T_s s,

$$\frac{T}{T_s} = 2N+1$$

Time-domain sample index must be changed to nT_s s. Here, fundamental frequency refers to the minimum frequency at which a periodic waveform exhibits repetition.

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{(2N+1)T_s}$$

radians per second. Substituting these changes, we get

$$\begin{aligned} p(nT_s) &= \sum_{k=-N}^{N} \left(\frac{1}{T} \sum_{m=-N}^{N} p(mT_s) e^{-j\frac{2\pi}{T}mT_s k} T_s \right) e^{j\frac{2\pi}{T}nT_s k} \\ &= \sum_{k=-N}^{N} \left(\frac{1}{T} \sum_{m=-N}^{N} p(mT_s) e^{-j\omega_0 mT_s k} T_s \right) e^{j\omega_0 nT_s k} \end{aligned}$$

ω_0 is fixed for a given T, independent of T_s. As sample period T_s drops, spectrum broadens and time-domain waveform gets more compressed. As T_s approaches zero, the variables mT_s and nT_s become continuous and are represented by the symbols τ and t, respectively. Within the range of $-T/2$ to $T/2$, the inner summation is replaced by an integral, and T_s is represented by the differential $d\tau$. An unlimited number of harmonics, namely $2N+1$ of them, are present. These changes result in the discrete periodic waveform changing into a continuous periodic waveform and the discrete periodic spectrum changing into an aperiodic discrete spectrum. Formula is converted to

$$p(t) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{T} \int_{-T/2}^{T/2} p(\tau) e^{-j\omega_0 \tau k} d\tau \right) e^{j\omega_0 t k}$$

Periodicity of Fourier Series

The periodicity of a Fourier series refers to the recurrence of the sum of sinusoidal components generated by representing a periodic signal with a Fourier series, where the period of the signal being analyzed is equal to the fundamental period.For any time shift that is a whole number multiple of the fundamental period, the addition of the sinusoidal components utilizing the Fourier series formula results in a signal that is identical to the original periodic signal. Put another way, a periodic signal's Fourier series representation is periodic as well, having the same fundamental period as the original signal. This characteristic is necessary for Fourier analysis and other signal processing methods to comprehend and work with periodic signals.

Existence of Fourier Series

It is not necessarily certain that a particular periodic signal has a Fourier series. The Dirichlet conditions are the circumstances in which a Fourier series representation is present. In order to meet these requirements, the signal must be absolutely integrable over a single period, which means that its magnitude's integral over that period must be finite. The signal must exhibit limited number of maximum and minimum points within each specified interval, as well as a limited number of points where it is not continuous.

If these conditions are met, the Fourier series representation of the signal exists, and it converges to the original signal on all points of continuity and to either the left or the right limit on all discontinuities. The Fourier series can fail to exist, or can converge to a value that is different from the original signal, if some conditions are violated. These can be represented using other forms of expansions, such as the generalized Fourier series, or other expansion types.

2.3 Fourier Transforms

The Fourier transform (FT) is the most commonly employed method of Fourier analysis. Main purpose of this technique - to depict continuous signals that do not have a recurring pattern, together with their accompanying continuous spectra. Moreover, it serves as a tool for examining mixed classes of signals as it may depict signals that are characterized by various versions of Fourier analysis. One can conceptualize it as a Discrete-Time Fourier Transform (DTFT) expansion. As the sample interval approaches zero, time-domain signal becomes continuous, and theere, periodic spectrum becomes continuous and aperiodic. Considering that the period of the periodic signal is tending towards infinity, it can be perceived as an expansion of the Fourier Series (FS). Periodicity is a defining characteristic of a time-domain signal that corresponds to a discrete spectrum. Nevertheless, the time-domain signal corresponding to a continuous spectrum lacks periodicity.

Fourier Transform as a Limiting case of Fourier Series

The Fourier series is a mathematical representation of a periodic function that can be expressed as the combination of sinusoids with different frequencies and amplitudes. Conversely, the Fourier transform is a mathematical operation that converts a function dependent on time (or space) into a function dependent on frequency (or wavenumber).

The Fourier series can be thought of as a limiting case of the Fourier transform under specific situations. This is the result of the function becoming non-periodic as its period gets closer to infinity. The Fourier series becomes the Fourier transform in this limit.

Let's consider a function that is zero outside of a finite interval and non-zero within one to see this. Such a function is called a time-limited or windowed signal. Its Fourier transform is called the short-time Fourier transform, or STFT.

Let us take a signal and window it with a width of T seconds, for example. As $T \implies \infty$, then our signal becomes more and more periodic, with period T seconds. As $T \implies \infty$, the Fourier series of the signal approaches its STFT. This is because the STFT is essentially the window function times the Fourier transform of the windowed signal.

In conclusion, the Fourier series is non-periodic and can be understood as a limiting case of the Fourier transform as the period approaches infinity. The relationship between the two ideas demonstrates their essential function in signal processing and offers information on the range of applications in which they can be applied to the analysis and manipulation of signals.

Examine the FS synthesis and analysis formulas for a continuous periodic signal p(t) which has period T -

$$p(t) = \sum_{k=-\infty}^{\infty} P_{fs}(k) e^{j\omega_0 tk}$$

and,

$$P_{fs}(k) = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-j\omega_0 tk} dt = \frac{\omega_0}{2\pi} \int_{-T/2}^{T/2} p(t) e^{-j\omega_0 tk} dt$$

$\delta(k\omega_0)$ represents the frequency increment, or the fundamental frequency ω_0. At the extreme,

$$T \to \infty, \omega_0 \to 0, k \to \infty, \delta(k\omega_0) \to d\omega, TP_{fs}(k) \to P(j\omega)$$

At the end of the limiting process, there we get fourier Transformation, also Inverse Fourier Transformation Expression.
The Fourier Transformation $P(j\omega)$ of p(t) is defined by

$$P(j\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt$$

The Inverse Fourier Transformation p(t) of $P(j\omega)$ is defined by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

The given signal is a square pulse with a period of 2π and whose Fourier Series spectrum consists of harmonics plus a fundamental frequency of ω_0 = 1 radian. Furthermore, the equivalent scaled FT is also presented as a continuous line. The square pulse has a period of 4π and whose Fourier series spectrum is scaled with a frequency of $\omega_0 = 0.5$ radians. A denser spectrum results from a drop in the fundamental frequency with an increase in period. As the limit approaches, the ratio of $P_{fs}(k)/\omega_0$ converges to a finite limiting function. Ultimately, the fundamental frequency tends towards 0 and the period tends towards infinity. Consequently, the signal becomes uninterrupted and lacking a specific pattern, just as its spectrum. Here, square pulse has a period of 8π. and also we can see that whose Fourier series spectrum is scaled with $\omega_0 = 0.25$ rad.

The limiting process can be conceptualized as the time T being doubled and then doubled again. When a period doubles, the order k of a certain frequency component also doubles. Frequency is divided by two when the period is twice. Consequently, $k\omega_0$ does not change. $TP_{fs}(k)$ is multiplied to get a finite function $P(j\omega)$.

Each frequency component has an infinitesimal amplitude, which is $P(j\omega)d\omega/(2\pi)$. On the other hand, $P(j\omega)$ is equal to $P(j\omega)d\omega/(2\pi)$. The plot of $P(j\omega)$ versus ω represents the Fourier Transform spectrum since $P(j\omega)$ is finite. The relative fluctuations of a harmonic amplitudes versus frequency are displayed by the FT spectrum, which is a relative amplitude spectrum.

Fourier Transformation using Orthogonality

Unlike in the discrete Fourier transform (DFT), where the signal is reconstructed by summing finite exponentials, in this case, the signal is reconstructed by integrating continuous-time complex exponentials with all frequencies, along with their corresponding coefficients. This is because the original signal and the spectrum are continuous and extend infinitely. The continuous aperiodic signal is created as

$$p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(j\omega)e^{j\omega t}d\omega$$

Existence of Fourier Transformation

A mathematical idea known as the Fourier transformation has been proven via exacting mathematical demonstrations. The following criteria must be met for the Fourier transformation to exist:

1. Over the whole real line, the function to be changed must be integrable. This implies that across every finite interval, the function must have a finite integral.

2. The integral of the function multiplied by an exponential function with a complex argument is the definition of Fourier transform of function. The order for the Fourier transform to exist, the integral needs to converge.

3. The Riemann-Lebesgue lemma, which states that if a function is integrable, then its integral over an infinite interval goes to zero as the interval goes to infinity, guarantees the existence of the Fourier transform, which is a complex-valued function of frequency.

4. A familiar mathematical concept is that the Fourier transform of the product of two functions is the convolution of the two respective Fourier transforms.

In summary, the functions which are integrable over the entire real line and converge under certain conditions can be transformed using the Fourier transform. Its existence depends upon well-established mathematical concepts, which have been rigorously examined and applied in numerous fields of science and engineering.

Determination of Fourier Series From Fourier Transformation

Consider a periodic signal $p_s(t)$ with period T. Let us define an aperiodic signal, $p(t)$, where t_1 is an arbitrary time. This signal is identical to $p_s(t)$ over one period, beginning at t_1 and ending at $t_1 + T$, and zero otherwise. The FT for this signal is

$$P(j\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt = \int_{t_1}^{t_1+T} p(t)e^{-j\omega t}dt$$

The Spectrum Fourier Series for $p_s(t)$ is

$$P_{fs}(k) = \frac{1}{T}\int_{t_1}^{t_1+T} p(t)e^{-jk\omega_0 t}dt, \omega_0 = \frac{2\pi}{T}$$

Comparing the definitions of the signals by means of Fourier Series and by means of Fourier Transformation, we get there

$$P_{fs}(k) = \frac{1}{T}P(j\omega)|_{\omega=k\omega_0} = \frac{1}{T}P(jk\omega_0)$$

Thus, the Fourier series spectrum consists of samples of $frac1T(j$ $omega)$ spaced at increments of $omega_0$ and obtained from a periodic signal, $p_s(t)$. To perform the inverse Fourier transform, spectral values must be known for a continuum of frequencies in order to reconstruct a single period of a periodic waveform or infinite extent of zero values of an aperiodic waveform. However, to perform the inverse Fourier series, spectral values are only required at discrete frequencies in order to reconstruct a single period of a periodic waveform. There is an analogous correspondence between the DFT and the DTFT.

Chapter 3

DISCRETE FOURIER TRANSFORM

In the field of signal analysis, transformations stand as powerful mathematical tools that make the detection and understanding of signals in various fields possible. A transformation changes the signal from one representation to another, thus revealing unique information which may otherwise stay hidden in its original form. The transformations help untangle patterns, harmonics, and characteristics embedded within the signal by converting signals between domains such as time, frequency, and spatial dimensions.

In that line, these tools are used extensively for noise reduction, compression, feature extraction, and data visualization. The analysis has a flexible toolkit to decipher the complex signaling language and unlock the valuable insights hidden within using signal transduction.

The DFT—Discrete Fourier Transform—is a mathematical technique and is of pivotal importance for signal processing. It provides a way to analyze and convert the digital signal to time from the frequency domain. The DFT brings out more characteristics of a signal by representing a sequence of discrete data points in terms of its constituent frequencies, thereby allowing huge flexibility in applications that span from processing and signal processing to audio management, image analysis, communication, and several others.

Unlike a continuous signal, a digital signal can be represented by the finite set of discrete values recorded at specific time intervals. The DFT acts as a bridge between these discrete signals and their frequency components, allowing us to explore the oscillations, harmonics, and fundamental patterns that define signal behavior. Using DFT, complex data sequences can be broken down into simpler sinusoidal components, each of which is associated with a unique frequency and amplitude. The heart of the DFT lies in its transform formula, which calculates the contribution of the different frequency components present in the input signal.

Although the direct application of this formula involves computational challenges, the development of efficient algorithms, especially the fast Fourier transform (FFT), has revolutionized the real implementation economy DFT. FFT significantly reduces computational complexity, making it possible to analyze and process large data sets in real-time applications.

In whole, the main perspective of the transformation is to approximate the practical signals, which usually in their original forms are more complex, with arbitrary amplitude profiles, which makes it difficult to analyse.

Fourier transforms effectively convert signals into clearly defined basis signals, such as sine and cosine. In addition to other benefits that are mostly related to the fact that this decomposition makes it possible to determine a system's output faster than any other technique

1. Orthogonality of Sinusoids

2. Fast algorithms are available for its practical implementation..

There are four alternative variants of Fourier analysis, but DFT is the only one that has fast algorithms available for its implementation and can represent signals in both discrete and finite form in both domains.

The Exponential Function

In Fourier analysis, any periodic signal can be expressed as a sum (or integral) of sinusoidal functions with different frequencies, amplitudes, and phases. These sinusoidal functions are often represented using complex exponential functions. The Exponent is of the form

$$x(n) = b^n$$

and

$$\log_b x = n \text{ iff } x = b^n$$

e.g. $\log_2 8 = 3$ as $8 = 2^3$, $\log_{10} 100 = 2$ as $100 = 10^2$,
$e = \lim_{n \Longrightarrow \infty} (1 + \frac{1}{n})^n \sim 2.71828$

The multiplication operation is reduced to an addition operation in the exponential form of the integers, if that is possible to describe.

The Complex Exponential Function

The complex exponential is of the form $e^{j\phi}$ and it can be written as the $p + jq$ where p is the real part coefficient and q is the imaginary part coefficient, and j is the imaginary unit.

The signals can be represented in their complex exponential forms using this representation technique, which reduces the convolution action to a multiplication operation.

Euler's Formula

It is discovered that the complex exponential facilitates the signals' examination. In essence, Euler's formula provides the relationship between complex exponential and sinusoidal functions.

$$e^{j\phi} = \cos(\phi) + \sin(\phi)$$

Real Sinusoids in terms of Comlpex Exponentials

On solving the equations of Euler's formula

$$e^{j\phi} = \cos(\phi) + \sin(\phi) \text{ and } e^{-j\phi} = \cos(\phi) - \sin(\phi)$$

we get

$$\cos(\phi) = \frac{e^{j\phi} + e^{-j\phi}}{2},$$
$$\sin(\phi) = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

The DFT and IDFT

Essential ideas in Fourier analysis are the Discrete Fourier Transform (DFT) and its inverse, the Inverse Discrete Fourier Transform (IDFT). They serve as the foundation for transforming signals between the frequency and time domains, which enables us to process and analyze the signal in a number of ways.

3.1 Discrete Fourier Transform - DFT

A discrete sample sequence in the time domain can be transformed mathematically into its matching sequence in the frequency domain using the DFT.. It reveals the frequency components present in the signal as well as their respective amplitudes and phases. DFT is especially useful for analyzing periodic or time-limited signals.

Mathematically, for a sequence of N samples $p[n]$ in the time domain, the DFT $P[k]$ is calculated using the formula:

$$P[k] = \sum_{n=0}^{N-1} p[n] \cdot e^{-i2\pi kn/N}$$

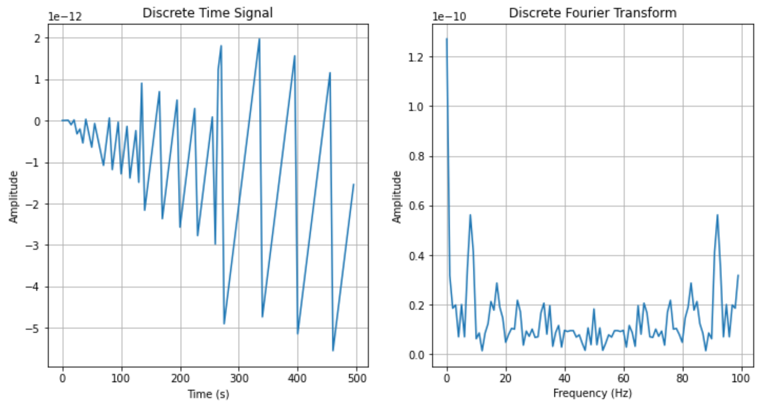

Figure 3.1: DFT

Here N is the number of samples, $p[n]$ is the input sample at time n and $P[k]$ is the frequency component corresponding to just frequency number k.

Here is the example -

Let

$$p(n) = 16, 28, 28, 24, 46, 27, 23, 29, 38, 2, 44$$

on Solving - 305 + 0j, -16.88 - 15.117j, -1.835 + 9.95j, 33.075 - 1.787j, -34.453 + 44.245j, -44.407 + 40.649j, -44.407 - 40.649j, -34.453 - 44.245j, 33.075 + 1.787j, -1.835 - 9.95j, -16.88 + 15.117j

Graphical representation of the Discrete Fourier Transformation has been depicted in the figure 3.1

3.2 Inverse Discrete Fourier Transform - IDFT

IDFT is the reverse of DFT. It takes a sequence of frequency domain components and reconstructs the original time domain sequence. In other words, it transforms the signal from the frequency domain to the time domain.

Mathematically, for a sequence of N frequency domain components $P[k]$, the IDFT $p[n]$ is calculated using the formula:

$$p[n] = \frac{1}{N} \sum_{k=0}^{N-1} P[k] \cdot e^{i2\pi kn/N}$$

Here, N is the number of samples, $P[k]$ is the frequency component at frequency index k and $p[n]$ is the signal in the domain time is rebuilt . .

The combination of DFT and IDFT allows for the analysis of signals in both the frequency and time domains. They are extensively employed in many different domains, including as image analysis, audio processing, telecommunications, and signal processing. Algorithms like the Fast Fourier Transform (FFT) for the DFT and its inverse for the IDFT improve computational performance, making these transformations a crucial tool for comprehending and handling data.

Example

Identifying objects in a picture is a necessary in the image processing step. The image is segmented using the object characteristics for that reason. Compact representation of the split objects is required. An item can be described using its border representation. The coordinates of a boundary can be used to characterize it. Reducing the amount of storage needed to represent it is the goal. One useful tool for representing an object's boundary is the Fourier boundary descriptor.

A set of coordinates represents a closed boundary in the spatial domain. Each point on the border creates the p-coordinate on x axis, which is the imaginary portion of a complex number, and the q-coordinate on y axis, which is the real part. The number of points on the boundary determines the period of the complex number set, which is a periodic complex data collection.

Let us consider an example for the same

3.3 Properties Of DFT

1. Linearity

   Let us consider $p(n) \leftrightarrow P(k)$ and $q(n) \leftrightarrow Q(k)$, be the sequences which has period N. Then,

   $$ap(n) + bq(n) \leftrightarrow aP(k) + bQ(k)$$

   where a,b are arbitrary constants, ∀ a,b ∈ Constant. The DFT of the individual signals is the same as the DFT(Discrete Fourier Transform) of a linear combination of a collection of signals.
   Let's look at a straight forward example to demonstrate this feature.
   Assume we have two signals with discrete times: where
   p[n] = [1, -2, 3, 4] ans q[n] = [2, 4, 5, 0]as well.
   Taking p[n] and q[n]'s DFT, we obtain:

[6, -4 + 2j, -2, -4 - 2j] is what P[k] is. Q[k] is equal to [11, 4 + 2j, -1, 4 - 2j]. Let's now calculate the DFT of a*p[n] + q[n] using the scalar variable a = -0.5:

-0.5, 3, 3.5, 2 is equal to a*p[n] + q[n].
By calculating this linear combination's DFT, we get:

DFT of [9, -4 + 2j, -4, -4 - 2j] for a*p[n] + q[n]
Let's now compute p*X[k] + Y[k]:

-2.5, -2 - 2j, -3.5, -2 - 2j = a*P[k] + Q[k]
It is evident that the DFT of a*p[n] + q[n] equals the linear combination of a*P[k] + Q[k], the DFTs of the individual components.

2. Priodicity
   Let us consider $p(n) \leftrightarrow P(k)$ be the sequence which has period N. Then,

$$p(n+aN) = p(n)$$

   and

$$P(k+aN) = P(k)$$

   $\forall$ n,k $\in$, where a is any arbitrary integer. When a signal p(n) exhibits periodicity with a period of N samples, its values remain constant throughout any subsequent N samples.

   In Fourier analysis, periodicity property of the Discrete Fourier Transform (DFT) asserts that if a discrete-time signal p[n] has a period of N, then its DFT, P[k], will also have a period of N.
   To demonstrate this characteristic, let us examine an example:
   Assume that p[n] = [1, 2, 3, 4, 5] is a discrete-time signal with period N = 5. The DFT for p[n] is as follows:$P[k] = \sum p[n] * exp(-j * 2\pi kn/N)$, where n = 0 to N-1 is the summation. When we calculate the DFT, we obtain
   P[0] = 15.
   P[1] $\approx$ 1.081j - -2.688
   P[2] $\approx$ 0.475j - 0.688
   P[3] $\approx$ 0.475j + -0.688
   P[4] $\approx$ 1.581j + 2.688
   It is evident that P[k] recurs after each N = 5 index. Now, let's compute the DFT of a shifted version of p[n] to confirm the periodicity feature. The shifted signal p.shifted[n] = [4, 5, 1, 2, 3] is to be considered. P.shifted[k]

= ∑p.shifted[n]*exp(-j*2πkn/N) is the DFT of p.shifted[n], where the summation is across n = 0 to N-1.
P.shifted[0] = 15 is the result of computing the DFT of p.shifted[n].
P.shifted[1] 0.081j - -2.688
P.shifted[2] 0.475j - 0.688
P.shifted[3] 0.475j + -0.688
P shifted[4] 1.081j + 2.688
It is evident that the DFT of p.shifted[n], P.shifted[k] and the DFT of p[n], P[k] are identical.
This proves the periodicity property of the DFT, which states that if a discrete-time signal p[n] has a period of N, then so will its DFT P[k].

3. Circular Time shifting
Let us consider $p(n) \leftrightarrow P(k)$, be the sequence which has period N. Then,

$$p(n \pm n_o) \leftrightarrow \exp^{\pm \frac{2\pi}{N} kn_o} P(k)$$

where n_o is any random sampling interval. A sinusoidal waveform's shift alone modifies its phase. The magnitude has not changed. .
As a result, when a waveform is shifted in the time domain, the frequency component's phase is increased, and this increase is linearly proportional to the corresponding frequency indices.
Aim: The computational cost of computing the DFT of p(n) can be reduced by shifting p(n) to pS(n), calculating its DFT PS(k), and then inferring the DFT of p(n) from PS(k) using the shift theorem.

When a discrete-time signal p[n] is circularly shifted by a certain number of samples, the corresponding DFT P[k] will also be circularly shifted by the same number of samples, according to the circular time shifting property of the Discrete Fourier Transform (DFT) in Fourier analysis.
To demonstrate this characteristic, let us examine an example:
Assume p[n] = [1, 2, 3, 4, 5] is a discrete-time signal. Using p[n]'s DFT, we obtain:
P[k] = ∑p[n]*exp(-j*2πkn/N), where n = 0 to N-1 is the summation range.
When we compute the DFT of p[n], we obtain: P[0] = 15
P[1] $\approx$ 1.081j - -2.688
P[2] $\approx$ 0.475j - 0.688
P[3] $\approx$ 0.475j + -0.688
P[4] $\approx$ 1.581j + 2.688

Let's now shift x[n] two samples to the right in a circular time shift. [4, 5, 1, 2, 3] is the signal p.shifted[n] that results.
The DFT of p.shifted[n], let's call it P.shifted[k], will likewise be circularly shifted by the same amount in accordance with the circular time shifting property.
Let's compute the DFT of p.shifted[n] to confirm this:
When we compute the DFT of p.shifted[n], we obtain: P.shifted[0] $\approx$ 15
P.shifted[1] $\approx$ 1.081j + 2.688
P.shifted[2] $\approx$ 0.475j + -0.688
P.shifted[3] $\approx$1.081j - -2.688
P.shifted[4] $\approx$ 0.475j - 0.688
As we can see, in comparison to the original DFT P[k], the DFT of p.shifted[n], P.shifted[k], is circularly shifted by two samples.
In other words, if we circularly shift a discrete-time signal p[n] by a certain number of samples, the corresponding DFT P[k] will likewise be circularly shifted by the same number of samples. This illustrates the circular time shifting property of the DFT.

4. Circular Frequency Shifting
   Let us consider $p(n) \overset{P}{\leftrightarrow} (k)$ be the sequence which has period N, Then,

$$e^{\pm j\frac{2\pi}{N}k_0 n}p(n) \leftrightarrow P\left(k \mp k_0\right)$$

   where k0 is an sampling intervals' arbitrary number, In the definition of DFT, The spectral values occur at frequency index k + k0 and are delayed by k0 sampling intervals.

   Assume the following: a 10 Hz sinusoidal signal (radian frequency: 2π f = 62.83 rad/s) p(t) equals sin(62.83t).

   We increase the Fourier transform of this signal by a complex exponential at this frequency in order to shift it up by 5 Hz, or 31.41 rad/s, the radian frequency:
   $e^{j31.41\pi} * P(f) = P(f - 31.41) + P(f + 31.41)$

   Two components will make up the final spectrum: one at the original frequency, which has been shifted down by -5 Hz, and one at the newly shifted frequency, which is now at +5 Hz. The initial spectrum and the complex exponential's phase will determine the size and phase of these components.

5. Circular Time Reversal
   Let $p(n) \leftrightarrow P(k)$ with period N. Then,

$$p(N-n) \leftrightarrow P(N-k)$$

   For a DFT with $N = 8$, the function $\mod(nk, 8)$ for each nk in $e^{-j\frac{2\pi}{8}nk}$ in the DFT definition, with $k = \{0, 1, 2, 3, 4, 5, 6, 7\}$, yields

$$\mod(nk, 8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 2 & 4 & 6 & 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \\ 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 \\ 0 & 5 & 2 & 7 & 4 & 1 & 6 & 3 \\ 0 & 6 & 4 & 2 & 0 & 6 & 4 & 2 \\ 0 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

   The residual of nk divided by 8 is returned by the mod function. Every row of values represents the time-reversal of that of (Nk) for a given k. Therefore, $p(N-n) \leftrightarrow P(N-k)$

   Assume the following: a 10 Hz sinusoidal signal (radian frequency: 2πf = 62.83 rad/s)

   $p(t) = sin(62.83t)$.

   We multiply this signal's Fourier transform by a complex exponential with a phase shift of $-\pi$ radians in order to reverse it in time and then wrap it around:

   $e^{-j\pi} * P(f) = P(-f)$

   For frequencies above the Nyquist frequency, which is half the sampling rate, the resulting spectrum will have the same magnitude but the opposite phase. This is because the original signal's positive frequencies are matched by negative frequencies introduced by the time reversal procedure.

6. Duality
   The basic Fourier analysis theory known as duality states that the Fourier transform of a time-domain function is equivalent to the inverse Fourier transform of its frequency-domain representation. Let us consider $p(n) \leftrightarrow P(k)$ with period N. Then,

$$P(n) \leftrightarrow Np(N-k)$$

A signal p(n) can be calculated twice in succession using the DFT to get N times its time reversal. If we multiply the input vector by the transform matrix, then we get a scaled and time-reversed version of the input because the product of the transform matrix by itself, gives a matrix with N elements (except for the first entry) below the reverse diagonal.

To illustrate the duality property of Fourier analysis, let's consider a simple example using a discrete signal of length 4(
$mathrm\ N = 4$).
The definition the signal is $p[n] = \{1,2,3,4\}$.

We can now use the formula for the DFT to compute its Fourier transform:

For every $k = 0,1,\dots,N-1$, $P[k] = \sum_{n=0}^{N-1} p[n]e^{-j2\pi \mathrm{kn}/N)}$

Now let's compute our signal's DFT:
$P[0] = \sum_{n=0}^{3} p[n] = 10$
$P[1] = \sum_{n=0}^{3} p[n]e^{-j2\pi \mathrm{kn}/4)} = -j(1-2i)$
$P[2] = \sum_{n=0}^{3} p[n]e^{-j2\pi \mathrm{kn}/2} = -j(1+2i)$
$P[3] = \sum_{n=0}^{3} p[n]e^{-j2\pi \mathrm{kn}} = -4$

Let's now determine X[k]'s inverse DFT: For each $\mathrm{n} = 0,1,\dots,\mathrm{N}-1$,

$$pinv[n] = (1/4)\sum_{k=0}^{3} P[k]e^{j2\pi kn/4}$$

Let's figure out the DFT in reverse: For every $pinv[0] = (1/4)(P[0]+P[1]+P[2]+P[3]) = 1$
$pinv[1] = (1/4)(P[0]+P[1]-P[2]-P[3]) = -2$
$pinv[2] = (1/4)(P[0]-P[1]+P[2]-P[3]) = 1$
$pinv[3] = (1/4)(P[0]-P[1]-P[2]+P[3]) = 1$
As you can see, the duality property in Fourier analysis for discrete signals is confirmed by the fact that the inverse DFT of P(k) is equal to the original signal p(n). This feature is very important in a number of applications ranging from digital signal processing to communication systems, all of which are very important in understanding and changing signals.

7. Transform of Complex Conjugate
   Let us consider $p(n) \leftrightarrow P(k)$ with period N. Then,

$$p^*(n) \leftrightarrow P^*(N-k) \text{ and } p^*(N-n) \leftrightarrow P^*(k)$$

Conjugating both sides of Eq. and replacing k by $N-k$, we get

$$P^*(N-k) = \sum_{n=0}^{N-1} p^*(n)e^{-j\frac{2\pi}{N}nk}$$

For example,
Let's take a simple example of a real and even function $f(t) = e^{-at}cos(bt)$‘, here a,b are positive constants.

To find the Fourier transform of the complex conjugate of f(t), we first find the complex conjugate:

$$f^*(t) = e^{-at}cos(bt)$$

The Fourier transform of $f^*(t)$ is:

$$F^*(\omega) = \int_{-\infty}^{\infty} e^{-at}cos(bt)e^{-i\omega t}dt$$

Now, let's substitute -t for t in the integral:

$$F^*(\omega) = \int_{-\infty}^{\infty} e^{-at}cos(bt)e^{i\omega t}dt$$

Next, we use the fact that cos(-bt) = cos(bt) and change the sign of ω in the exponent:

$$F^*(\omega) = \int_{-\infty}^{\infty} e^{-at}cos(bt)e^{-i(-\omega)t}dt$$

Finally, we recognize that this is the Fourier transform of the original function with a frequency shift of $-\omega$, which is:

$$F^*(\omega) = F(-\omega)$$

Thus, we have demonstrated that the Fourier transform of a real and even function's complex conjugate is equal to the Fourier transform of the original function with a $-\omega$ frequency shift. We call this characteristic conjugate symmetry.

8. Circular Convulation and Correlation

Circular Convolution of Time-Domain Sequence
Digital signal processing uses circular convolution, a type of convolution procedure, for signals with finite length that are wrapped around a circle or periodic boundary. It entails taking the inverse Fourier transform of the product after multiplying the Fourier transforms of two sequences. Circular convolution is employed in filtering, coding, and pattern recognition for periodic or cyclic signals. Its length is equal to that of the input sequences.

Circular Convolution of the Frequency-Domain Sequence
Circular convolution is a procedure that takes two complex-valued sequences, multiplies them in the frequency domain, and then does an inverse Fourier transform. In short, it is the time domain Fourier transform of circular convolution. This procedure is very often applied in digital signal processing, usually for signals of finite length, which are taken to wrap around some circular or periodic boundary. It may be used in pattern recognition, coding, and filtering for periodic or cyclic signals.

Circular Correlation of the Time-Domain Sequences
The kind of correlation operation that uses digital signal processing is called circular correlation for signals of finite length, wrapped around a circle or periodic boundary. It is a process that involves the calculation of the cross-correlation between two sequences wrapped around a circle, but with circular boundary conditions. In signal processing, for periodic or cyclic signals, filtering, and pattern recognition, circular correlation does enjoy the same length as the input sequences. Circular correlation, in a nutshell, is the time domain analogue of the cross-correlation operation.

9. Sum and Difference of Sequence

   $P(0)$ is the total of the values in the input sequence, $p(n)$, as all of the transform matrix coefficients have a value of unity when $k = 0$. The transform matrix coefficients produce the alternating sequence $\{1, 1, 1, 1, ..., 1\}$ when N is even and $k = N/2$. The difference between the sum of the even and odd indexed values of $p(n)$ is therefore $P(N/2)$.

10. Upsampling of the Sequence
    Upsampling, commonly referred to as interpolation, is a Fourier analysis technique that raises a signal's sampling rate without altering its frequency content. The sampling rate controls the frequency resolution of the spectrum when a signal is subjected to the discrete Fourier transform (DFT).

Upsampling boosts the frequency resolution and sampling rate by sandwiching fresh samples between old ones. Upsampling, however, can cause distortion of the signal's frequency content if done incorrectly, increasing the DFT's processing complexity and introducing aliasing. Making sure the upsampling factor is a power of two and applying the proper anti-aliasing filters prior to upsampling are crucial steps in preventing aliasing.

Let $p(n) \leftrightarrow P(k)$ n,k=0,1,.....,N-1. Upsampling x(n) with zeros is defined as

$$p_u(n) = \begin{cases} p\left(\frac{n}{L}\right) & \text{for } n = 0, L, 2L, \ldots, L(N-1) \\ 0 & \text{otherwise} \end{cases}$$

where L $\in \mathbf{Z}$, then

$$P_u(k) = P(k \bmod N), wherek = 0, 1, \ldots, LN-1$$

Where $p_u(n)$ is given by,

$$P_u(k) = \sum_{n=0}^{LN-1} p_u(n) e^{-j\frac{2\pi}{LN}nk}, wherek = 0, 1, \ldots, LN-1$$

We can replace n = m L since we only have nonzero input values at intervals of L. then we will get,

$$\begin{aligned} P_u(k) &= \sum_{m=0}^{N-1} p_u(mL) e^{-j\frac{2\pi}{LN}mLk} \\ &= \sum_{m=0}^{N-1} p(m) e^{-j\frac{2\pi}{N}mk} = P(k \bmod N), wherek = 0, 1, \ldots, LN-1 \end{aligned}$$

As N-point DFT is periodic of period $N.P_u(k)$ is the L-times repetition of $P(k)$ is seen.

11. Zero Padding of Data
In Fourier analysis, zero padding is a method for lengthening a signal without altering its frequency content. The resolution of the frequency spectrum is determined by the length of the signal when it is subjected to the discrete Fourier transform (DFT). By adding zeros to the end of the signal, or zero padding, we can extend its length and improve the frequency spectrum's resolution. When analyzing signals at higher frequencies or with finer resolution than what is achievable with the original signal length, this can be

helpful. It should be noted, however, that zero padding may also increase DFT computational expenses since processing of the longer signal requires more complex computations. Let $p(n) \leftrightarrow P(k), n, k = 0, 1, \ldots, N-1$. If we append $p(n)$ with zeros to get $p_z(n), n = 0, 1, \ldots, LN-1$ defined as

$$p_z(n) = \begin{cases} p(n) & \text{for } n = 0, 1, \ldots, N-1 \\ 0 & \text{otherwise} \end{cases}$$

where L is any positive integer, then

$$P_z(Lk) = P(k), k = 0, 1, \ldots, N-1$$

The DFT of the signal $p_z(n)$ is given by

$$P_z(k) = \sum_{n=0}^{LN-1} p_z(n) e^{-j\frac{2\pi}{LN}nk}, k = 0, 1, \ldots, LN-1$$

Since $p_z(n)$ is zero for $n > N-1$, we get

$$P_z(k) = \sum_{n=0}^{N-1} p_z(n) e^{-j\frac{2\pi}{LN}nk}, k = 0, 1, \ldots, LN-1$$

Replacing k by Lk, we get

$$P_z(Lk) = \sum_{n=0}^{N-1} p_z(n) e^{-j\frac{2\pi}{N}nk} = P(k), k = 0, 1, \ldots, N-1$$

12. Symmetry Properties

    In Fourier analysis, symmetry properties describe how a function behaves and how its Fourier transforms under specific symmetries.

    There are various kinds of symmetries that can influence a function's Fourier transform:

    1. Real-valued functions: In the time domain, a real-valued function is symmetric about the y-axis. In the frequency domain, its Fourier transform is also symmetric about the origin and real-valued.

    2. Even functions: In the time domain, an even function is symmetric about the origin. In the frequency domain, its Fourier transform is also symmetric about the origin and real-valued.

3. Odd functions: In the time domain, an odd function is antisymmetric about the origin. In the frequency domain, its Fourier transform is entirely imaginary and antisymmetric about the origin.

4. Periodic functions: In the time domain, a periodic function repeats itself at regular intervals. Each term in the Fourier transform represents a frequency component of the signal, and it is equal to the sum of complex exponentials. The amplitude of the appropriate term in the Fourier series expansion of the periodic function determines the magnitudes of these components.

Because they permit simplifications and reductions in computational complexity, these symmetry features can have significant effects on signal processing applications, including filter design and signal compression.

13. Parseval's Theorem

The frequency domain representation of the signal p(n) is determined by the DFT coefficients P(k), which are obtained through an orthogonal transform. Consequently, P(k) is a precise and thorough depiction of p(n) in every aspect. It is essentially an alternative way of representing p(n) in a new domain or a straightforward modification to the independent variable. Consequently, in both representations, the power of the signal during a single period can be accurately measured.Let N represent the length of the sequence $p(n) \leftrightarrow P(k)$. The Discrete Fourier Transform (DFT) utilizes complex exponentials with harmonic frequencies to precisely depict signals. When the samples of a complex exponential are placed on the unit circle, the magnitude of the exponential throughout one complete cycle is denoted as N. Therefore,

$$\sum_{n=0}^{N-1} |p(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |P(k)|^2$$

Fourier analysis and other orthogonal transforms has the property of power preservation.

3.4 2 Dimensional DFT

The 1D-DFT, Discrete Fourier Transform, is extended to the two dimensions in the 2D DFT. It is employed in the analysis and processing of two-dimensional

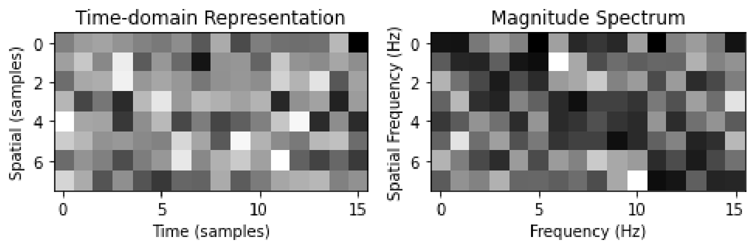

Figure 3.2: 2 Dimensional-DFT

signals and images, such as those from digital photography or 2D spectroscopy.

A 2D array of complex numbers represents a signal or image, which is converted into a 2D array of complex numbers in the frequency domain via the 2D DFT. The spatial frequencies contained in the original image or signal are revealed by the frequency domain representation. The 2D DFT formula -

$$F(u,v) = \frac{1}{MN}\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} f(p,q)e^{-j2\pi(up/M+vq/N)}$$

where M and N are the original array's dimensions, u is spatial frequencies in the horizontal and v is spatial frequencies in the vertical directions, and f(p, q) is the original image or signal and F(u, v) is converted image or signal.

whose inverse 2D DFT is:

$$f(p,q) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(up/M+vp/N)}$$

Figure 3.2 depicts the 2 Dimensional Discrete Fourier Transformation

Chapter 4

4.1 Convolution

Convolution is a mathematical procedure used in Fourier analysis that creates a new signal by combining two existing signals. It shows what would happen if the input signals were convolved in the time domain. Convolution is a fundamental idea in several disciplines, including engineering, physics, and signal processing, since it enables us to examine and modify information in many ways.

For continuous time signal, convolution formula is:

$$q(t) = p(t) * i(t) = \int_{-\infty}^{\infty} p(\tau)i((t-\tau)d\tau = \int_{-\infty}^{\infty} p(t-\tau)i(\tau)d\tau$$

Properties of Convulation

1. Commutative Property
   $q_1(t) * q_2(t) = q_2(t) * q_1(t)$
2. Distributive Property
   $q_1(t) * [q_2(t) + q_3(t)] = q_1(t) * q_2(t) + q_1(t) * q_3(t)$
3. Associative Property
   $q_1(t) * [q_2(t) * q_3(t)] = [q_1(t) * q_2(t)] * q_3(t)$
4. Shifting property

$$q_1(t) * q_2(t) = w(t)$$
$$q_1(t) * q_2(t-t_1) = w(t-t_1)$$
$$q_1(t-t_1) * q_2(t) = w(t-t_1)$$
$$q_1(t-t_1) * q_2(t-t_2) = w(t-t_1-t_2)$$

5. Convulation with Impulse
   q(t)*δ(t)=q(t)
   q(t)*$\delta(t-t_1) = q(t-t_1)$ with shifting property
   $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$
   Additional Properties of Convulation

6. p(t)*h(t)=q(t)
   $\frac{dq(t)}{dt} = \frac{dp(t)}{dt} * h(t) = p(t) * \frac{dh(t)}{dt}$

7. $u(t) * u(t) = r(t)$
   Convolution of two unit signals is ramp signal.

8. $\frac{d^n p(t)}{dt^n} * \frac{d^m i(t)}{dt^m} = \frac{d^{n+m} q(t)}{dt^{n+m}}$

9. Scaling of Convolution
   Whenever the input and impulse signals are scaled by α then the output will be
   For $p(t) * i(t) = q(t)$
   $= p(\alpha t) * i(\alpha t) = \frac{1}{|\alpha|}q(\alpha t)$

10. Area of the convoluted signals
    Let the input signal p(t) has the area A_i
    Let the other input signal i(t) has the area A_n
    Then the area A of the convoluted signal

$$A = A_i A_n$$

Limits of the Convoluted Signal
Let $w(t) = p(t) * q(t)$
Then,
Lowerlimit of w(t) = Lowerlimit of p(t) + Lowerlimit of q(t)
Upperlimit of w(t) = UpperLimit of p(t) + Upperlimit of q(t)
So, limit of w(t) = Lowerlimit of w(t)≤t≤Upperlimit of w(t)

Convolution using Slide and Shift Method
The slide and shift method provides a visual picture of convolution in its context. In order to calculate the product of the overlapping sections at each time shift, one signal—referred to as the input signal or impulse response—is slid over another—referred to as the input signal.

The slide and shift method operates as follows: 1. Slide the impulse response: To form a new function i(t tau), where tau represents the time shift, we first shift impulse response—i(t)—across input signal—p(t). This is equivalent to saying it's the same asof convolving i(t) with p(t) at some time shift tau.

2. Calculate the product: For every time shift tau, we now calculate the product of $i(t\tau)$ and p(tau). This gives us a new function, which represents the way i(t tau) and p(tau) overlap with one another.

3. Sum all products: Finally, we sum up all these products for all possible time shifts tau in order to obtain the output signal q(t). Mathematically, this summing up is represented by integration, which denotes the sum of all overlapping products between i(t tau) and p(tau).

4.2 Correlation

Correlation is a mathematical process used in Fourier analysis that quantifies the degree of linear dependency or resemblance between two signals. It is a signal-specific extension of vector algebra's dot product idea.

The correlation coefficient between two discrete-time signals, p[n] and q[n], is equal to:

$$r_{pq}(m) = \sum_{n=-\infty}^{\infty} p(n)q^*(n-m) = \sum_{n=-\infty}^{\infty} p(n+m)q^*(n), \quad m = 0, \pm 1, \pm 2, \ldots$$

where the summation is over all values of n, and * denotes correlation. Using this formula, one signal (q[n]) is slid over another signal (p[n]), and at each time shift, m, the product of the overlapping parts is computed. The final output correlation function, R[m], is then determined by adding the resulting products.

Here is a step-by-step breakdown of how the correlation functions:

1. Slide q[n] over p[n]: To begin with, we first slide q[n] over p[n], which time-shifts q[n] to the left or right by different times m. A new function, q[n-m], is thus created, where m is the time shift.

2. Multiply p[n] and q[n-m]: Then we multiply p[n] by q[n-m], which is the time shift, giving us the time-shifted form of q[n]. This gives us a product function that represents how p[n] and q[n-m] overlap with each other.

3. Add up all products: To get the final output correlation function, R[m], we add up this product function over all possible time shifts, m. This sum indicates the total of all the overlapping products between p[n] and q[n-m].

One interesting property of correlation in Fourier analysis is that, for discrete-time signals, it is equivalent to complex conjugation in the frequency domain. Consequently, the act of complex conjugating the pertinent frequency components in the frequency domain is tantamount to doing a Discrete Fourier Transform (DFT) of a correlation operation. Due to this property, correlation becomes a good way to analyze the relationship of the phases and amplitudes in digital data.

Properties of Correlation

Correlation in Fourier analysis is a helpful method for studying digital signals because of a number of significant properties:

1. Linearity: Since correlation is a linear operation, it produces the same linear combination of correlations when applied to a linear combination of signals. Commutativity: Since correlation is commutative, it is unaffected by the signals' chronological order. This means that $q[n] * p[n] = p[n] * q[n]$.

2. Time reversal: Correlation is symmetric with regard to time reversal. This implies that if we correlate one signal with the original signal and another with the order of samples reversed, the outcome will be the same as if we correlate the original signal with the reversed signal. $p[-n] * q[n] = p[n] * q[-n]$, in other words.

3. Conjugate symmetry: In the frequency domain, correlation is conjugate symmetric. This means that the corresponding frequency components can be obtained using the complex conjugation of the discrete Fourier transform (DFT) of a correlation operation. This characteristic enables us to extract phase and amplitude information from signals using correlation.

4. Normalization: By dividing by the product of the lengths of the correlated signals, one can normalize correlation. When the signals are identical, this normalization guarantees that the correlation function's maximum value is 1.

Due to these attributes, correlation is a versatile tool that may be employed for various tasks in digital signal analysis, such as extracting features, matching signals, and recognizing patterns.

4.3 Aliasing Effect

The Fourier analysis is available in four different forms. The only one that can be implemented with a digital system is DFT since it is discrete and finite in both domains. It is necessary to make sure that data is accurately represented by DFT in anyof given timeperiod while approaching other Fourier analysis versions. Physical devices can only produce signals for a limited amount of time and finite order frequency components, which makes it feasible. As a result, it is necessary to select the sample interval and record duration carefully. Then, the DFT may effectively represent all waveforms produced by physical devices, practically speaking. This chapter instructs us on how to choose the suitable sampling interval and measure its duration.

When we sample a continuous-time signal at a rate that is less than twice the highest frequency component in the signal, we experience a phenomena known as aliasing. In this instance, the sampling procedure results in frequency components in the discrete-time signal that appear lower than their true frequencies. This phenomenon is called aliasing, and it can lead to spectral estimates that are inaccurate or deceptive.

In order to comprehend aliasing, let's look at a basic illustration. Assume for the moment that we sample a 10 Hz sinusoidal signal at a rate of 5 Hz, or 10 samples per second. The sampled signal seems to oscillate at a lower frequency of 2 Hz when shown. That is, ten samples every five seconds. This is because extra frequency components, or "aliases," were added to the signal during the sampling process.

The Nyquist sampling theorem dictates that for precise reconstruction of a continuous-time signal from its samples, the sampling rate must be no less than twice the frequency of the highest component in the signal. The reason for this is that when sampling at a lower rate, aliasing occurs as certain high-frequency elements of the signal are compressed into the baseband or frequencies that are lower than half of the sampling rate.

Consider a sinusoidal signal with a frequency of 20 Hz and a sampling rate of 10 Hz, or 10 samples per second, to demonstrate this. The recorded signal appears to fluctuate at two distinct frequencies when plotted, which are 2 Hz and 18 Hz (10 samples per 5 seconds and 90 samples per second). This is because the high-frequency component at 20 Hz caused the sampling process to generate an alias at -18 Hz (i.e., below half the sampling rate).

In the real world, where signals are not strictly periodic or where they are sampled at rates that are not high enough, aliasing can have drastic effects on the accuracy and reliability of inferences made from a Fourier analysis.

For this reason, this concept is critical to understand, and measures must be taken that will help to mitigate its effects on real-world applications of Fourier analysis. One way to achieve this is by employing anti-aliasing filters to eliminate high-frequency elements prior to data sampling, or by increasing the sampling rate.

4.4 Leakage Effect

When we compute the discrete Fourier transform of a signal that has a finite duration but is not precisely periodic, we get a phenomenon called leakage in Fourier analysis. Because of its finite duration in this case, the resulting spectral estimate has energy coming from neighboring frequency components. This loss of energy means that the spectral estimates can be distorted or erroneous, particularly for signals with small spectral peaks.

For a basic understanding of leakage, let us look at a simple example. Suppose we have a one-second sinusoidal signal at a frequency of 10 Hz. When we compute the DFT of the signal, we find that instead of the spectral estimate at 10 Hz being a perfect delta function—or a single spike at the right frequency—it has a wider shape, with energy spreading out over the nearby frequency components. This is due to leakage, energy spreading from the nearby frequency components, caused by the signal's finite duration and its lack of periodicity.

From a mathematical standpoint, the most common reason for leakage is that the DFT assumes a perfectly periodic signal. However, signals that are not necessarily periodic—that is, signals that have a finite duration—constitute real-life signals. This assumption brings about spectrum leakage that can lead to spectral estimations which are inaccurate or misleading, particularly for signals with small spectral peaks.

For example, let's take a sinusoidal signal at a frequency of 20 Hz that lasts for one second. We can see that the spectral estimate at 20 Hz is not a perfect delta function but has energy spreading out over the nearby frequency components due to leakage. We can demonstrate this by computing the DFT of such a signal. However, the leakage can be reduced by tapering the signal using windowing techniques before calculating the DFT. This will depress the energy of nearby frequency components and thereby make the spectral estimate more precise.

Chapter 5

APPLICATIONS

5.1 Image Restoration

Image restoration in Fourier transforms refers to the act of **restoring the original image from a distorted or corrupted version**.
The Fourier transform, which changes a picture from the spatial domain to the frequency domain, serves as the foundation for this method.
A blurry or noisy image is the result of some frequency components being lost or attenuated when an image is distorted.
Estimating the absent frequency components and reconstructing the original image are the objectives of image restoration.
Using **Wiener filtering, deconvolution, and iterative techniques**, the original image is restored by extrapolating the noise and blur parameters from the deteriorated image.

Image degradation or restoration **degradation phenomenon**
Let f(x, y) be an original function.Let h(x,y) be a degradation function.Let g(x,y) represent a deteriorated image.Let η (x,y) represent the noise.The user's text is simply two backslashes.

When an image is aquired by the imaging system, the process of degradation starts

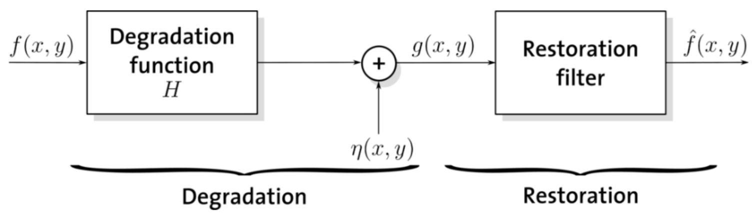

Figure 5.1: Image degradation Restoration Block Diagram

Additive Noise -

$$g(p,q) = f(p,q) + n(p,q)$$

Linear blurring -

$$g(p,q) = f(p,q) * h(p,q)$$

Degraded image is expressed as -

$$g(p,q) = f(p,q) * h(p,q) + n(p,q)$$

Now applying the Fourier Transformation, the equation in frequency domain becomes as -

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

The original image can be restored by rearranging this equation -

$$F(u,v) = G(u,v)/H(u,v) - N(u,v)/H(u,v)$$

After this, f(p,q) can be obtained by applying an Inverse Fourier Transformation to F(u,v).(Deconvolution or Inverse Filtering)

5.2 Time Series Analysis using Discrete Fourier Transform for Financial Forecasting

Time series analysis is an essential undertaking in finance for comprehending and forecasting the behavior of financial markets. An effective method for analyzing financial time series is through the utilization of Discrete Fourier Transforms (DFTs), which enable the breakdown of a time series into its individual frequencies. This explanation will examine the application of Discrete Fourier Transforms (DFTs) in the classification of financial time series.

What is the rationale for employing Discrete Fourier Transforms (DFTs) in the analysis of financial time series?
Financial time series frequently display intricate patterns, including trends, cycles, and noise. The Discrete Fourier Transform (DFT) is highly advantageous for evaluating signals of this nature due to its specific characteristics:

1. Periodic components are extracted by dividing the time series into its frequency components. This allows us to spot recurring patterns, such as daily or weekly cycles.

2. Distinguishes between the underlying trend and random fluctuations: Here Discrete Fourier Transform (DFT) enables extraction of fundamental pattern in data by isolating it from the random fluctuations, hence facilitating the identification of patterns.

3. Frequency band identification: Through analysis of the frequency spectrum, we can discern particular frequency bands that hold significance in the financial market, such as those associated with high-frequency trading.

Applying Discrete Fourier Transforms (DFTs) to categorize financial time series To categorize financial time series using Discrete Fourier Transforms (DFTs), we can proceed as follows:

1. Calculate the Discrete Fourier Transform (DFT): Utilize the

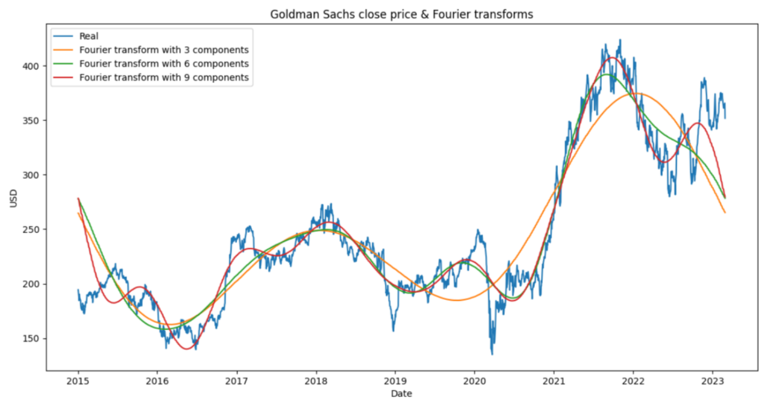

Figure 5.2: This figure is taken from ARIMA (AutoRegressive Integrated Moving Average) and Fourier Transform Analysis of predicting stock prices.

DFT on the financial time series to acquire its representation in the frequency domain.

2. Retrieve characteristics: Identify and isolate important characteristics from the representation of the frequency domain, such as:

    - Peak frequencies refer to the frequencies in a spectrum that have the highest amplitude.
    - Frequency bands: Determine the particular frequency bands that are pertinent to the financial market.
    - Spectral power: Determine the overall power present in the spectrum

3. Apply machine learning methods, such as classification trees, random forests, and neural networks, to categorize the financial time series according to the extracted attributes.

Example:
Categorizing stock values using Discrete Fourier Transforms (DFTs).

Let us examine a scenario in which we aim to categorize stock prices as either "bullish" or "bearish" by analyzing their frequency domain characteristics.

1. Calculate the Discrete Fourier Transform (DFT) of the time series data representing the stock prices.

2. Derive features from representation of signal in frequency domain:

   - Highest frequencies: Determine frequencies that exhibit the greatest magnitude (e.g., daily or weekly patterns).
   - Frequency bands: Determine the particular frequency bands that are pertinent to the stock price, such as those associated with high-frequency trading.

3. Apply a machine learning algorithm, such as a classification tree, to categorize the stock price.

   - Classify as "bullish" if the peak frequencies are linked to high-amplitude values.
   - Assign the label "bearish" to instances where the highest frequencies are linked to low levels of amplitude.

By implementing this methodology, we can construct a classification model that utilizes Discrete Fourier Transforms (DFTs) to scrutinize and forecast the behavior of stock prices.
In conclusion, To summarize, Discrete Fourier Transforms (DFTs) can be an effective method for categorizing financial time series by extracting significant characteristics from their frequency domain representations. By utilizing machine learning algorithms on these characteristics, we can create precise categorization models that forecast the behavior of stock prices and other trends in the financial market.

5.3 5G and future advancements: The significance of Fourier Transformations in wireless communications

Wireless communication systems have been transformed by the fifth generation (5G), offering faster data throughput, reduced latency, and improved connectivity. As we progress towards the upcoming era of wireless communications, referred to as 6G, the significance of Fourier transforms will inevitably increase.

Utilizations of Fourier Transformations in 5G and future technologies

1. Spectrum Analysis: Fourier transformation is employed to examine the frequency spectrum of wireless signals, facilitating the identification of undesired interference and the enhancement of signal transmission.

2. Channel Estimation: The channel properties, such as the channel impulse response, are estimated using Fourier transformation. This estimation is crucial for ensuring high-quality connections.

3. Pulse shaping involves the use of Fourier transformation to manipulate the pulse of transmitted signals, guaranteeing their accurate modulation and demodulation.

4. Modulation Analysis: Fourier transformation is employed to examine the modulation scheme employed by wireless signals, facilitating the identification of unlicensed communications and the enhancement of signal transmission.

5. Synthetic Aperture Radar (SAR) technology utilizes Fourier transformation to produce high-resolution pictures of targets.

6. Fourier transformation is employed to enhance the efficiency of huge multiple-input multiple-output (MIMO) systems, which necessitate intricate signal processing methods.

7. Machine Learning: Fourier transformation is employed in machine learning algorithms to handle vast quantities of data and enhance the efficacy of wireless communication networks.

Fourier transformations' benefits in 5G and beyond increased

1. Signal Quality: By lowering noise and interference, Fourier transforms can enhance signal quality.

2. Enhanced Efficiency: Signal processing can be optimised and computing complexity can be lowered by using Fourier transforms.

3. Enhanced Security: Unauthorised signals can be found and cyber attacks can be defended against by Fourier transforms.

4. Enhanced Capacity: By maximizing spectral efficiency, Fourier transforms can be applied to enhance the capacity of wireless communication networks.

5. Increased Reliability: By lowering mistakes and raising signal quality, Fourier transforms can be applied to raise the reliability of wireless communication systems.

Problems and Prospects

There are several opportunities and problems that need to be addressed even if Fourier transforms provide several advantages in 5G and beyond:

1. Complexity: Fourier transforms are one of the more sophisticated signal processing methods needed by the growing complexity of wireless communication systems.

2. New Interference situations: New signal processing methods that can manage these issues are needed as new interference situations, such as the use of unlicensed spectrum and Internet of Things devices, emerge.

3. Modulation Schemes New: Development of new signal processing algorithms is necessary to construct novel modulation schemes, such as orthogonal frequency division multiplexing (OFDM).

4. More Bandwidth: The growing need for bandwidth calls for more sophisticated signal processing methods that can manage bigger data sets and higher frequencies.

Conclusion

Ultimately, Fourier transformations are essential to 5G and beyond wireless communication systems since they allow for better capacity, reliability, security, efficiency, and signal quality. The significance of Fourier transforms will only increase as we approach the 6G wireless communications technology.

5.4 Fourier Transform Spectroscopy

Nowadays, a method for measuring a substance's spectrum that involves computing the Fourier transform of physical data as a necessary step in the process is called Fourier transform spectroscopy. The fundamental ideas behind this method date back to Michelsont, who created the interferometer in 1880 and showed how to use it for spectroscopic measurements.

It seems clear that Michelson could have created Fourier transform spectroscopy (FE) almost a century ago if he had access to modern computers and other electrical devices at the time. The advancements in specialized mini-computers and related hardware have been the main cause of the explosive growth in FTS applications during the past ten years.

One kind of spectroscopy called Fourier Transform Spectroscopy (FTS) analyzes a sample's spectrum characteristics by use of the Fourier transform. With several benefits above conventional techniques, this potent approach has completely changed the area of spectroscopy.

Principle In conventional spectroscopy, a spectrometer analyzes the transmitted or reflected light after a light source illuminates the sample. The content and characteristics of the sample are then extracted by processing the spectrum that results.
A time-varying signal is produced in Fourier Transform Spectroscopy by modulating the light source used to illuminate the sample. Following measurement of this signal with a detector, the Fourier transform technique is used to process the data.
The spectrum characteristics of the sample are correlated with a frequency domain representation of time-dependent signal obtained by Fourier transform. This makes composition and characteristics of the sample more precisely and efficiently analyzed.

Features
FTS surpasses conventional spectroscopic techniques in a number of ways:

1. Greater Sensitivity: FTS can identify lower concentrations of analytes than conventional spectroscopic techniques because of its greater sensitivity.

2. Faster Acquisition: Real-time analysis benefits greatly from FTS's ability to acquire spectra more quickly than conventional techniques.

3. Greater Resolution: FTS enables the study of minute molecular characteristics by achieving better resolution than conventional techniques.

4. Less Sample Preparation: Generally speaking, FTS is more convenient and lowers the chance of contamination than conventional techniques.

Category of FTS

Among the numerous forms of Fourier Transform Spectroscopy are:

1. Molecular vibrations in the infrared are analysed using infrared (IR) FTS.

2. FTS for Nuclear Magnetic Resonance (NMR): Used for solution molecule structure analysis.

3. Raman FTS: Molecular vibration analysis in the Raman scattering area.

4. Mass Spectrometry (MS) FTS: Gas phase molecular mass analysis.

usages

Applications of FTS are many and include:

1. The purity and composition of materials are monitored in quality control using FTS.

2. Materials science is the study of material characteristics and behavior under various circumstances using FTS.

3. FTS is applied to biological research to investigate the composition and operation of biological molecules like proteins and nucleic acids.

4. Environment Monitoring: FTS is applied to track changes in ecosystems and environmental contaminants.

Problems
There are certain difficulties with FTS even if it has many benefits:

1. Information extraction from the data via FTS calls for advanced data analysis methods.

2. Spectrophroscopy and chemical knowledge are necessary for the often difficult interpretation of FTS results.

3. Equipment cost: Some researchers are unable to afford the frequently costly FTS equipment.

Conclusion
Fourier Transform - The discipline of spectroscopy has been completely transformed by the potent approach. Its benefits over more conventional spectroscopic techniques include faster acquisition, greater resolution, and less sample preparation. Despite certain difficulties, this approach has many uses in many domains and is nevertheless crucial to contemporary research and development.

Chapter 6

Conclusion

This thesis examines the fundamental ideas and current advancements in Fourier analysis and signal processing, which have been pivotal in the progress of modern science and engineering. For almost two centuries, Fourier analysis has served as a fundamental tool in signal processing by offering a mathematical framework to break down signals into their individual frequencies.

Since its initial implementation in physics and engineering, Fourier analysis has undergone continuous development and adjustment to cater to the requirements of emerging technologies and applications, including audio processing, image compression, and biomedical signal processing. Over the past ten years, there have been notable advancements in Fourier analysis and signal processing, largely due to the growing accessibility of extensive datasets and powerful computational resources.

These advancements have facilitated the creation of novel algorithms and methodologies for examining and manipulating signals in diverse fields, such as audio, image, and biological data. Recent advancements have been made in the field of signal processing, specifically in the development of sophisticated approaches that are capable of properly managing complicated and non-stationary data. The demand for increasingly advanced techniques to analyze and process signals in practical scenarios, such as audio processing, voice recognition, and biological signal processing, has prompted this development.

Furthermore, notable progress has been made in the field of computational complexity and parallelization, with the aforementioned advancements in signal processing techniques. These technological advancements have facilitated the creation of more effective algorithms for Fourier analysis and signal processing, which may be simultaneously implemented across several CPU cores.

Recent research has focused on developing advanced algorithms and strategies to perform Fourier analysis on huge datasets. The rise of enormous datasets in domains such as genomics, proteomics, and other areas of biological study has been the driving force behind this phenomenon. This thesis examines significant advancements in Fourier analysis and signal processing, encompassing the creation of novel algorithms and methodologies for the analysis and manipulation of signals across many domains.

We have also examined the obstacles and constraints that arise from these advancements, such as concerns over computational complexity and scalability. Ultimately, Fourier analysis and signal processing remain essential in various scientific and engineering domains. The recent advancements outlined in this thesis highlight the continuous significance of these techniques in various domains, including audio processing, image compression, biomedical signal processing, and others.

As we progress in this domain, it is probable that we will witness substantial advancements in both the theoretical underpinnings of Fourier analysis and signal processing, as well as the creation of novel algorithms and approaches for implementing these methods in practical situations. The growing accessibility of powerful computing resources and extensive datasets will surely fuel additional advancements in this domain.

In conclusion, the ongoing progress in Fourier analysis and signal processing has significant potential to enhance our comprehension of intricate phenomena across various disciplines, including biology, medicine, physics, and engineering. By persistently challenging the limitations of these methodologies, we might uncover fresh perspectives on the surrounding universe and stimulate advancement in various scientific and engineering domains.

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