An Extended multilevel Transportation Problem with Multiple Inputs and Outputs

A Project Dissertation submitted in partial fulfilment of the requirements for the degree of

MASTERS OF SCIENCE IN MATHEMATICS

Submitted by

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CERTIFICATE

I hereby certify that the Project Dissertation titles " an extended multilevel transportation problem with multiple inputs and outputs " which is submitted by Yogesh Bhardwaj (2K19/MSCMAT/11) and Kanchan Kumari (2K19/MSCMAT/03) to the Department of Applied Mathematics, this document, submitted to Delhi Technological University as part of the requirements for the Master of Science degree, is a record of the project work completed by students under my supervision. This thesis has not been applied in part or in full for any degree or diploma at this university or elsewhere, to the best of my knowledge.

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ABSTRACT

We have suggested an extension to the classic multilevel transportation problem in which for each transportation problem, each shipment connection has several incompatible input and output. The relative efficiency definition is established for each shipment connection. To evaluate the most efficient transportation strategy, two linear programming problem is solved, one is direct transportation and second one is multilevel transportation. A numerical is illustrated to explain the method.

Keywords: Multilevel Transportation problems (MTP); Decision making unit (DMUs); Data envelopment analysis (DEA); Relative efficiency.

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This is to enlighten you that the above manuscript has been appraised by the editors of LNME and is accepted for the **6th International Conference on Advanced Production and Industrial Engineering (ICAPIE)- 2021 to be held during June 18-19, 2021**. The paper is recommended for publication in **Lecture Notes in Mechanical Engineering (Scopus Indexed publication of Springer Nature), ISSN: 2195-4356**. The manuscript has been submitted to Springer in May 2021 and will be online during October 2021.

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Prof. Ranganath M Singari
(Conference Chair, ICAPIE-2021)

Dr. Harish Kumar
(Convener, ICAPIE-2021)

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Chapter 1
Introduction

In numerous research domains, such as the examination of micro unit production possibilities, economics and operations research share common interests. The stochastic frontier approach (SFA) and data envelopment analysis (DEA) have arisen as two alternate developments of Farrell's theories (1957). According to Grosskopf (1986), the parametric technique was created primarily by economists, whereas the nonparametric approach was created by operations researchers. The advantage of DEA over econometric techniques is its ability to easily accommodate the presence of various input and output without making any assumptions about the functional form.

The following are the truth about DEA which make it a significantly priority tool for evaluating efficiency: To begin, Central tendency method (a common statistical strategy), it analyses manufacturer in comparison to the average manufacturer. DEA, on the other hand, is a tipping point. technique, and only compares the greatest producers (s). Second, DEA does not imply that inputs and outputs have a functional shape. It creates its own functional shape from a set of inputs and outputs from several businesses. As a result, there is little risk of frontier technology being misspecified. The econometric approach, on the other hand, presupposes a functional form for inputs and outputs, such as Cobb-Douglas or Translog. Third, efficiency of businesses that produce a single output from a collection of many outputs is assessing by the parametric approach, on the other side DEA easily accommodates presence of numerous outputs. Fourth, for datasets with fewer than 100 observations, the parametric approach's subdivided the error term into two parts, one indicates inefficiency and other indicates stochastic approach, is ineffective (Aigner, Lovell, & Smith, 1997). In contrast, DEA works effectively with a small sample size.The minimal sample size for DEA analysis is 3 times then the total input and output, as a rule of thumb (Nunamaker, 1985; Raab & Lichty, 2002).

In recent years there are many application of DEA in different areas. Many Authors also has reviewed developing scenario and general DEA literature for the evolution of DEA methodology in various time interval for a variety of situations. The rapid rise of DEA research in some last year have resulted in a massive expansion in the DEA literature. Seiford's rapid rise in popularity can be extrapolated from the fact that (1994) Some were from as far back as 1992. Emrouznejad et al. (2008) offered a complete introduction of DEA literature published in many journals/book chapters/proceedings since 1978, demonstrating that the literature has grown exponentially. Liu et al. (2012) indicated in one of the most recent survey reports that, up until 2009, In the ISI Web of Science Database, the topic includes roughly 4,500 papers.

Origin of Data Envelopment Analysis:

The efficiency of Decision making units is measured with " Charnes , Cooper and Rhodes" and for measuring the production units in the study of operational research a specialised research stand were organised. (Frsund & Sarafoglou, 2002) as the seminal paper. The intellectual origins of DEA in economics, on the other hand,may be observed till back to the starting 1950s. Following WWII, linear programming (LP) became widely granted as a useful method for economics anlaysis

In the field of operations analysis, the transportation issue is frequently used. It's a form of linear programming problem. The key goal of the transportation problem is that reduces cost of shipping comparable commodities from different sources to different target with different rim requirements. During the formulation of the classic transportation dilemma, only the cost or benefit for each potential shipment connection is considered.

For each potential shipment connection, many types of variables such as cost, distance, shipment value, manpower, benefit, and so on can be involved in several real-world applications and must be considered during the shipment plan.For each potential shipment connection, the decision makers can have different goals in mind. In such a scenario, we are interested in determining the most efficient transportation strategy possible.

Charnes et al.[3] is the first to incorporate (DEA) data envelopment analysis into the literature. DEA is a mathematical method for comparing the effectiveness of group of DMU for example: airlines, railway, bank, car manufacturers, hospitals,and so on.

When the financial perspective is not the dominant concern, Charne and Rhodes (1978) [4] created DEA to calculate the effectiveness of efficient unit i.e Decision

Making Units . The DEA methodology assesses each DMU's efficiency [9-15] by looking at its resources (inputs) and the outcomes it produces (outputs).

In the practise and study of efficiency analysis, DEA has gained popularity. Banker et al. [3] have been at the forefront of many advances in DEA principles and methodologies. The CCR model was proposed by Charnes et al. [4] to compute relative efficiency of various DMUs. The Banker, Charnes, and Cooper (BCC) model suggests a different variant of DEA. The treatment of returns to scale differs significantly between the BCC and CCR models [6][14]. The constant returns to scale are used to evaluate efficiency in the CCR model. The BCC model is more adaptable and allows for variable scale returns.

As with any mathematical programming model, classic DEA models often present dual formulations. As a result, there are two DEA formulations [12] that are identical . Simply set, among one of formulation of the Envelope model establishes a suitable or achievable production area then calculate gap or distance between every DMU [8] and the area's boundary. The Multipliers model, on other hand, operates with weighted sums of goods and capital. The weighing factors are the most desirable options available for each DMU under particular conditions.

Since the two formulations are dual problems, [11] they will obviously have the same efficiency for each DMU. Furthermore, the two models may provide additional details in addition to performance [7]. To construct a effective DMU which is used to compare inefficient DMUs an envelope model uses the weight from every DMU (denoted as A). What occur in effective DMUs is particularly interesting: they are their own standard, so the framework Linear Programming Problem [13] gives 1 for A refers to that DMU and naught for all others. As a result, the LPP becomes severely degraded..

The multiplier model specifies which valuation coefficients that each DMU can assign to each inputs and outputs. The very essence of DEA is that each DMU offers different values for these multipliers [10]. Every DMU is untied to place a higher value on whatever it excels at while ignoring the variable in which it falls . This freedom should be preserved in some form or another in every DEA model.

There is a scarcity of literature on transportation problems with numerous inputs and outputs. We take a quick look at some related literature. Chen and Lu [2] broadened the scope of the assignment problem by taking into account various inputs and outputs. Alireza Amirteimoori [1] has used a DEA-based approach to expand the transportation issue. As far as we know, there is no work in the literature that applies to our proposed method. We use the BCC model for each shipment connection to expand the transportation problem by considering several inputs and outputs. The relative efficiency of each shipment connection is determined, and the most efficient shipment plan is considered the best solution to the transportation problem.

Aside from the freedom for each DMU to select its own weights, some DMUs can select multiple sets of weights. This can lead to issues,as a result, the aim of our work is to propose a method for selecting a single set of multiplier to all DMUS.

In his seminal work The Measurement of Productive Efficiency, Farrell (1957) created a path-breaking contribution by building linear programming model with the help of actual input and actual output for a selected sample firm , its solution offering a numerical approach of technical efficiency for each firm in the selected sample. Farrwell showed, economic efficiencies may broken down in two categories: allocative efficiency and technological efficiency. Technological efficiency referring a company's capacity to get best result from the given value , whereas allocative efficiencies are referring to a company's capacity to employ inputs in the best possible proportions, given resource pricing. Farrell's (1957) concept can be shown with a simple scenario with firms producing a single output (Y) using two inputs (X1 and X2) through premise of continuous returns to scale (CRS). According to the CRS assumption, a radial increase in the input vector induces a proportional rise in output vector.

The efficient production function is also assumed to be known. The unit isoquant of an efficient producer is represented by the curve SS′ in Figure 1-1. Allow a corporation to employ the number of input specified by point A to give return of one unit .Drawing a line between origin and A, the efficient isoquant will meet at B. Implies the point is efficient point B, that also lie on efficient isoquant SS′ if inputs can be lowered proportionally. As a result, point B represents the same proportion of inputs as A, even so with less number of each input to make a unit amount of output. To produce the same amount of output, an OB/OA percentage of input is required, or, we can say OA/OB times of output will be created with given amount of both inputs. This proportion can be used to determine the firm's technological efficiency.

$$TE=OA/OB$$

Where TE=1 suggests that the company is technically efficient

Where as TE<1 suggest that company is technically inefficient

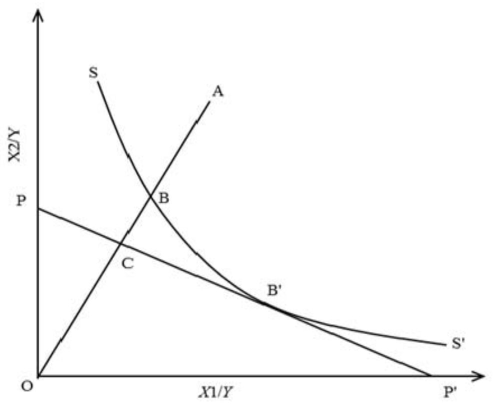


fig 1.1, Allocative, economic and technical efficiency

Here, above the main importance of input price while we measures the efficiency is not examined while defining efficiency . Now take the price line or iso cost line PP′ in (fig 1.1) to obtain the efficient allotment of inputs in terms of input price. The prices of input is given which represents the cheapest option for using the equal ratio of inputs as at point B. As a result, the OC/OB ratio is used to calculate pricing efficiency or allocative efficiency.

AE=OC/OB

where BC denotes the cost savings that would occur if production took place at the (allocatively and technally) Avoid starting with technically efficient but allocatively inefficient point B , start at more efficient point B'.

The proportion OC/OA will be the measure of economic efficiency or overall efficiency if the organisation is both technically and allocatively efficient (EE).

EE=OC/OA

Here , the distance AC will be seen as a cost-cutting measure. It's worth noting that the product of allocative and technical efficiency equals economic efficiency, with all three metrics falling between 0 and 1.

TE*AE= OB/OA *OC/OB= OC/OA=EE

Based on the preceding principles of technical and allocative efficiency, there are two empirical techniques to measuring efficiency. Most economists can choose deterministic or stochastic approach, in which the form of the production function (or isoquant) is either assumed to be known or statistically inferred. In many circumstances functional form of the production function (or isoquant) is unknown.experimentally piecewise linear convex isoquant is created with the help of observable input and output in the nonparametric technique, as (fig 1.2)

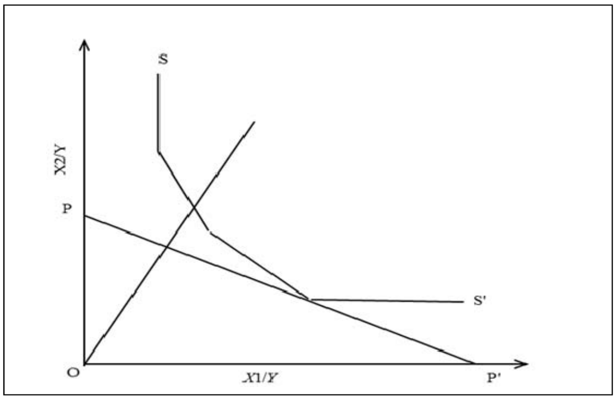


fig 1.2, Piecewise linear convex isosquant

If Farrell's essay from 1957 is considered important, the foundational research published in 1978 is unquestionably the foundation for subsequent improvements in nonparametric approach to measuring technical efficiency. formal definition of efficiency was established by Charnes and Cooper (1985) in their later work:

Only when a production unit is completely efficient can it be said to be 100% efficient.

a) Increasing any of the outputs requires either reduce some other output or increase some of its input.

b) Any of the outputs could be reduced without reducing any of its other outputs or increasing any of its other inputs.

The economist's concept of Pareto (Pareto-Koopmans) optimality is reflected in this term. There is no path to develop a correct or theoretical model of efficiency, i.e an absolute standard, the definition must be modified to refer to efficiency levels relative to known levels achieved elsewhere in similar conditions. As a result, Charnes and Cooper (1985) added the following definition:

Any (unit) achieves 100 percent relative efficiency only when comparisons with other relevant (units) show no indication of inefficiency in the usage of any input or output.

Chapter 2
Basic Models

2.1 BCC Model

The Banker, Charnes, and Cooper [1] (BCC) versatile performance model is as follows:

$$Maximise \qquad \frac{v_0 + \sum_{r=1}^{s} v_r x_{r0}}{\sum_{i=1}^{m} w_i y_{i0}}$$

Subject to:

$$\frac{v_0+\sum_{r=1}^{s} v_r x_{rj}}{\sum_{i=1}^{m} w_i y_{ij}} \leq 1; j= 1,2,3\ldots..n$$

$$\frac{v_0+v_r}{\sum_{i=1}^{m} w_i y_{ij}} \geq \epsilon \; ; r = 1,2,3,\ldots.s$$

$$\frac{v_0+w_i}{\sum_{i=1}^{m} w_i y_{ij}} \geq \epsilon \; ; i = 1,2,3,\ldots.m$$

$v_r, w_i \geq \epsilon$ and v_0 is unrestricted -----------(M1)

Where $\epsilon > 0$ is the non-Archimedean infinitesimal number. The decision variables v_r, w_i denotes weights. The x_{rj}, denotes the observed amount of the r^{th} outputs (r=1,2,….,s) whereas y_{ij} denotes the observed amount of i^{th} input (i=1,2,3……,m) for j^{th} DMU represented as DMU_j ; j=1,2,……,n.

The limit of the ratio of DMUo's weighted sum of outputs to weighted sum of inputs as $e_0 = \frac{v_0+\sum_{r=1}^{s} v_r x_{r0}}{\sum_{i=1}^{m} w_i y_{i0}}$

is used to calculate its efficiency where the limit is sought, subject to the condition that the ratio

$e_j = \frac{v_0+\sum_{r=1}^{s} v_r x_{rj}}{\sum_{i=1}^{m} w_i y_{ij}}$ should not exceed the value one for any DMU_j , j= 1,2,……n.

These m-inputs are used by the DMU_j to generate s-outputs. One of the DMUs, denoted DMUo, is put in the objective function of M(1), which also takes into account the constraints. It would be computationally difficult to solve the problem in (M1). Fortunately, Charnes and Cooper [3] provide a theory of fractional programming that allows (M1) to be replaced with an equivalent linear programming problem. Charnes et. al. [2] describe the transition needed to achieve this. Using the results of the transformation [2], we rewrite (M1) as

$$Maximise \qquad v_0 + \sum_{r=1}^{s} v_r x_{r0}$$

Subject to:

$$\sum_{i=1}^{m} w_i y_{i0} = 1$$

$$v_0 + \sum_{r=1}^{s} v_r x_{rj} - \sum_{i=1}^{m} w_i y_{ij} \leq 0 \qquad ;j=1,2…..,n$$

$v_r, w_i \geq \epsilon$ and v_0 is unrestricted ------------------------(M2)

The restriction $\sum_{i=1}^{m} w_i y_{i0} = 1$ ensures that moving from (M2) to (M1), as well as from (M1) to (M2), is possible . if the objective function's optimal value is unit we say that a DMUo is relatively efficient; otherwise, it is said relatively inefficient.

2. CLASSICAL TRANSPORTATION PROBLEM:

We assume there are m-warehouses with various amounts of a homogeneous commodity that must be transported to n-destinations.The i^{th} warehouse has supply a_i (i=1,2,.....m), and the j^{th} destination has demand b_j (j= 1,2,....n). $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Weassume that the total amount of available supply equals the total amount of demand.C_{ij} is the cost of shipping a unit product from warehouse i to destination j. The number of units delivered to the jth destination from the ith warehouse is zij. The goal is to devise a shipping strategy that reduces the total cost of transportation from warehouses to destinations.

We solve the following transportation problem,

$$Minimise \quad \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} z_{ij}$$

Subject to :

$$\sum_{j=1}^{n} z_{ij} = a_i \qquad ; i=1,2,\ldots\ldots,m$$

$$\sum_{i=1}^{m} z_{ij} = b_i \qquad ; j=1,2,\ldots\ldots,n$$

$$z_{ij} \geq 0 \text{, for all (i,j) .} \qquad \text{---------(M3)}$$

The above transportation problem can be solved using the simplex algorithm**.**

we have extend a classic transportation problems by assuming several incommensurate input and output for every shipping connection. Let's look at m-warehouses with a_i supply units at i[th] warehouse. Consider n-destinations with $\mathbf{b}_j$ units of a demand for the j[th] destination. Let's call the inputs and outputs for each shipping connection (i,j) as $Y_{ij} = \left(yij(1), yij(2), \cdots\cdots yij(s)\right)$ and $X_{ij} = \left(xij(1), xij(2), \cdots\cdots xij(t)\right)$ respectively. There are s+t attributes for each shipment relation (i,j), with s-inputs $Y_{ij}^{(k)}, k = 1,2 \cdot \cdots s$ and t-outputs

$x_{ij}^{(l)}$, $l = 1,2, \ldots, t$. The DEA technique is used to construct the solution protocol in this situation.For each warehouse i we consider all destinations j (j = 1, 2,...n) and each shipment relation (i, j) as a DMU. Using (M2), we can measure the relative efficiency of the i[th]warehouse with the shipment relation (i, j) as follows:

$$e_{ij}^{(\prime)} = Max\left\{v_0 + \sum_{r=1}^{t} v_r x_{ij}^{(r)}\right\}$$

Subject to:

$$\sum_{k=1}^{s} w_k y_{ij}^{(k)} = 1$$

$$v_0 + \sum_{r=1}^{t} v_r x_{ij}^{(r)} - \sum_{k=1}^{s} w_k y_{ij}^{(k)} \leq 0 \qquad ,j=1,2....,n$$

$V_r \geq \epsilon$; r = 1,2,3,....,t

$W_k \geq \epsilon$; k = 1,2,3,....,s

$\epsilon > 0$ and V_0 is unrestricted --------------(M4)

By adjusting the target warehouse in the model, we can get the relative efficiency for each ith warehouse as $e_{i1}^{(\prime)}, e_{i2}^{(\prime)} \ldots\ldots\ldots e_{in}^{(\prime)}$.

For each destination j, we consider all warehouses i (i= 1,2,...m) and each possible relation (i,j) as DMU$^{(2)}$. Using relation (i,j), we can measure the relative efficiency of the jth destination for destination j.

$$e_{ij}^{('')} = Max\left\{v_0 + \sum_{r=1}^{t} v_r x_{ij}^{(r)}\right\}$$

Subject to:

$$\sum_{k=1}^{s} w_k y_{ij}^{(k)} = 1$$

$$v_0 + \sum_{r=1}^{t} v_r x_{ij}^{(r)} - \sum_{k=1}^{s} w_k y_{ij}^{(k)} \leq 0 \qquad ,i=1,2....,m$$

$V_r \geq \epsilon$, r=1,2,3,....,t

$W_k \geq \epsilon$,k 1,2,3,....,s

$\epsilon > 0$ and V_0 is unrestricted -----------------(M5)

We get the relative efficiency of jth warehouse as $e_{j1}^{('')}, e_{j2}^{('')} \ldots\ldots\ldots e_{jn}^{('')}$ using (M5) by modifying the target warehouse in the model.

To evaluate the relative efficiency of any warehouse to each destination, we used a collection of decision-making units. Each warehouse follows a similar procedure. The two classes of relative efficiencies are measured from either the warehouse or the destinations side for comparative purposes. When dealing with a transportation issue with multiple inputs and outputs, we need to maximise overall efficiency for the entire shipment [10]. We calculate the composite efficiency index that includes all types of relative efficiencies, which is given below.

$e_{ij} = \frac{e_{ij}^{(')} + e_{ij}^{('')}}{2}$; i=1,2,……,m ; j=1,2,……..,n ----------(M6)

The composite efficiency index values are used as a performance metric for shipment ties (i ,j).

We solve the following transcortation problem,

$$Minimise \quad \sum_{i=1}^{m} \sum_{j=1}^{n} (1 - e_{ij}) y_{ij}$$

Subject to :

$\sum_{j=1}^{n} y_{ij} = a_i$; i=1,2,……,m

$\sum_{i=1}^{m} y_{ij} = b_j$; j=1,2,……,n

$y_{ij} \geq 0$, for all (i,j) . -------------(M7)

The problems mentioned above are classic transportation problems that can be solved using the simplex algorithm.

Chapter 3

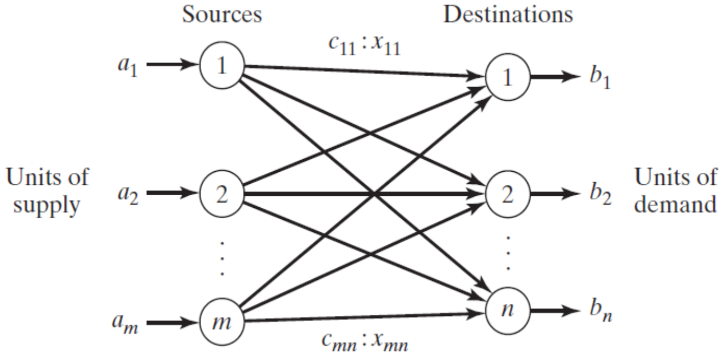
Transportation Model

Transportation theory is the study in supply chain management of the optimal transportation of commodity.The problem was first discussed in 1781 by the scientist named Gaspard Monge . It is a special case of min cost ow problem .The motivation of this topic is to understand the dynamics of supply chain better and the role of transportation or logistics in it. A transportation problem can be considered as an example of a network problem , hence it is represented on a network model. A simple transportation problem consists of nodes and directed edges.The weights in the arcs represent the unit cost of transportation from ith source to jth destination.

DEFINITION: (TRANSPORTATION MODEL)

Consider, sources and destination in the transportation model and let *m* and *n* represent sources and destinations respectively, each is represented by a **node** in the below figure. The routes that joins the sources and destinations are represented by Arcs. (*i*, *j*)th arc joins the *i* source with *j* destination that gives two information : cost of transportation per unit i.e, c_{ij} , and the shipping amount i.e, x_{ij}. *ai* represent amount of supply to the source *i* , and bj denotes the amount of demand to destination *j* . The main motive of this model is to minimize the total transportation cost which satisfy all the supply and demand.

Transportation Model and Its Variants



DEFINITION: (Balancing the transportation model)
The transportation tableau representation assumes that model is balanced, meaning that the total demand equals to the total supply. If the model is unbalanced, a dummy source or a dummy destination must be added to restore balance.

3.1 METHODOLOGY

Transportation algorithm for minimization problem

Step 1 : 1st, construct a transportation table entering the origin capacities a_i , the destination requirements b_j and the costs c_{ij} .

Step 2 : Find an initial basic feasible solution by vogel's method or by any of the given method. Enter the solution at the centre of the basic cells.

Step 3 : For all the basic variables x_{ij} , solve the system of equations $u_i+v_j=c_{ij}$ for all I , j for which cell (i,j) is in the basis, starting initially with some $u_i=0$ and entering successively the value of u_i and v_j on the transportation table.

Step 4 : Compute the cost difference $d_{ij} =(u_i+v_j)-c_{ij}$ for all the non basic cells and enter them in the upper right corners of the corresponding cells.

Step 5 : Apply optimality test by examining the sign of each d_{ij} :

(i) If all $d_{ij} \leq 0$, the current basic feasible solution is an optimum.
(ii) If at least one $d_{ij} > 0$, select the variable x_{rs} to enter the basis.

Step 6 : Let the variable x_{rs} enter the basis . Allocate an unknown quantity say ɵ , to the cell (r,s). Then construct a loop that starts and ends at the cell (r,s) and connect some of the basic cells. The amount ɵ is added to and subtracted from the transition cells of the loop in such a manner that the availability and requirements remain satisfied.

Step 7 : Assign the largest possible value to ɵ in such a way that the value of at least one basic variable becomes zero and other basic variable remain non negative . The basic cell whose allocation to been made zero will leave the basic.

Step 8 : Now, return to step 3 and then repeat the process until an optimum basic feasible solution is obtained.

After setting up a transportation table which is balanced , the next step is to find **initial basic feasible solution.**

The initial basic feasible solution can be found by the following three algorithms

- Northwest corner method
- Least cost method
- Vogal approximation method (penalty method)

NORTHWEST CORNER METHOD

Step 1) : Select the northwest or left upper cell of the transportation table . The maximum quantity that can be transported in this cell would be the maximum of the supply and demand in that cell.

Step 2) : Make changes in the supply and demand values by subtracting the minimum value from both of these. The Quantity allocated is written in the left upper box in the respective cell.

Step 3): If the supply is exhausted then strike that row and move to the next northwest corner of the transportation table.

Step 4): If the demand is met then Strike that column and move to the next northwest corner of the transportation table.

Step 5): Repeat all the above steps until the entire supply is exhausted and entire demand is met.

Example 1 : There is a furniture company in UK which is responsible for manufacturing office desks for various corporate offices. The company manufactures these goods (office desks) at factories in Belfast, Edinburgh and London. Then it distributes the office desks in the nearby locations through regional warehouses which are located in Newport, Oban and Nottingham. There needs to be a monthly supply of desks in this company and monthly demands for desks in the warehouses.

Also, the company has arranged the production in such a way that the costs per office desk are same in each of the factories so the costs of our concern is only the shipment cost from the factories to the warehouses. The figures of the supply and demand of goods at each of these locations would be mentioned in the project along with cost incurred on each shipment route. Also, the shipment is managed in such a way that shipment cost is constant between two locations irrespective of the volume of shipment.

It is now where we formulate this case as a transportation model and we wish to find the number of these office desks which when shipped from each of these routes would minimize the total transportation cost. Solve using northwest corner method.

	D_1	D_2	D_3	a_i
O_1	5	4	3	100
O_2	8	4	3	300
O_3	9	7	5	450
b_j	300	200	200	

Solution: Total supply = 950

Total demand = 700

Since total suppy ≠ total demand.

Therefore the problem is not balanced and we need to add a dummy destination with a demand of 150.

	D_1	D_2	D_3	D_4	a_i
O_1	5	4	3	0	100
O_2	8	4	3	0	200
O_3	9	7	5	0	450
b_j	300	200	200	150	

	D_1	D_2	D_3	D_4	a_i
O_1	100 **5**	**4**	**3**	**0**	100
O_2	200 **8**	100 **4**	**3**	**0**	200
O_3	**9**	100 **7**	200 **5**	150 **0**	450
b_j	300	200	200	150	

x_{11} =100, x_{21}=200 , x_{22}=100 , x_{32} = 100, x_{33} =200 , x_{34} = 150.

Therefore there are m+n-1=6 independent allocations.

Total cost = (100*5)+(200*8)+(100*4)+(100*7)+(200*5)+(150*0) = 4200.

The minimum transportation cost obtained through northwest corner method is 4200. This is certainly a feasible solution but we don't know yet whether it is optimal or not.

LEAST COST METHOD

Step 1): Select the cell having minimum cost and allocate as much as possible , that is minimum of the supply and demand of that cell.

Step 2) : Make allocation in the cell and make changes in the supply and demand values of the respective cell. If the supply is exhausted then strike off that row and if the demand is met then strike off the column.

Step 3) : If the minimum cost is not unique then allocation can be made in omly of the least cost cell.

Step 4) : Repeat the above cell until all the supply is exhausted and demand is met.

Example 2 : Solve example 1 using least cost method.

	D_1	D_2	D_3	a_i
O_1	**5**	**4**	**3**	100
O_2	**8**	**4**	**3**	300
O_3	**9**	**7**	**5**	450
b_j	300	200	200	

Solution: Total supply = 950

Total demand = 700

Since total suppy ≠ total demand.

Therefore the problem is not balanced and we need to add a dummy destination with a demand of 150.

	D_1	D_2	D_3	D_4	a_i
O_1	**5**	**4**	**3**	**0**	100
O_2	**8**	**4**	**3**	**0**	200
O_3	**9**	**7**	**5**	**0**	450
b_j	300	200	200	150	

	D_1	D_2	D_3	D_4	a_i
O_1	**5**	**4**	**3**	[100] **0**	100
O_2	**8**	[50] **4**	[200] **3**	[50] **0**	200
O_3	[300] **9**	[150] **7**	**5**	**0**	450
b_j	300	200	200	150	

x14 =100, x22=50 , x23=200 , x24 = 50, x31 =300 , x32= 150.

Total cost = (100*0)+(50*4)+(200*3)+(50*0)+(300*9)+(150*7)=4550.

The minimum transportation cost obtained through least cost method is 4550.

VOGAL APPROXIMATION METHOD

Step 1) : Find the penalty cost for each row and each column of the transportation table , penalty cost is the difference between the smallest and next to smallest cost.

Step 2) : Select the row or column which has column penalty. In this row or column select the least cost cell and allocate or much possible in this cell.

Step 3) : After the allocation is done make changes in the supply and demand quantities of the respective cell . If there is a in the values of the penalties , select the cost in such a way that maximum allocation is possible in the least cost cell.

Step 4) : As soon as the supply get exhausted, strike off that row, and as soon as demand gets met , strike off met column.

Step 5) : Repeat the above steps till all the supply and demand penalty is zero.

Example 3 : Solve example 1 using vogal approximation method.

	D_1	D_2	D_3	a_i
O_1	**5**	**4**	**3**	100
O_2	**8**	**4**	**3**	300
O_3	**9**	**7**	**5**	450
b_j	300	200	200	

Solution: Total supply = 950

Total demand = 700

Since total suppy ≠ total demand.

Therefore the problem is not balanced and we need to add a dummy destination with a demand of 150.

	D_1	D_2	D_3	D_4	a_i
O_1	**5**	**4**	**3**	**0**	100
O_2	**8**	**4**	**3**	**0**	200
O_3	**9**	**7**	**5**	**0**	450
b_j	300	200	200	150	

	D_1	D_2	D_3	D_4	a_i	Row Penalty
O_1	[100] **5**	**4**	**3**	**0**	100	3 1 - -
O_2	**8**	[200] **4**	[100] **3**	**0**	200	3 1 1 5
O_3	[200] **9**	**7**	[100] **5**	[150] **0**	450	5 2 2 4
b_j	300	200	200	150		
Column Penalty	3	0	0	0		
	3	0	0	-		
	1	3	2	-		
	1	-	2	-		

x_{11} =100, x_{22}=200 , x_{23}=100 , x_{31} = 200, x_{33} =100 , x_{34} = 150.

Total cost = (5*100)+(200*4)+(3*100)+(200*9)+(100*5)+(150*0) = 3900.

The minimum transportation cost obtained through vogal approximation method is 3900.

The next challenge would be to check if this initial basic feasible solution is optimal or not, and if it is not optimal, then how to reach optimality. This problem will be taken care with the help of Modified Distribution (MODI) mention in algorithm or Stepping stone method.

3.2 NUMERICAL EXAMPLE:

K,L,M,N,O,P,Q,R are the eight towns where an automotive company has assembly plants. The bikes are assembled and shipped to eight warehouses: S, T, U, V, W, X, Y, Z, and then to ten customers: A, B, C, D, E, F, G, H, I, J. The manufacturer considers one input, namely shipping costs, as well as two outputs, namely shipment value and benefit. Each ordered triplet (x1, y1, y2) is a symbol for (shipping cost, value of shipment, profit). In each Table 1,2 and 3, the appropriate input-output, supply ai, and demand bj are specified.We have solved the problems M4, M5 and M6 and calculated the optimal values of $eij^{(')}$and $eij^{('')}$.

The values of $eij^{(')}$and $eij^{('')}$are listed in Table 4 , 5 and 6 cossesponding to each table 1,2 and 3.Composite efficiency (eij) is listed in table 7,8 and 9 .After calculating efficiencies ,we have solved the transportation problem for the data indicated in Table 10,11 and 12, to determine the transportation plan with maximum efficiency.The entry in each cell is 1- eij which represent the value of inefficiency associated with particular arc in each table.

We used the simplex method to solve transportation problem of table 7,8 and 9 .The optimal solution as X_{KS}=3 , X_{KZ}=7,X_{LU} = 12, X_{LV} = 12, X_{MY}=13, X_{MZ} = 3, X_{NW} =12 , X_{OS} = 18, X_{OX} =12, X_{PZ} = 15 , X_{QT} = 19 , X_{QV} =2 , X_{QW} = 4 and X_{RZ} = 18 .Optimal **objective value** is 3.72 with maximum **efficiency** 96.28 for table 10 .

The optimal solution as X_{SG}=13 , X_{SH}=8,X_{TA} = 15, X_{TD} = 14, X_{VE}=11, X_{UI} = 7, X_{UJ} =12 , X_{YB} = 14, X_{WF} =5, X_{WI} = 11 , X_{XH} = 12 , X_{YA} =3 , X_{YB} = 5, X_{YF} =5 , X_{ZA} = 3 and X_{ZC} = 22. Optimal objective value is 3.52 with maximum efficiency 96.48 for table 11 .

The optimal solution as X_{KG}=10 , X_{LD}=2,X_{LF} = 10, X_{LJ} = 12, X_{MG}=3, X_{MI} = 13, X_{ND} =2 , X_{NE} = 5, X_{NI} =5, X_{OC} = 22 , X_{OD} = 3 , X_{OI} =5 , X_{PH} = 15, X_{QB} =19 , X_{QE} = 6 ,X_{RA}=11 and X_{RD} = 7. Optimal objective value is 4.11 with maximum efficiency 95.89 for table 12 .

Maximum efficiency in transportation from company to warehouse is 96.28 and maximum efficiency in transportation from company to warehouse is 96.48.so total maximum efficiency for transportation from company to customer is average of these two i.e. 96.38 .Maximum efficiency in direct transportation from company to customer is 95.89. Therefore from result we can say we should use multilevel transportation method as it is more efficient .

Table 1 Company to warehouse

	S	T	U	V	W	X	Y	Z	ai
K	(20,151,100)	(19,131,150)	(25,160,160)	(27,168,180)	(22,158,94)	(55,255,230)	(33,235,220)	(31,206,152)	10
L	(30,125,110)	(21,121,220)	(23,159,258)	(25,169,254)	(36,236,152)	(35,256,247)	(40,158,268)	(33,147,258)	24
M	(44,134,180)	(45,126,116)	(25,167,198)	(34,156,251)	(42,232,164)	(26,198,254)	(25,147,265)	(28,156,259)	16
N	(19,160,180)	(25,225,250)	(36,154,123)	(25,154,254)	(28,256,159)	(34,125,252)	(39,157,241)	(39,298,187)	12
O	(33,245,230)	(30,110,245)	(32,264,194)	(25,147,265)	(45,256,195)	(21,138,258)	(23,152,241)	(48,129,249)	30
P	(45,160,100)	(28,180,230)	(41,123,145)	(23,129,264)	(31,194,267)	(46,198,265)	(35,254,197)	(23,147,264)	15
Q	(44,139,163)	(24,230,120)	(28,259,152)	(34,258,169)	(27,167,252)	(23,175,269)	(35,159,247)	(27,160,180)	25
R	(37,255,256)	(34,130,167)	(24,264,168)	(31,264,195)	(36,264,147)	(29,164,294)	(30,140,250)	(40,190,170)	18
bj	21	19	30	14	16	12	13	25	

Table 2 Warehouse to customer

	A	B	C	D	E	F	G	H	I	J	ai
S	(30,161,160)	(29,151,170)	(35,180,160)	(37,188,200)	(32,178,114)	(55,285,250)	(43,255,240)	(41,246,172)	(40,264,200)	(50,288,270)	21
T	(20,135,140)	(31,161,240)	(33,179,258)	(25,189,274)	(26,256,172)	(45,276,267)	(50,178,288)	(43,167,278)	(46,174,256)	(36,267,281)	19
U	(54,144,190)	(55,146,136)	(35,187,198)	(44,176,271)	(52,252,184)	(36,198,274)	(35,187,285)	(38,176,279)	(29,256,209)	(21,276,179)	30
V	(29,150,190)	(35,255,270)	(46,174,123)	(35,174,274)	(28,276,179)	(44,155,272)	(49,177,261)	(39,268,197)	(41,185,167)	(56,148,277)	14
W	(43,255,250)	(40,130,265)	(42,284,194)	(35,167,285)	(45,276,215)	(31,148,278)	(33,172,261)	(45,149,269)	(35,172,288)	(52,287,185)	16
X	(25,150,140)	(48,190,250)	(51,143,145)	(33,149,284)	(41,164,287)	(56,198,285)	(45,274,197)	(33,167,274)	(41,189,261)	(55,146,179)	12
Y	(24,149,173)	(33,250,140)	(38,279,152)	(44,278,189)	(27,187,272)	(33,175,299)	(45,179,267)	(37,180,200)	(44,184,267)	(36,227,271)	13
Z	(47,285,276)	(44,150,197)	(34,284,188)	(41,284,195)	(46,284,167)	(39,184,254)	(40,160,270)	(50,210,190)	(44,180,260)	(48,276,145)	25
bj	11	19	22	14	11	10	13	20	18	12	

Table 3 Company to customer

	A	B	C	D	E	F	G	H	I	J	ai
K	(25,156,130)	(24,141,160)	(30,170,160)	(32,178,190)	(27,168,104)	(55,270,240)	(38,245,230)	(36,226,162)	(35,254,195)	(50,278,260)	10
L	(25,130,125)	(26,141,230)	(28,169,258)	(25,179,264)	(31,246,162)	(40,266,257)	(45,168,278)	(48,157,268)	(41,164,246)	(31,257,271)	24
M	(49,139,185)	(50,166,126)	(30,177,198)	(39,166,261)	(47,242,174)	(31,198,264)	(30,167,275)	(33,166,269)	(29,246,249)	(21,266,169)	16
N	(25,157,183)	(30,240,260)	(41,164,123)	(30,164,674)	(28,266,169)	(39,140,262)	(44,167,251)	(39,283,192)	(41,175,177)	(51,138,267)	12
O	(38,250,240)	(35,120,255)	(37,274,194)	(30,157,275)	(45,266,205)	(26,143,268)	(38,162,251)	(47,139,259)	(35,162,293)	(42,277,175)	30
P	(35,155,120)	(38,185,240)	(46,133,145)	(28,139,274)	(46,179,277)	(51,198,275)	(40,264,197)	(28,157,269)	(36,179,251)	(50,136,169)	15
Q	(34,144,168)	(29,240,130)	(35,269,152)	(39,268,179)	(27,177,262)	(38,175,289)	(40,169,257)	(32,170,190)	(39,174,257)	(31,222,261)	25
R	(42,270,266)	(39,140,182)	(29,274,178)	(46,274,195)	(41,274,157)	(34,174,274)	(35,150,260)	(45,200,180)	(39,170,250)	(43,266,135)	18
bj	11	19	22	14	11	10	13	20	18	12	

Table 4 Efficiency for company to warehouse

	S	T	U	V	W	X	Y	Z
K	(1.0000,0.8966)	(1.0000,0.7867)	(0.9040,0.7086)	(0.8905,0.8133)	(0.9512,0.7855)	(0.6431,0.6088)	(0.9441,1.0000)	(0.8933,0.9436)
L	(0.5928,0.4948)	(0.9339,1.0000)	(1.0000,1.0000)	(0.9748,1.0000)	(0.9327,0.7259)	(1.0000,0.9605)	(0.5973,0.6354)	(0.6970,0.6970)
M	(0.4037,0.4318)	(0.2865,0.3050)	(0.8032,0.8072)	(0.7078,0.7059)	(0.4852,0.6321)	(0.9720,1.0000)	(1.0000,1.0000)	(0.8827,0.8555)
N	(0.9468,1.0000)	(1.0000,1.0000)	(0.4706,0.4267)	(1.0000,0.9616)	(1.0000,1.0000)	(0.7295,0.6033)	(0.6097,0.6057)	(0.8359,1.0000)
O	(0.9412,0.8816)	(0.6647,0.7795)	(1.0000,0.7943)	(0.8841,0.9683)	(0.6937,0.6693)	(1.0000,1.0000)	(0.9545,1.0000)	(0.4222,0.4792)
P	(0.4899,0.4222)	(0.9449,0.8089)	(0.4355,0.3614)	(1.0000,1.0000)	(0.9311,0.9629)	(0.6374,0.5653)	(1.0000,1.0000)	(1.0000,1.0000)
Q	(0.3845,0.3910)	(1.0000,1.0000)	(0.9813,0.8409)	(0.8188,0.08910)	(0.8083,1.0000)	(1.0000,1.0000)	(0.6034,0.685)	(0.7137,0.9029)
R	(0.8148,0.8184)	(0.5318,0.4834)	(1.0000,1.0000)	(0.8389,1.0000)	(0.6667,0.8021)	(1.0000,0.8334)	(0.8228,0.7868)	(0.5230,0.6889)

Table 5 Efficiency for warehouse to customer

	A	B	C	D	E	F	G	H	I	J
K	(0.9485,0.8960)	(0.7671,1.0000)	(0.8147,0.8677)	(0.7769,0.9137)	(0.6550,0.874)	(0.7382,0.7290)	(1.0000,0.9858)	(0.8746,0.8650)	(0.8555,1.0000)	(0.6048,0.8484)
L	(0.7907,0.6272)	(1.0000,0.8377)	(1.0000,0.8726)	(1.0000,1.0000)	(0.8456,0.9572)	(1.0000,0.8021)	(0.6739,0.5850)	(0.5825,0.5287)	(0.6988,0.5682)	(1.0000,1.0000)
M	(0.5158,0.4204)	(0.4067,0.3030)	(0.8874,0.7497)	(0.6013,0.7331)	(0.5661,0.4533)	(1.0000,0.9481)	(1.0000,1.0000)	(0.8788,0.8905)	(1.0000,0.9880)	(1.0000,1.0000)
N	(1.0000,0.7355)	(1.0000,0.9222)	(0.5464,0.4313)	(1.0000,1.0000)	(1.0000,1.0000)	(0.6522,0.4771)	(0.6629,0.4731)	(1.0000,0.7662)	(0.5032,0.4852)	(0.5989,0.3634)
O	(1.0000,0.9459)	(0.8231,0.7948)	(1.0000,1.0000)	(0.7457,1.0000)	(0.6670,0.7831)	(1.0000,0.8581)	(0.7515,0.7631)	(0.5736,0.6012)	(0.9750,0.9132)	(0.55207,0.8300)
P	(0.6731,0.6744)	(0.7215,0.8121)	(0.4318,0.4657)	(0.7338,1.0000)	(0.6206,0.6761)	(0.8787,0.6574)	(0.6732,1.0000)	(1.0000,1.0000)	(0.8120,0.8439)	(0.3866,0.4496)
Q	(0.6750,0.5903)	(1.0000,1.0000)	(1.0000,0.6347)	(0.9597,0.5874)	(1.0000,1.0000)	(0.7894,0.7837)	(0.7405,0.6621)	(0.8235,0.7327)	(0.7675,0.6800)	(0.9631,1.0000)
R	(0.9828,0.8892)	(0.5330,0.6060)	(0.8810,1.0000)	(0.8319,0.6620)	(0.7035,0.7073)	(0.8589,1.0000)	(0.8104,0.9218)	(0.6511,0.5781)	(0.7466,0.8078)	(0.4884,0.6547)

Table 6 Efficiency for company to customer

	A	B	C	D	E	F	G	H	I	J
S	(0.7951,0.9257)	(0.7593,1.0000)	(0.6157,0.8076)	(0.6721,1.0000)	(0.5643,0.7146)	(0.8449,0.8048)	(0.9665,1.0000)	(0.8731,0.7750)	(0.7477,0.9091)	(0.6335,0.9456)
T	(1.0000,0.7829)	(1.0000,0.7064)	(0.6494,0.7157)	(1.0000,1.0000)	(1.0000,1.0000)	(1.0000,0.6945)	(0.7074,0.5255)	(0.7786,0.5899)	(0.6889,0.5078)	(0.9157,0.8647)
U	(0.4881,0.3284)	(0.3608,0.2547)	(0.6396,0.6735)	(0.5640,1.0000)	(0.5076,0.3965)	(0.9756,0.7025)	(1.0000,0.7430)	(0.9004,0.669)	(1.0000,0.7756)	(1.0000,1.0000)
V	(0.9089,0.8431)	(1.0000,1.0000)	(0.4528,0.4030)	(0.7169,1.0000)	(1.0000,1.0000)	(0.6823,0.7896)	(0.6710,0.6818)	(1.0000,0.7490)	(0.5521,0.5578)	(0.5803,0.6318)
W	(0.8786,0.9678)	(0.8557,0.7388)	(0.8095,1.0000)	(0.7457,0.9557)	(0.6589,0.9373)	(0.9828,1.0000)	(0.9745,0.9948)	(0.7200,0.6841)	(1.0000,0.9777)	(0.4199,0.8162)
X	(0.8889,1.0000)	(0.6735,0.7259)	(0.3357,0.7825)	(0.7881,1.0000)	(0.6949,0.8297)	(0.6340,0.6675)	(1.0000,1.0000)	(1.0000,1.0000)	(0.7928,0.8585)	(0.4038,0.4764)
Y	(1.0000,0.8712)	(1.0000,1.0000)	(0.8790,0.9692)	(0.8357,0.8445)	(1.0000,1.0000)	(1.0000,0.8994)	(0.7408,0.5890)	(0.8132,0.6793)	(0.7528,0.6036)	(0.8831,0.8877)
Z	(0.8983,0.9620)	(0.5790,0.6902)	(1.0000,1.0000)	(0.9162,0.8105)	(0.6263,0.7391)	(0.8363,0.9954)	(0.8289,1.0000)	(0.6522,0.6324)	(0.7335,0.8964)	(0.4375,0.6884)

Table 7 Composite Efficiency for company to warehouse

	S	T	U	V	W	X	Y	Z
K	0.9483	0.89995	0.8083	0.8519	0.86835	0.62595	0.97205	0.91845
L	0.5428	0.96695	1.0000	0.9608	0.708	0.8652	0.61635	0.697
M	0.41775	0.29575	0.8052	0.70685	0.55865	0.986	1.0000	0.8691
N	0.9734	1.0000	0.44865	0.9808	1.0000	0.6664	0.6077	0.91795
O	0.9114	0.7221	0.89715	0.9262	0.6815	1.0000	0.97725	0.4507
P	0.45605	0.8769	0.39845	1.0000	0.947	0.60135	1.0000	1.0000
Q	0.38775	1.0000	0.9111	0.8549	0.90415	1.0000	0.6442	0.8083
R	0.8166	0.5076	1.0000	0.91945	0.7344	0.9167	0.8048	0.60595

Table 8 Composite Efficiency for warehouse to customer

	A	B	C	D	E	F	G	H	I	J
S	0.8603	0.8796	0.7116	0.836	0.6394	0.8248	0.9832	0.824	0.8284	0.7895
T	0.8914	0.8532	0.6825	1.000	1.000	0.8472	0.6164	0.6842	0.5983	0.8902
U	0.4082	0.3077	0.6565	0.782	1.000	0.7359	0.8715	0.7851	0.8878	1.000
V	0.876	1.000	0.4279	0.8584	1.000	0.7395	0.6764	0.872	0.5521	0.606
W	0.9232	0.7972	0.9047	0.8507	0.7981	0.9914	0.9846	0.702	0.9888	0.618
X	0.9444	0.6997	0.4409	0.1059	0.2377	0.3492	0.000	0.000	0.1743	0.5599
Y	0.9356	1.000	0.9241	0.8401	0.8401	1.000	0.9497	0.6649	0.6782	0.8854
Z	0.9301	0.6346	1.000	0.8633	0.6827	0.9158	0.9144	0.6423	0.8149	0.5629

Table 9 Composite Efficiency for company to customer

	A	B	C	D	E	F	G	H	I	J
K	0.9222	0.8835	0.8412	0.8453	0.7562	0.7336	0.9929	0.8698	0.9227	0.7266
L	0.7089	0.9188	0.9363	1.000	0.9014	0.9740	0.6294	0.5596	0.6335	1.000
M	0.4681	0.3548	0.8185	0.6672	0.5097	0.974	1.000	0.8846	0.994	1.000
N	0.8677	0.9611	0.4888	1.000	1.000	0.5646	0.568	0.8831	0.4952	0.4811
O	0.9729	0.8089	1.000	0.8773	0.725	0.929	0.7573	0.5874	0.9441	0.6753
P	0.6737	0.7668	0.4487	0.8669	0.6483	0.768	0.8366	1.000	0.8279	0.4181
Q	0.6326	1.000	0.8173	0.7735	1.000	0.7865	0.7013	0.7781	0.7237	0.9815
R	0.936	0.5695	0.9405	0.7469	0.7054	0.9249	0.8661	0.6146	0.7772	0.5115

Table 10 (1- Composite Efficiency) for company to warehouse

	S	T	U	V	W	X	Y	Z
K	0.0517	0.10665	0.1917	0.1481	0.13165	0.37405	0.02795	0.06155
L	0.4572	0.0331	0.0000	0.0392	0.2920	0.1348	0.3836	0.3030
M	0.5822	0.7042	0.1948	0.2931	0.4413	0.0140	0.0000	0.1309
N	0.0266	0.0000	0.5513	0.0192	0.0000	0.3336	0.3923	0.0820
O	0.0886	0.2779	0.1028	0.0738	0.3185	0.0000	0.0227	0.5493
P	0.543	0.1231	0.6015	0.0000	0.0530	0.3986	0.0000	0.0000
Q	0.6122	0.0000	0.0889	0.1451	0.0958	0.000	0.3558	0.1917
R	0.1834	0.4924	0.0000	0.080	0.2656	0.0833	0.1952	0.3940

Table 11 (1- Composite Efficiency) for warehouse to customer

	A	B	C	D	E	F	G	H	I	J
S	0.1396	0.1203	0.2883	0.1659	0.3605	0.1751	0.0167	0.1759	0.1716	0.2104
T	0.1085	0.1468	0.3174	0.0000	0.0000	0.1524	0.3825	0.3157	0.4016	0.1098
U	0.5917	0.6922	0.3434	0.218	0.0000	0.2641	0.1285	0.2148	0.1122	0.0000
V	0.124	0.0000	0.5721	0.1415	0.0000	0.264	0.3236	0.128	0.445	0.3939
W	0.0768	0.2027	0.0952	0.1493	0.2019	0.0086	0.0153	0.2979	0.0111	0.3819
X	0.0555	0.3003	0.4409	0.1059	0.2377	0.3492	0.0000	0.0000	0.1743	0.5599
Y	0.0644	0.0000	0.0759	0.1599	0.1599	0.0000	0.0503	0.3351	0.3218	0.1146
Z	0.0698	0.3654	0.0000	0.1366	0.3173	0.0841	0.0855	0.3577	0.185	0.4370

Table 12 (1- Composite Efficiency) for company to customer

	A	B	C	D	E	F	G	H	I	J
K	0.0780	0.1164	0.1588	0.1547	0.2438	0.2664	0.0071	0.1302	0.0722	0.2734
L	0.291	0.0811	0.0637	0.0000	0.0986	0.0259	0.371	0.444	0.3665	0.0000
M	0.5319	0.6451	0.1814	0.3328	0.4903	0.025	0.0000	0.1153	0.006	0.0000
N	0.1322	0.0389	0.5111	0.0000	0.0000	0.4353	0.432	0.1169	0.5058	0.5188
O	0.027	0.1910	0.0000	0.1226	0.2749	0.0709	0.2427	0.4126	0.0559	0.3246
P	0.3262	0.2332	0.5512	0.1331	0.3516	0.2318	0.1634	0.0000	0.172	0.5819
Q	0.3673	0.0000	0.1826	0.2264	0.0000	0.2134	0.2987	0.2219	0.2762	0.0184
R	0.064	0.4305	0.0595	0.253	0.2946	0.0705	0.1339	0.3854	0.2228	0.4284

CONCLUSION

The classic multilevel transportation problem was expanded in this paper by concerning multiple input and multiple flexible output for each shipment connection. DEA based proposed solution, with a BCC model based on the relative efficiencies of each potential connection as a performance measure to determine the most efficient transportation strategy. Decision makers use different approach to get goals with every conflict potential shipment connection, and these goals may conflict with one another in the case of multilevel transportation, the proposed approach is useful. We can see from this illustration that a multilevel transportation approach is more effective than a direct transportation approach. To solve a transportation issue with multiple inputs and multiple versatile outputs, we recommend using multilevel transportation rather than direct transportation.

REFERENCES

[1] Alireza Amirteimoori (2012):An extended transportation problem: a DEA -based approach.: Central European Journal of Operations research 19:513-521.

[2] Chen L.H., Lu H.W(2007):An extended assignment problem considering multiple inputs and outputs. Applied Mathematical modelling31:2239-2248.

[3] Banker R.D., Charnes A., Cooper W.W.(1984):Some methods for estimating technical and scale inefficiencies in data envelopment analysis. Management Sci.30(9):1078-1092.

[4] Charnes A., Cooper W.W., Rhodes E.(1978):Measuring the efficiency of decision making units. European Journal of Operational Research 2(6):429-444.

[5] Cooper, W.W., Seiford, L.M. & Tone, K., (2006) . Introduction to Data Envelopment Analysis and its uses. New York : Springer-Verlag.

[6] Andersen, P., and N.C. Petersen. 1993. A procedure for ranking efficient units in data envelopment analysis. Management Science 39(10): 1261-1265.

[7] Ali, I. and Seiford, L.(1990). Translation invariance in Data Envelopment Analysis. Operations Research Letters, 9(6), 403-405.

[8] Amirteimoori, A. (2012). An extended shortest path problem: A Data Envelopment Analysis approach. Applied Mathematics letters. 25(11), 1839-1843.

[9] Cardiilo , D.D.L. and Fortuna, T.(2000) . A DEA model for the efficiency evaluation of nondominated paths on a road network. European J. Oper. Res., 121(3), 549-558.

[10] Boychuk L.M. and Ovchinnikov V.O. (1973): Principal methods for solution of multi criteria optimization problems (survey), Soviet Automatic Control 6,1-4

[11] Charnes A., and cooper W.W.(1962): Programming with linear fractional. Naval Research Logistic quarterly9:181-186.

[12] Cooper W. W., Seriford L.M., Zhu J(2004):Handbook of data envelopment analysis. Kluwer Academic Publishers. Norwell.
[13] Mokhtar S. Bazaraa ,Johon J. Jarvis , Hanif D. Shevali (2011):Linear programming and network flows. 3rd edn. Willey, New York.

[14] Cooper W. W., Seriford L.M., Tone K(2007):Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software.2nd edn. Springer. Berlin.

[15] Coelli,T.J, D.S.P. Rao, C.J.O'Donnell, and G.E..Battese. 2005. An introduction to efficiency and productivity analysis. New York , NY:Springer.