Klein Bottle and Black Hole Geometry in Understanding Quantum Gravity

A PROJECT REPORT

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CANDIDATE'S DECLARATION

I, DIPA DAS, Roll No(s). 2K22/MSCPHY/64 student of M.Sc. Physics hereby declares that the project Dissertation titled "Klein Bottle and Black Hole Geometry in Understanding Quantum Gravity" which is submitted by me to the Department of Applied Physics & Department of Applied Mathematics, of Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Science is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

Dipa Das

Place: Delhi
Date: 6/6/24

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CERTIFICATE

I hereby certify that the Project Dissertation titled "Klein Bottle and Black Hole Geometry in Understanding Quantum Gravity" which is submitted by DIPA DAS, Roll No 2K22/MSCPHY/64, Department of Applied Physics & Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Science, is a record of the project work carried out by the student under our supervisions. To the best of our knowledge, this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi

Date: 6/6/24

07.06.2024
Prof. S.C. Sharma (SUPERVISOR)

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ABSTRACT

There are two types of curvature in geometry, intrinsic & and extrinsic curvature of space, extrinsic curvature depends on which space or dimension it is embedded into. To create a geometrical structure that can model or be compatible with the Quantum and macroscopic (GR) nature of gravity, we need to be able to connect the topological & Riemannian curvature or metric space. Here we are trying to integrate the topological i.e. Quantum Property of the geometry & the Macroscopic geometrical nature of a Geometrical Space to model Quantum Gravity theory with it. There can be many mathematical approaches to finding quantum geometry. Here we are concentrating on drawing new mathematical structures or approaches from different modified gravity theory that leads to quantum gravity namely Loop Quantum Gravity, Teleparrall gravity, F(g) Modified gravity etc. We are also investigating many mathematical aspects of creating a Quantum gravity theory like the background independence for gravitation force. In LQG there is no background independence, but for the quantum properties of space to exist, there has to be a background dependency in the theory, which can be derived from taking a Klein bottle as a quanta of space rather than the space itself like as the weave of loops creates the space itself for LQG. Also, the concept of time can be derived from the topological property of Klein bottle namely for its sidedness or no boundary for odd numbers which is not in LQG as it doesn't consider time to be a coordinate as it doesn't count the space-time to be 4 dimensional. It can not have a physical time component as it has for the space. Time is generated by the increased number of nodes in the spin network of the weave, which is a mathematical construct.

For unifying the Topological space with the metric space we can form a mathematical structure where the topology of a Klein bottle is multiplied by the metric tensor of a 5

particular space to impose a topological variable on the geometry itself so that it has a quantum property. Correction in Riemannian geometry is optional. There is an attempt to model the concept of particle & anti particle in hawking radiation through kelin bottle geometry. Also, the concept of time can be derived from the topological property of Klein bottle namely for its one-sidedness or no boundary for odd numbers.

Many aspects of black hole geometry can be explain or model by Klein bottle geometry. Using Nash embedding concept to describe the singularity & wormhole i.e. er=epr paradox & a new TQFT model to describe Hawking radiation we can understand more about the possible quantum gravity model. The metric of Klein bottle hole is compared with both Kerr metric & schrchild metric of blackhole to find out a time dimension. The isometric embedding theory of hawking temperature predicts the hawking temperature. Here we tried to do the same with the Klein bottle to prove the connection between the Klein bottle geometry & black hole geometry by predicting the Hawking temperature. In Nash embedding the the minimum possible dimension is possible to describe the klein bottle is 5 dimensions is the same as the klein bottle hole metric, as it also has the 5 component. Some other geometrical description of wormholes is shown here also, along with a different explanation of blackhole creation because of the high gravity along with the explanation of the creation of singularity in plank length.

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Chapter 1

Introduction

1.1 Overview

Here we are describing a black hole geometry as it is the perfect object to study or model for quantum gravity, as it is the object where the general theory of relativity & the quantum field theory both apply. We see in a black hole two main metrics can describe its geometry, one is the Schwarchilds metric & the other is the Kerr metric for a rotating blackhole, here we can correct the Penrose diagram according to klein bottle geometry of a black hole. Nash embedding is a big part of this new model of the klein bottle geometry ofblack hole, it describes the instability of the wormhole, along with the singularity. The Heaviside step function is a type of step function that can be use for describing the singularity.

1.1.1 A Mathematical Overview of General theory of relativity

General Relativity is a proposition of gravity and a set of physical and geometric principles attained from Newtonian gravity through a transition from the conception of ‘ space ’ to ‘ space-time ’ and the transition from flat figure to curved figure, which lead to a set of field equations that determine the gravitational field, and to the geodesic equations that describe light propagation and curvature of spacetime due to any energy momentum tensor

- The Formulation of geodesics, which are the least path taken from one point to another in a curved space, by the parallel transport system

● Defining gaussian normal co-ordinates for a curved space to model in a flat coordinate approximately as much as possible without introducing the intrinsic curvature in space, In two

dimension it can be thought like a tangent plane which can lie in a curved manifold

● Defining Tensor Equation: The Derivation of the vector will be same for every reference frame with respect to the co-ordinates.

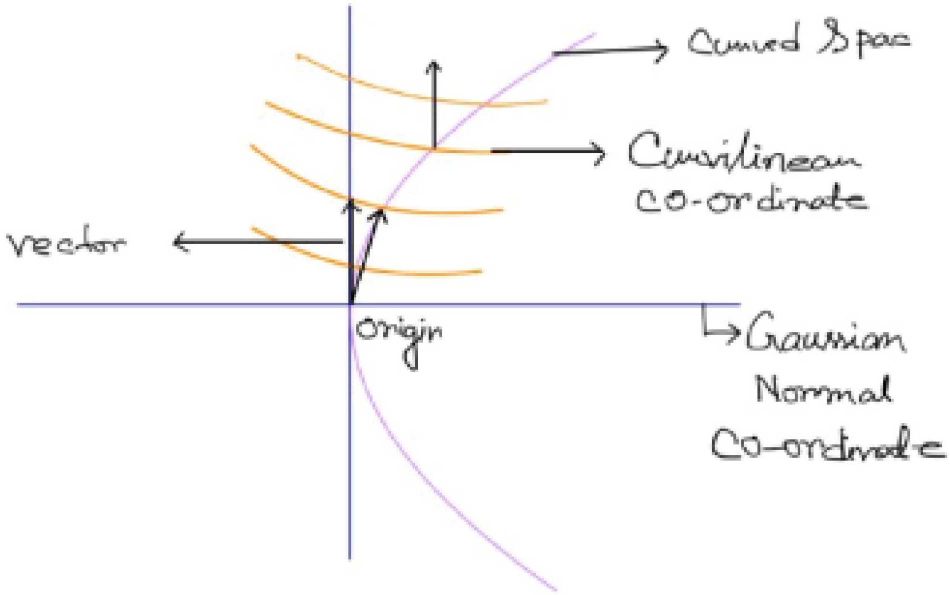
● We need to define derivative of a vector which is not just a derivation w.r.t the co-ordinate as it will not be same for curved spaces.

● Deriving the co variant & contravariant tensor

● Derivative of a point: we need to look at the nearby points, construct Gaussian normal coordinates near that point, Then shift the vector to the Gaussian normal co-ordinate Origin from the curved coordinate or space, the same vector. The difference between this to vector is the Derivative of that vector and we can find it by comparing this two vector.

Meaning differentiate the vector in curved space we need to differentiate it in

the normal co-ordinate. So the term which comes because of the change in co-ordinate is the Christopher symbol Γ^{r}_{rm}



● Defining Cristofer symbol symmetry of the indices $\Gamma^{r}_{rm} = \Gamma^{r}_{mr}$ which are not same for torsional geometry

● Defining the energy-momentum tensor for a given mass or energy

● Adding the Ricci tensor to equate the dimension in the equation which describes volume changes due to curvature and spacetime curvature due to tidal forces. ● Equating the curvature of space as the Riemann & Ricci tensor with the

energy-momentum tensor which gives

● Riemann tensor we can find how to calculate the length of every infinitesimal curve. And from that we can build up the geometry as a whole. That information in turn is captured in the metric tensor, four by four, tensor in four spacetime dimensions. And that tensor has a curvature characterised by the Riemann tensor. So in other words, the metric tensor and the Riemann tensor aren't completely separate. The whole point of the metric tensor is it tells you everything you need to know as long as you can extract it. So the Riemann tensor is defined in terms of the metric tensor.

● Find Field equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

1.1.2 A Mathematical Overview of Quantum Mechanics & Topology

● Quantum mechanics is define by the wavefuncion of a system, which describes the distribution of possibilities of the quantum states.

● The energy is discrete

● The fundamental group is defined by the number of holes created between two points, which is the topology of that manifold

● We can define infinite point sets that are superimposed in a manifold ● These points in the form of groups like S(U)2, S(U)3

● Which Define the extrinsic curvature of that point of that manifold ● Merging these two theories can give us the possible quantum theory of gravity or a geometrical structure to model it.

1.1.3 Topology of a Klein bottle

The paramatric equations describe a smooth, closed surface that passes through itself once. The surface is not orientable, which means that there is no way to define a consistent "inside" and "outside."

Unlike parametric equations, which describe the geometry of a surface point by point, a topological equation captures the overall connectivity and structure of a space. Unfortunately, there isn't a single, simple equation that defines the Klein bottle's topology in the same way we have for simpler shapes like spheres or tori.

$Fx(\theta, \varphi) = R(\cos \theta\ 2 \cos \varphi - \sin \theta\ 2 \sin 2\varphi)$ (1)

$Fy(\theta, \varphi) = R(\sin \theta\ 2 \cos \varphi - \cos \theta\ 2 \sin 2\varphi)$ (2)

$Fz(\theta, \varphi) = P \cos \theta(1 + \sin \varphi)$ $Fw(\theta, \varphi) = P \sin \theta(1 + \sin \varphi)$ (3)

F is an embedding of the Klein Bottle in to $\mathbb{R}^4$

However, there are several ways to represent the Klein bottle's topology using mathematical tools:

1. CW complex: We can build the Klein bottle as a CW complex, a space constructed by glueing together cells of different dimensions. The Klein bottle

requires one 0-cell (a point), two 1-cells (lines), and one 2-cell (a surface). The specific way these cells are glued together defines the Klein bottle's non-orientability and self-intersection.

2. Fundamental group: The fundamental group of a space describes its "loopiness" and how paths within the space can be continuously deformed into one another. The Klein bottle's fundamental group is Z×Z/2Z, indicating its connection to two Möbius strips and its non-orientability.

3. Homology groups: Homology groups capture the "holes" and "tunnels" in a space. The Klein bottle has non-zero homology groups in dimensions 0 and 1, reflecting its closedness and single, non-contractible loop.

4. Cross-cap sum: The Klein bottle can also be described as the connected sum of two projective planes, meaning two copies of the projective plane glued together in a specific way. This representation emphasizes the self-intersection and non-orientability.

These are just some examples, and the choice of representation depends on the specific context and desired properties to highlight. Overall, understanding the Klein bottle's topology requires using a combination of these different tools and concepts, rather than a single equation.

The Klein bottle is a mathematical object with a unique topology. It has unusual properties that set it apart from everyday surfaces. Here's a breakdown of its key topological features like onesidedness, closedness, and self-intersection in 3d as Klein bottle hole.

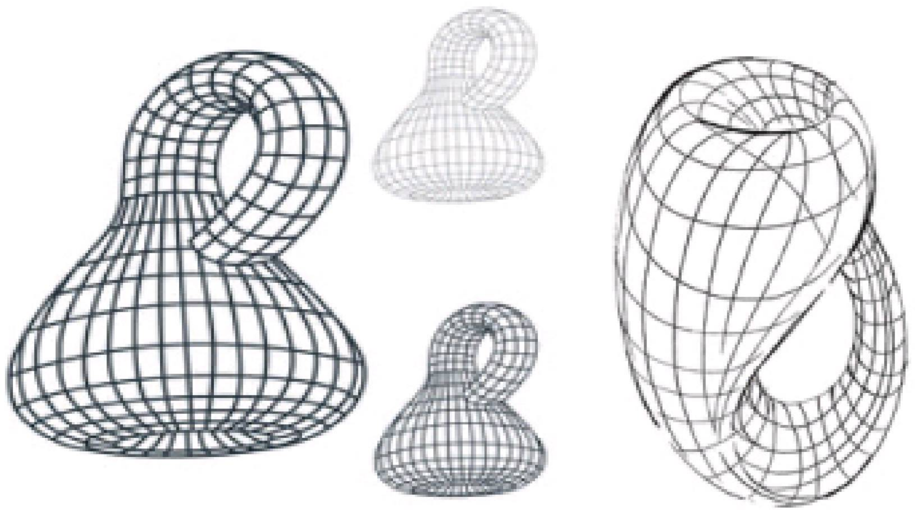


Fig 1: Klein Bottle

One-sidedness: Unlike a sphere or a cylinder, the Klein bottle has only one side. Imagine an ant crawling on its surface; it could travel from any point to any other without crossing an edge or boundary. This non-orientability is a defining characteristic of the Klein bottle.

Closedness: Despite having only one side. This means it has no holes or gaps, forming a continuous loop. Think of it like a closed loop of Mobius strips glued together in a certain way.

Self-intersection: Interestingly, the Klein bottle also intersects itself. This happens because the neck of the bottle passes through its own body, creating a unique crossing point in the topology.

Euler characteristic: The Euler characteristic is a topological invariant that helps distinguish different shapes. For the Klein bottle, it's equal to 0, which differentiates it from a sphere (2) and a torus (0).

Non-embeddability in 3D: While the Klein bottle can be visualized and described mathematically, it cannot be perfectly represented in three-dimensional Euclidean space. This means you cannot create a physical model of the Klein bottle without some form of distortion or self-intersection.

Additionally:

The Klein bottle can be constructed in various ways, including glueing two Mobius strips together or using geometric transformations.

2. Mathematical Models

2.1 Klein bottle metric & black hole metric

There is two main types of black hole metric, one is metric & the other is kerr metric, here we are trying to model kerr metric with klein bottle metric.

(ving other universe through ringholes and Klein-bottle holes Pedro F. González-Díaz #)

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

Is the Schwarzschild metric

$$s^2 = -e^{2\phi} dt^2 + \theta\left(2\pi - \varphi_1\right)\left(\frac{dr_1^2}{1 - K\left(b_1\right)/b_1} + d\Omega_1^2\right) + \theta\left(\varphi_1 - 2\pi\right)\left(\frac{dr_2^2}{1 - K\left(b_2\right)/b_2} + d\Omega_2^2\right) \quad [1]$$

Is the Klein bottle metric

& The Kerr metric is

$$ds^2 = \left(1 - \frac{2m\rho}{\rho^2 + a^2\cos^2\theta}\right) c^2 \left(dt^2\right) - \frac{\rho^2 + a^2\cos^2\theta}{\rho^2 + a^2 - 2m\rho}(d\rho)^2 - \left(\rho^2 + a^2\cos^2\theta\right)(d\theta)^2$$

$$- \frac{4m\rho a \sin^2\theta}{\rho^2 + a^2\cos^2\theta} c\, dt\, d\phi \;-\left(\left(\rho^2 + a^2\right)\sin^2\theta + \frac{2m\rho a \sin^4\theta}{\rho^2 + a^2\cos^2\theta}\right)(d\phi)^2$$

Here we can see that the kerr metric has the same dimensions as the Klein bottle hole metric, It is also an 5- 5-dimensional object like kerr black hole. Here we can compare & find out any connection between two metrics & approximate or simplify it to the other metric.

First Term

From the first term we can see a connection of blolzman constant with the phi angle, which is the angle the nash embedding happens, to indicate a connection of plank law of blackbody radiation which is the starting point of theoritical quantum mechanics & the quantum energy state.

From the first term of Schwarzschild metric & k.b hole metric we get,

$$-c^2 = -e^{2\phi}$$

$$or,\ c^2 = e^{2\phi}$$

$$or,\ 2\phi = \log\left(c^2\right)$$

$$or,\ \phi = \log\left(c^2\right) \div 2$$

$$\phi = 8.47712125472$$

$$\phi \approx\ k_b\ \times 10^5 \qquad Here\, k_b\ is\, in\, ev$$

2.2 Calculating Geodesics for Klein Bottle Hole Metric

Space-time metric for klein bottle hole to be-

$$ds^2 =\ -e^{2\phi}dt^2\ + \theta\left(2\pi - \varphi_1\right)\left(\frac{dr_1^2}{1 - K\left(b_1\right)/b_1} + d\Omega_1^2\right) + \theta\left(\varphi_1 - 2\pi\right)\left(\frac{dr_2^2}{1 - K\left(b_2\right)/b_2} + d\Omega_2^2\right) \qquad [1]$$

Consider 2d slices (t=constant, $\varphi_1 = \frac{\pi}{2}$) Then

$$dt^2 = 0,\ \theta\left(\varphi_1 - 2\pi\right) = \theta\left(\frac{\pi}{2} - 2\right) = \theta\left(-\frac{3\pi}{2}\right) = 0\ (By\ def)$$

$$d\Omega^2 = 0,\ \text{since}\ \ \varphi = \frac{\pi}{2}\ :\ Hence\ (1)\ becomes$$

$$ds^2 = \theta\left(\frac{3\pi}{2}\right)\left(\frac{dr_1^2}{1 - k\left(b_1\right)/b_1}\right) + b_1^2 d\varphi_2^2$$

$$ds^2 = \left(\frac{dr_1^2}{1 - k\left(b_1\right)/b_1}\right) + b_1^2 d\varphi_2^2 \quad \left(as,\ \theta\left(\frac{3\pi}{2}\right) = 1,\ by\, def\right)$$

Comparing it with cylindrical co-ordinates we get-

$$\left(\rho\left(r\right), \varphi, z\left(r\right)\right)$$

$$ds^2 = d\rho^2 + \rho^2 d\varphi^2 + dz^2$$

$$= \left[\left(\frac{dz}{dr}\right)^2 + \left(\frac{d\rho}{dr}\right)^2\right] dr^2\ +\ \rho^2\left(r\right) d\varphi^2$$

Now we get,

$$\rho^2 = b_1^2$$

$$\rho = \pm\, b_1$$

$$\frac{d\rho}{dr} = \pm\frac{db_1}{dr}$$

Hence we have,

$$\left(\frac{dz}{dr}\right)^2 + \left(\frac{db_1}{dr}\right)^2 = \frac{1}{1-k\left(b_1\right)/b_1}$$

This is the geodesic equation

Geodesic equation from Lagrangian

We know a Klein bottle hole metric is -

$$ds^2 = -e^{2\phi}dt^2 + \theta\left(2\pi-\varphi_1\right)\left(\frac{dr_1^2}{1-K\left(b_1\right)/b_1} + d\Omega_1^2\right) + \theta\left(\varphi_1-2\pi\right)\left(\frac{dr_2^2}{1-K\left(b_2\right)/b_2} + d\Omega_2^2\right) \quad [1]$$

$$L = \sqrt{e^{-2\phi}\left(\frac{dt}{d\lambda}\right)^2 + \left(\frac{dr_1}{d\lambda}\right)^2\theta\left(2\pi-\varphi_1\right)\left(\frac{\left(\frac{dr_1}{d\lambda}\right)^2}{1-k\left(b_1\right)/b_1}\right) + \left(\frac{d\Omega_1}{d\lambda}\right)^2 + \theta\left(\varphi_1-2\pi\right)\left(\frac{\left(\frac{dr_2}{d\lambda}\right)^2}{1-k\left(b_2\right)/b_2}\right) + \left(\frac{d\Omega_2}{d\lambda}\right)^2}$$

Now For the Geodesic equation for t,

$$\frac{d}{d\lambda}\left(\frac{\delta L}{\delta \dot{t}}\right) = \frac{\delta L}{\delta t} \qquad \left(\dot{t} = \frac{dt}{d\lambda}\right)$$

$$Now, \quad \frac{dL}{d\dot{t}} = \frac{1}{2L}\,2\frac{dt}{d\lambda}e^{-2\phi}$$

$$\frac{d}{d\lambda}\left(\frac{2}{2L}\frac{dt}{d\lambda}e^{-2\phi}\right) = 0$$

$$or, \qquad L\frac{d}{dr}\left(\frac{1}{L}\frac{dt}{d\lambda}\right)e^{-2\phi} = 0$$

3. Observational Evidence

3.1 Isometric embedding of Hawking temperature

The Hawking temperature is equal to the Unruh temperature in an isometric embedding extending through the killing horizon. In Paper, we can find it is also true for the conformal isometric embedding of Schwarzschild in R 4,1. But there are some conditions

The Hawking temperature described by the Schwarzschild metric aligns with the temperature experienced by an observer moving along conformal circles in five dimensions, as per the Unruh temperature. So we can say that for the conformal isometric embedding of a klein bottle will also produce a hawking temperature which will be equal with the mapping of Unruh temperature if it can be embedded in 5 dimensions.

Also we can find a connection of thermal radiation which is analogous to Unruh temperature. T = −κ|bi=b0i /2π, we know Klein bottle hole emits thermal radiation at negative temperature & positive temperature in different regions. From these findings we can show a direct connection between hawking temperature so as hawking radiation from klien bottle hole geometry, which shows that the geometry of a blackhole can be found from a topological klein bottle geometry, we can say that because of the observational evidence.

4 Theoretical Models

4.1 Nash embedding & the geometry of a black hole

We are using nash embedding because it can potentially describe the ring singularity of black hole & creation of quantum states & wormhole creation.

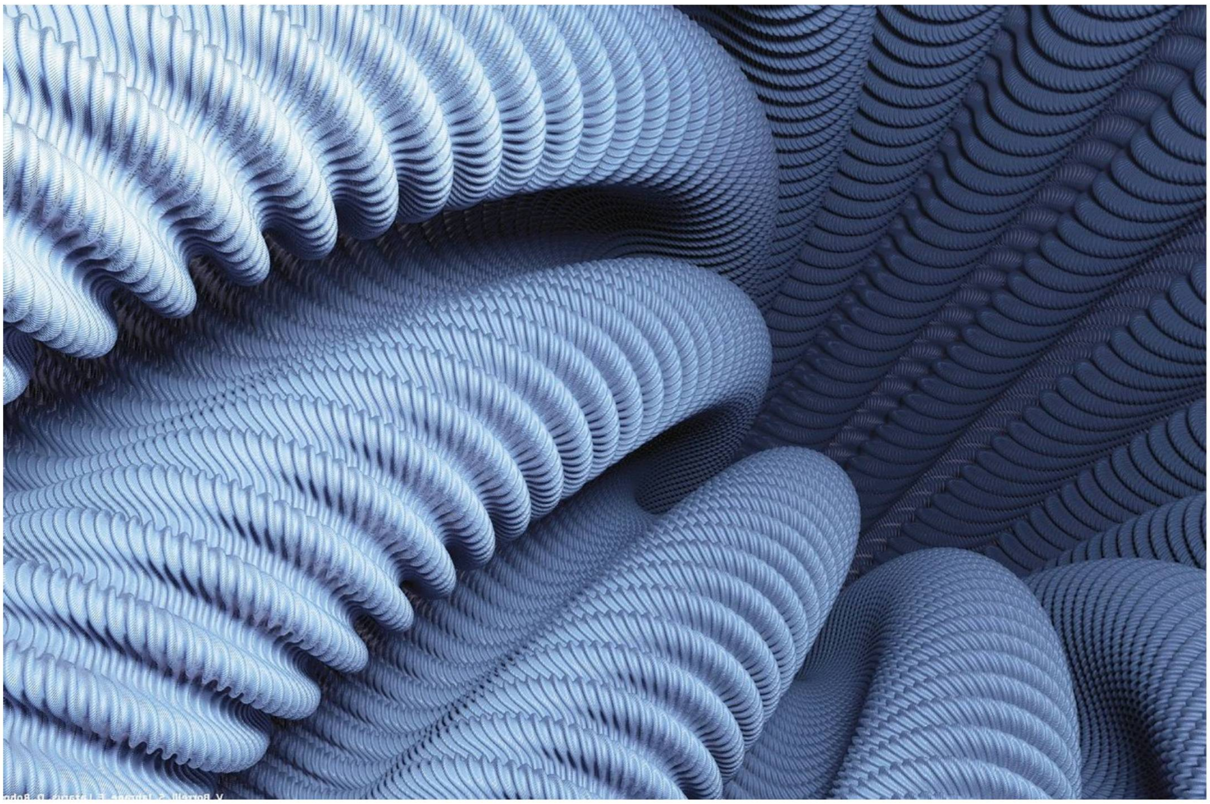
(Machiraju 2018)

Nash embedding on a torus

We Can apply the perturbation device in nash embedding on a klein bottle hole metric

$$\begin{pmatrix} -e^{2\phi} & 0 & 0 & 0 & 0 \\ 0 & \theta\left(2\pi-\varphi_1\right)\left(\frac{dr_1^2}{1-K(b_1)/b_1}\right) & 0 & 0 & 0 \\ 0 & 0 & \theta\left(2\pi-\varphi_1\right)d\Omega_1^2 & 0 & 0 \\ 0 & 0 & 0 & \theta\left(\varphi_1-2\pi\right)\left(\frac{dr_2^2}{1-K(b_2)/b_2}\right) & 0 \\ 0 & 0 & 0 & 0 & \theta\left(\varphi_1-2\pi\right)d\Omega_2^2 \end{pmatrix} = \sum_{\alpha} \frac{\delta z_\alpha}{\delta x_i}\frac{\delta z_\alpha}{\delta x_j}$$

4.2 Quantum nature of blackhole: The Creation of Quantum states from nash embedding in klein bottle blackhole geometry

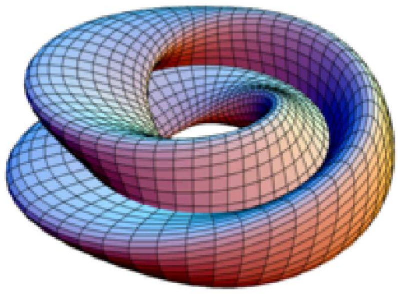


(ABDOUN 2012)

Inside of a blackhole singularity: In a klein bottle ring hole

There exist a lowest energy klein bottle, which can be taken as a first klein bottle quantum state to describe quantum energy phenomenon in klein bottle black hole. This is called a lawson klein bottle.

4.3 Willmore klein bottles & minimum energy state



Minimum energy klein bottle

There are three different regular homotopy classes exist of immersions of klein bottles into $\mathbb{R}^4$, These are the different topological classes of klein bottle, we know from Willmore klein bottle theory of minimum energy state is that as each of these classes contains an embedding and one specific class has the minimizer with the minimum Willmore energy, thus proved that klein bottle can be seen as a quantum object & it can be represented as the energy state of the black hole.

We also can see that there exist of infinitely many distinct embedded klein bottles in with willmore energy of 8π and euler normal number of ± 4

These surfaces remain distinct even under conformal transformations of $\mathbb{R}^4$

Willmore Surfaces: The Klein bottles in the willmore paper are Willmore surfaces as they minimize the Willmore energy within their regular homotopy class.

The properties and classification of these surfaces, particularly focusing on their geometric and topological characteristics in higher-dimensional spaces.

From these findings we can see that the the willmore energy can describe the quantum energy which will occur from nash embedding of the klein bottle if we define the perturbation of nash embedding as the modes of energy & each mode can contain a specific distinct quanta of energy, so the minimize klein bottle energy of willmore can be the sum of these energies.

Lower Bound of Willmore Energy:

- For Klein bottles which is immersed in $\mathbb{R}^3,\ W(f) \geq 8\pi$
- For Klein bottles which is immersed in $\mathbb{R}^4,\ W(f)\, can\, be\, less\, than\, 8\pi$

Existence of Minimizing Klein Bottles:

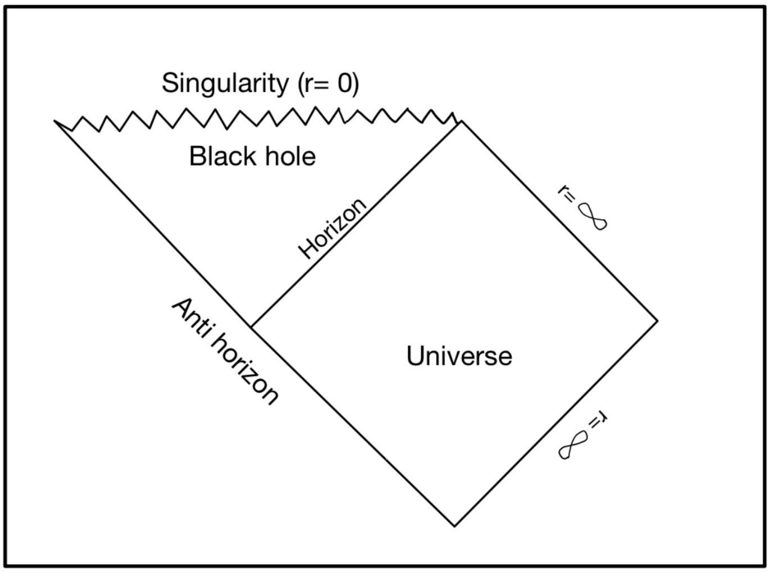
- There exists a smooth embedded Klein bottle in $\mathbb{R}^4$ that minimizes the Willmore energy, with the energy of strictly less than 8π.
- The minimal energy is achieved by embedding the Klein bottle in $\mathbb{R}^4$ which is the 4 dimensional space, which is the dimension of general theory of relativity.

4.4 Singularity & Penrose diagram

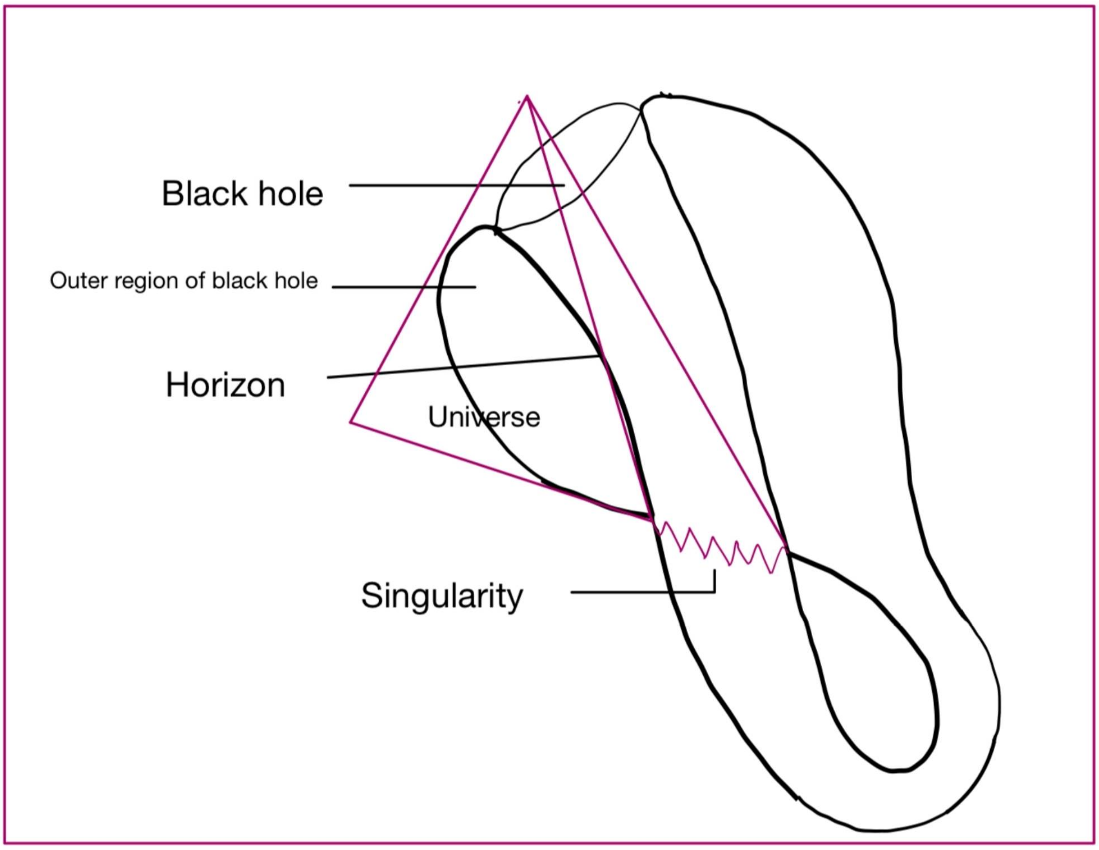
Singularity in a blackhole is the most mysterious problem we still have now, in kerr black hole the there exist singularity ring holes, like a klein bottle ringhole, which can create many ring holes from the perturbation in the nash embedding.

We also found out the Heaviside function in the klein bottle hole metric, which is a good mathematical indication of the existence of singularity for a static or non-nash embedded klein bottle which is approximate with Schwarzschild black hole geometry.

Another explanation of singularity can come from the fact that gravity doesn’t work in plank length scale, which can say a lot about how a singularity is created as it can be created when the curvature of space is at plank length because of the high density of mass in the black hole & then suddenly if the gravity stops working then there will be no curvature in that region, such as a singularity is created. This is for a Schwarzschild metric.

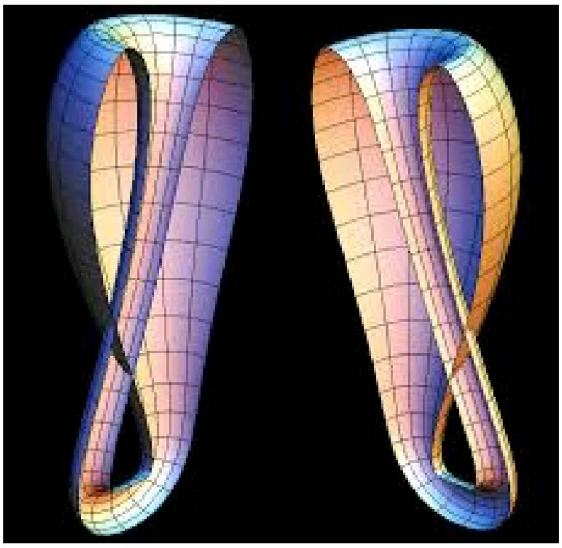


Penrose diagram for black hole



Mapping Penrose Diagram of black hole geometry with Klein bottle geometry

4.5 TQFT MODEL OF QUANTUM GRAVITY FROM KLEIN BOTTLE



(Terng 2000)

Two Mobius band in different directions: Particle & anti-particle pair

FOR HAWKING RADIATION

- We know that hawking radiation is created because of the pinch of the quantum field which creates the illusion of particle & anti-particle pair which is called hawking radiation when the virtual particle travels from the blackhole
- In klein bottle TQFT we can first model those particles & antiparticle pairs to be the two opposite directions mobius band of klein bottle
- Then to find the gauge connection field
- Define the symmetry of these Mobius bands locally to find the parallel transport vector
- To find the energy of the Hawking radiation or gravitons

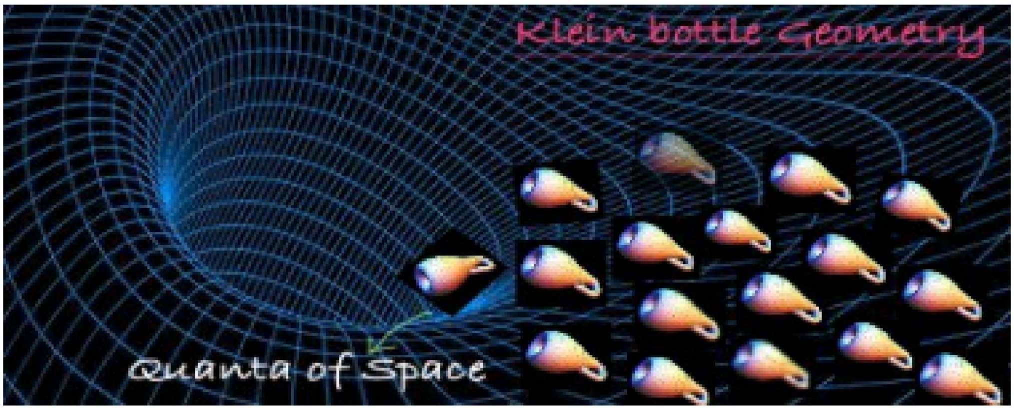
4.6 Blackhole, wormhole & quantum energy states

- In kerr black hole the singularity is a ring & because of it's rotation, there can be many ring singularity.
- Which can be described from klein bottle hole, such as ringhole which can also create a worm hole
- From nash embedding of klein bottle we can see the region of ring singularity can get very close to itself & when it reaches the plank length, the gravity stops working which can be the reason for the creation of the singularity.
- When the modes of perturbation get very close & overlap with itself there can't be any more modes of perturbation so it can create a closed surface which is the reason for the creation of singularity.
- These modes of perturbation acts as the quantum states
- Each state has a distinct energy, let's say E_k
- The overlapping of these energy states can create instability, which are very much known to exist in wormholes also

- With two klein bottles modes overlapping there can be a creation of wormhole
- From these theory we can hypothesize that the creation of entanglement between two black hole happens when the energy level of perturbation modes is the same & opposite & then it can create a wormhole in between, which is ER=EPR

5. Generalised Topological Quantum Geometry

5.1 Different mathematical approaches to connect the topological variable of the Klein bottle with the metric tensor by multiplying it with the metric tensor



For Creating a Generalised Geometry:

For a Given Generalised Geometry, we can write

Fx(θ, φ) , Fy(θ, φ), Fz(θ, φ)

$$\{F_x(\theta,\phi)\times F_y(\theta,\phi)\times F_z(\theta,\phi)\}\times\begin{bmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & \dots & a_{nn}\end{bmatrix}=\begin{bmatrix}F_{11} & F_{12} & \dots & F_{1n}\\ F_{21} & F_{22} & \dots & F_{2n}\\ \dots & \dots & \dots & \dots\\ F_{n1} & F_{n2} & \dots & F_{nn}\end{bmatrix}$$

With the Metric of the hole of a Klein bottle:

$$ds^2 = -e^{2\phi}dt^2 + \theta\left(2\pi-\varphi_1\right)\left(\frac{dr_1^2}{1-K\left(b_1\right)/b_1}+d\Omega_1^2\right)+\theta\left(\varphi_1-2\pi\right)\left(\frac{dr_2^2}{1-K\left(b_2\right)/b_2}+d\Omega_2^2\right) \qquad [1]$$

& The Schwarzschild metric is

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

If we equal these two equation, a tensor form which can result into a new topological metric space for the Klien bottle hole geometry.

For Klein bottle as a topological variable itself

A Topological theory of gravity is a theory where the physical manifold is a dynamical variable itself here that is a Klein bottle. Here we can define the klein bottle as the variable itself of the geometry.

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 - \frac{r_s}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$F_n(k.b) = g_{\mu\nu}$$

5.2 The Klien Bottle Hole Metric in 4 Dimension

The Klein bottle intersect itself in the 3 dimensional space which creates a klein bottle hole. But for 4-dimention this represents a geometry which is not intersected by itself. The metric for that geometry can be written as

$$ds^2 = -e^{2\phi}dt^2 + \theta\left(2\pi - \varphi_1\right)\left(\frac{dr_1^2}{1 - K\left(b_1\right)/b_1} + d\Omega_1^2\right) + \theta\left(\varphi_1 - 2\pi\right)\left(\frac{dr_2^2}{1 - K\left(b_2\right)/b_2} + d\Omega_2^2\right) \quad [1]$$

& The Schwarzschild metric is

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right)c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

There is also five components like a a Schwarzschild metric, this can results into creating a similar metric which is consistent with the black hole in a quantum manner as there will be a quantum or topological variable imposed either by the topology of each klein bottle or the very number itself with each point of this 4 dimensional geometry. Which finally will result into a both the topological & metric space of geometry.

5.3 Bringing Back the time as a 4-dimensional co-ordinate

There can be many ways to compute time with a klein bottle geometry

APPROACH 1

In LQG There is no time component in a physical sense like there is the quanta of space, it does exist as the change in number of nodes as the space is expanded in the spin network theory in a mathematical way. But because of the klein bottle manifold exists as a 4 dimensional component it can bring back the time variable as the 4-dimensional topological variable.

APPROACH 2

Or Because of the topological characteristic of one-sidedness or two-sidedness exists for a klein bottle. For which any odd number of klein bottle merging together can form another klein bottle which also has one-side only & for even number two-sides, these can cause a general change in number of side as we go from one point to another point in space for a curved space-time, where more than one klein bottle is merged cause of the intrinsic curvature of space.

APPROACH 3

Another idea of using time as a dimension to represent a Klein bottle is an intriguing one, but it's important to understand that it would be a tropical or abstract representation rather than a nonfictional bone .

In the standard fine description, a Klein bottle is a four- dimensional object with all confines being spatial. The fourth dimension is demanded to avoid the tone- crossroad that occurs when trying to represent the Klein bottle in three confines.

still, we'd be moving down from the standard fine description and into a more abstract interpretation, If we were to consider time as one of the confines. In this interpretation, the Klein bottle could" appear" and" vanish" over time, growing and shrinking as you suggested. This could be allowed of as a Klein bottle being in a four-dimensional spacetime, with its actuality being flash rather than endless.

still, this interpretation would not be a Klein bottle in the strict fine sense. It would be more like a Klein bottle-shaped process or event being in spacetime. The" bottle" in this case would not be a stationary object, but a dynamic event that changes over time.

5.4 LOOP QUANTUM GRAVITY

The amount proposition is erected on classical general reciprocity, which is expressed, Here we consider this manifold to be of klein bottle's manifold in 4 dimension, (compact and without boundaries).

Then considering a S(U)2 connection on the manifold & a vector density on it which will contain the point set in other words quantum properties as local data on the manifold using lie algebra.

$$g\, g^{ab} = \tilde{E}_i^a \tilde{E}_i^b,$$ (3)

$$A_a^i(x) = \Gamma_a^i(x) + \gamma k_a^i(x);$$ (4)

For finding loop algebra, Classical Amounts play a significant part in amount proposition. One of these is the trace of the holonomy, which measures the amount of resemblant transport around closed circles fails to save the geometrical data being transported. The trace of the holonomy is labeled by circles on the three- manifold and is an important geometrical consequence of the curve of the connection in discriminational figure.

we can define a graph which has n links immersed in the manifold M. Then define parallel transport operator of the connection.

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$$\psi_{\Gamma,f}(A) = f(U_1(A), \ldots, U_n(A)).$$ (3)

$$(\psi_{\Gamma,f}, \psi_{\Gamma,h}) = \int_{SU_{(2)^n}} dg_1 \ldots dg_n \overline{f(g_1 \ldots g_n)} h(g_1 \ldots g_n) \,. \quad (4)$$

The scalar product mentioned above can be extended to finite linear combinations of cylindrical functions.

5.5 Using Spin network theory to model Klein bottle manifold to create a Quantum geometry

For General theory of relativity the background independence principle is valid so for creating a generalised quantum theory it should also somehow become equivalent. That kind of generalisation creates a quantum gravity theory is LQG.

For Klien bottle geometry we can use graph like structure to map klein bottles in it, like LQG. For LQG a graph is just a network of one dimensional, lines which are linked as such that together at their end points form a mesh like structure. For klein bottle this structure can be like each klien bottle is staying in the point or zero dimensional dot point of that mesh.

There exist arrows which give each line a direction or an orientation, this can be same for the klein bottle geometrical structure as well. Each line is also labelled with a half- integral number, but for K.B Geometry we propose this number to be any integral number for the one- sidedness & more than one sidedness property of odd & even number of klein bottle merge together. The mathematical representation of these numbers are same as of spin numbers, which is a basic property of elementary particles. For LQG this number is named as spin label. There also exist nodes which are very important for K.B geometry as each node can carry a klein bottle to create a weave but this will not be the space itself like LQG but the geometry which will be able to produce some mathematical background independency for an intrinsic geometrical structure also. The whole structure together called a spin network. This type of structure represents the quantum state of space at a point in time.

For LQG the lines don't have a length, because there is no background geometry involved in it that can derive lengths. This is a quantum system But for K.B geometry it can be background independent but it can also create a quantum system in a topological mannar. To define geometrical structure like lengths or areas, it should be clear that how to measure these quantities.

In LQG the areas of the faces in the graph can only take on certain separate values, which is a typical amount miracle, this property will remain same forK.B figure as any integral value when taken the number of klein bottle as a variable itself for the topological figure. The separate jumps from one to coming value is veritably small, on the order of the Planck area. But for K.B figure the spin network form not inescapably creates a space which is distorted as for LQG a general spin network will represent a space that's malformed and twisted.

5.6 On the Background Indepency

The very reason quantum mechanics & general theory of relativity can not be merge together mathematically because of the very property of Background Independency. because for quantum mechanics any Quantum state of a particle is extremely background dependent.

But for general theory of relativity the theory has to be background independent because it describe how the very fabric of space time change because of any energy momentum tensor or any mass or energy in the space-time, the very background is the dyanamical quantity for it. Because of this the equation represents GR needs to be background independent, it should work inspite of the background.

But for Quantum mechanics some properties have uncertainty and only take fix values only after measurement.

there are many possibilities which are compute in a wavefunction for a system or the particle

Because quantum mechanics the wavefunction & operators itself are dependent on the background.

6. Discussion & Conclusion

A Topological geometric space can be created by taking the klein bottle as the variable itself or introducing klein bottle hole geometry in 4 dimension. We find the background independency problem can be solved by this way & integrate the general theory of relativity.

Farther mathemetical development & specially the schwarzschild metric & the klein bottle hole metric generalisation is neened in order to obtain such geometry. We can also find an approach to make the klein bottle as the quanta of space itself like LQG.

We can find many Mathematical & theoretical connections between klein bottle & blackhole geometry, which can be farther studied to advances our understanding of quantum gravity.

Klein bottle has minimal energy from it's minimum surface & can be used to describe the quantum nature of black holes and know more about the nature of how the gravity & quantum states are related. We also find a direct relationship of hawking temperature from isometric embedding & klein bottle thermal radiation. All these findings theoretically & mathematically nudge us towards the direction of understanding of black hole geometry & as well as the whole quantum gravity in a topological quantum gravity or field theory gravity.

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