SLIDING MODE CONTROL ON AN UNDERACTUATED TORA SYSTEM

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I ABHISHEK CHAUDHARY, Roll No – 2k16/C&I/01 student of M.Tech (Control & Instrumentation), hereby declare that the project Dissertation titled "Sliding Mode Control On An Underactuated Tora System" which is submitted by me to the Department of Electrical Engineering, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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ABSTRACT

An important technique used in control systems subject to parameter uncertainty is sliding mode control (SMC). This method alters the dynamics of a non-linear system and forces the system to slide along a cross-section of system's normal behaviour. The state feedback control law is not considered the continuous function with respect to time. It switch from one continuous structure to another. That's why SMC is also called variable structural control method (VSC). SMC is a robust control algorithm which can initiatively change the control structure to obtain the output of desire.

SMC has been applied to many non-linear systems, in this dissertation SMC is being applied to TORA system, which is also a non-linear system. Parameters of the TORA systems and SMC controller are carried on one platform in such a way that the TORA system will provide a sliding surface for SMC and the implementation of controller is made possible.

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List of Symbols

u	Control torque applied on cart
x	Disturbance force on cart
θ	Angular position of rotational actuator
m	Mass of rotational actuator
z_i	Normalized displacement of the platform
e_i	State error positioning
p_i	State matrix
q_i	State matrix
x_i	Input State Matrix
d_i	Disturbance Matrix

Abbreviations Used

VSC	Variable Structural Control
SMC	Sliding Mode Control
TORA	Translational Oscillator Rotational Actuator
MIMO	Multiple Input Multiple Output
RH	Routh Hurwitz
EV	Electric Vehicles
IBC	Interleaved Boost Converter
PI	Proportional Integral
NSMC	Neural Sliding Mode Control
FTS	Finite Time Stable
NLC	Non-Linear Control

1. INTRODUCTION

This dissertation presents a sliding mode control (SMC) approach on a Translational Oscillator Rotational Actuator (TORA) system using decoupling algorithm. The control input of the underactuated system is modified and made disturbance free. The first peak of control input in TORA system is reduced in order to make the system more stable. The equations of SMC and TORA systems are imposed into the algorithm form and then using MATLAB SIMULATION and CODING skills, the improved graph is obtained. The convergence of the proposed sliding mode control is validated by the MATLAB platform, to prove the stability of TORA system under the proposed controller.

1.1 Actuated System

The component of a system or machine which is responsible for the movement or control of the given system or machine is known as actuator i.e. an actuator is a “mover”. The control system which is effected by such an actuator is known as the actuated system. This actuator, after receiving any control signal, works as a converter of signal’s energy into mechanical motion.

Types of actuators – hydraulic, pneumatic, electric etc.

Electric valve actuator controlling a half needle valve is an example of such systems.

1.2 Underactuated System

A system which is not commanded to follow arbitrary trajectories in any given configuration space can be understandable as an underactuated system, in other words we can also define an underactuated system as a system which has lower number of actuators than the degree of freedom (DOF) i.e. the members of actuators are strictly less than the dimension of the system. Underactuated systems are also called non-holonomic systems. If the number of actuators are equal to the dimensions of system, the system is fully actuated not underactuated.

e.g. – the classical inverted pendulum which has two degree of freedom (one is for the motion of support in the horizontal plane and other for its angular motions), but only one degree is actuated and other is always indirectly controlled.

In the past decade we may observe increasing interest in underactuated systems. These systems are characterized by the fact that they have fewer actuators than the degrees of freedom to be controlled. Underactuated systems have important applications like free-flying space robots, underwater robots, surface vessels, manipulators with structural flexibility, etc. [1] They are used for reducing weight, cost or energy consumption, while still maintaining an adequate degree of dexterity without reducing the reachable configuration space. TORA system also includes in such category of applications. Some other advantages of underactuated system include tolerance for failure of actuators. [1], [2]

Control researchers have given considerable attention to many examples of control problems associated with underactuated mechanical systems and different control strategies have been proposed. Use of decoupling algorithm to design the different controllers for underactuated systems has given satisfied results in researches. [2]

1.3 Coupling of a System

The differential equation with two dependant and one independent variable will form a coupled system.

$$\frac{dx}{dt} = ax + by$$

Where,

a & b are constant.

If time variation $\frac{dx}{dt}$ of i^{th} state is dependent of the states other than this i^{th} state of any coupled system, this will be known as the coupling of the system. In a coupled system, if we try to change the parameters of one system, it will cause change into the parameters of other systems also i.e. it is confirmed that parameters of both systems are interrelated in one or more than one ways.

1.4 Decoupling of a System

If, one or more than one system are able to produce any transection without being actually connected or coupled, the technique will be known as decoupling of the system. If we make the i^{th} state of any coupled system $\frac{dx}{dt}$ independent of the states other than this i^{th} state then we will say that the decoupling of a system is performed.

In simple words, decoupled system will always allow changes to be made to any one system without actually disturbing any other system.

1.5 Sliding Mode Controller (SMC)

An important technique used in control systems subject to parameter uncertainty is sliding mode control (SMC). This method alters the dynamics of a non-linear system and forces the system to slide along a cross-section of system's normal behaviour. The state feedback control law is not considered the continuous function with respect to time. It switch from one continuous structure to another. That's why SMC is also called variable structural control method (VSC).

Variable structural control method (VSC) with sliding mode control was initially proposed and elaborated in the Soviet Union in early 1950s by Emelyanov and several co-researchers such as Utkins and Itkis [3]. In the control research community, considerable interest on VSC and SMC have been developed among the control researchers.

More than one control structures are designed in order to move the trajectories towards the adjacent region with a different control structure. SMC is a robust control algorithm which can initiatively change the control structure to obtain the output of desire.

This motion of systems, as they slides along the boundaries is known as sliding mode and locus of the boundaries long which system slides, is called sliding surface.

SMC has been applied to many non-linear systems, MIMO systems, discrete-time models, systems with infinite dimensions of large scale, and also to stochastic systems. It is mostly liked by researchers because it is completely insensitive to parametric uncertainty and external disturbances during sliding mode [38].

Let us consider a sliding surface:

$$s=c_1e_1 + c_2e_2+c_3e_3 + e_4$$

(this equation is further dealt with in chapter 3)

Where,

c_1, c_2 & c_3 are positive constant.

e_1, e_2, e_3&e_4 are error equations.

Then,

Sliding surface is called attractive in nature if:

$$s\dot{s}<0$$

(this is the compulsory satisfied equation in SMC)

The variety of application of this method is very wide due to its robustness, associated to disturbance rejection capacity and simplicity [3]. SMC is a robust method for the systems which possesses some non-linearity owing to dynamic response and is being used in converters for many years [4]. SMC based controller technique is applied to buck boost and Cuk converter circuits in [5, 6, 7].

1.6 Non-Linear Translational Oscillator with Rotational Actuators

A TORA system consists of one unactuated translational cart and one actuated rotor. This system was originally studied as the simplified model for the dual-spin space craft for the investigation of the resonance capture phenomenon. After that, it was studied for the investigation of the utility of the rotational proof-mass actuator for the stabilization translational motion.

The cart is allowed to have one dimensional travel. The motion occurs in the horizontal plane, along with the x axis. Which makes no gravitational force to be considered while the

TORA system is in action. The detailed Figure with detailed description is shown later on in Fig.1 of this dissertation.

Robert T. Bupp gave the general model of TORA so that any control strategy for this system may be applied on that benchmark model [28]. This system is one of the benchmarks of underactuated mechanical system, which is usually used to verify the control design techniques for nonlinear systems.

R.T. Bupp proposed the mathematical model of TORA system. Over the time, many authors developed many approaches to work on TORA system, in this dissertation SMC is applied on this benchmark problem of TORA system and built the actual testbed for the control systems, which is being used for the testing of the control algorithms on non-linear TORA system

Wan et al. [33] achieved the global stability of this TORA system by simply using partial feedback linearization and back stepping procedure technique. Jankovic et al. [34] proposed a cascade and passivity-based control design technique for the same TORA system.

In the Special Issue of International Journal of Robust and Nonlinear Control, a variety of non-linear control methods were designed for this TORA system [35~37].]. P. Tsiotras et al. [36] present a state-feedback nonlinear controller for TORA system by solving the Hamilton-Jacobi-Isaacs equation; and a similar control design technique is also employed in [37].

1.7 Lyapunov

Lyapunov's stability method is the most general and powerful stability criterion. A system is said to be stable if the starting of the system near its described operating point implies that it will stay around the stability, for example – simple pendulum.

Lyapunov stability method is furthers divided into two types – direct and indirect. Both are used for the analysis and design of the NLC systems.

The indirect method is used for the linearized approximation of a small ranged motion and works as the local stability i.e. if the linearized system is strictly stable then the equilibrium point is asymptotically stable. On the other hand, the direct method says that if the total energy of a system is continuously dissipated then the system must eventually settle down in its equilibrium state only.

Consider a system:

$$\dot{X} = f(x) \tag{1.1}$$

Then,

if there exist a scalar function (energy like function) V(x) such that,

(i) V(x) > 0.

For x > 0 & V(x) is differentiable.

(ii) V(x) = 0.

For x = 0

(iii) V(x) < 0.

Then, the system is asymptotically stable and V(x) is called Lyapunov function.

if,

V(x) = $X^T PX$, is a Lyapunov function for the given system.

Then,

$\dot{V}(x) < 0$

i.e.

$$\begin{aligned} \dot{V}(x) &= \dot{X}^T PX + X^T P\dot{X} \\ &= [AX]^T PX + X^T P[AX] \\ &= X^T [A^T P + PA]X \\ &= -X^T QX \end{aligned}$$

Which gives:

$$A^T P + PA = -Q \tag{1.2}$$

Equation (1.2) is known as the Lyapunov Equation. We can solve for P by choosing a positive definite matrix for Q, say an identity matrix.

If P is positive definite function, the Lyapunov for $\left(V(x) = \frac{x^T}{2} PX\right)$, will be global asymptotic stability of the system given in equation (1.1) and the stability of system A will be defined by Hurwitz criteria and convergence analysis will be performed for the same system as required.

1.8 Hurwitz

For the given system in equation (1.1):

$$\dot{X} = AX$$

With,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \quad (1.3)$$

Where,

The values of a,b & c are selected in such a way that the characteristic equation of the matrix and the given system will form a stable combination.

Then, the system in equation (1.3) will follow the Hurwitz criteria and a Lyapunov function will exist as shown in equation (1.2):

$$A^T P + PA = -Q$$

Where, $P = P^T > 0$ (1.4)

This process is helpful in the linearization of a non-linear system.

1.9 Literature Review

In [8], a sliding mode controller is combined to a Kalman filter in a three phase unity power factor rectifier. On the other hand, the discrete time sliding mode approach is very attractive due to the possibility of easy implementation in digital controllers, and this approach has been developed by several researchers [9],[10],[11]. In literature, some authors have reported the use of higher order sliding mode controllers for different machines [12],[13],[14],[15],[16],[17]. While most of this work used SMC as an observer [12],[13],[14], the authors in [15],[16],[17] used SMC as a controller.

In [19], the author presents a technique where conventional internal combustion engine based vehicle is being replace by electric vehicle (EV), as EV never produces greenhouse gasses and air pollutant. These Electric vehicles are also known as green vehicle. Fuel cell based EVs are in need of a step-up high power dc-dc chopper or converter. Interleaved boost converter (IBC) can be dealt with high power ratings application such as HEV. The paper at [19] presents control technique for boosting output voltage of FC using an interleaved boost converter and which is used for controlling DC bus voltage under load variation condition sliding mode controller (SMC) method.

In [20], a SMC approach is used to a high output gain SEPIC converter. In which power circuit of the conventional SEPIC converter is modified with an inclusion of inductor and capacitor to improve the efficiency and gain. Some PI type controllers are very common solutions for a large number of DC DC converters. In [20], the proposed SMC based equivalent controller overcomes this drawbacks. Added to this, a dynamic hysteresis constant switching frequency method is attempted to operate in switching frequency. Hence, SMC technique is used in collaboration with PI controller technique.

In [21], we may observe another unified approach to deal with SMC controllers used in induction motors drives, it also reveals fundamental limitations of hysteresis controllers those can be overcome by higher order controllers. Second order SMCs are investigated to achieve a performance which is disturbance rejected and chattering free. Extensive simulations are carried out on MATLAB platform. Implementation of such a drive becomes feasible due to their inherent parallel processing capability.

The discrete-time SMC is also a unique approach in itself, which can be used in a multi-loop framework for current control of grid connected voltage source inverters, as shown in [22]. This makes the converter to behave like a current source inverter with a capacitive and inductive (CL) type filter. Which results in second order system of current control from third order. In this work, a virtual resistor with resonant controller was applied. Hence, it was quite clear that the SMC was used in RLC circuit in discrete form as a controller.

Not only in discrete time, SMC is also used with neuro systems. In [23], the author represented an approach of co-operative control that was based on the combination of neural network and SMC methodology. In this approach, two parallel neural networks were utilized to realise a neuro sliding mode control (NSMC), where the equivalent control and the corrective controls were used as the output of these two parallel neural networks. Which makes it possible to implement a sliding mode controller in collaboration with neural networks also.

SMC approach is also used in Lorentz systems based on conversion and integral sliding mode surfaces, [24]. The system explanations are divided into the unmatched and the matched parts. A discontinuous control is designed for the unmatched part, which will never be amplified and the continuous control is used to stabilize the error state in the Lorentz system. The used method is proven to guarantee the stability and robustness in the system, which is finally proved by the numerical simulation method.

A twisted SMC controller is shown in [25], which is robust to bounded disturbances. Using Lyapunov function, a control algorithm is also derived. From paper [25] we can see that for the derivation of control algorithms, the Lyapunov function plays essential role. We applied the same approach in implementing the algorithm for our proposed thesis also. The efficacy of twisted control is [25] is also verified on mass spring damper system.

In [26], the sliding mode control is used on context investigation of features of finite time stable (FTS) homogeneous differential inclusions. Here, continuous features are considered with respect to settling time function of FTS homogeneous differential inclusions, which results into the correct calculations of system asymptotic accuracy in the presence of disturbances, noises and delays. Hence, SMC helped in the analysing of the kinematics of car model relative degree bifurcations.

In this dissertation, a SMC modelling is derived into the MATLAB programming and then implemented onto the TORA system by using MATLAB SIMULATION, before implementing it onto the TORA system, the modelling of TORA system is also derived into the MATLAB programming and then the parameters of both SMC and TORA are matched in a way to give us a control input with wide range of operation. The design of SMC approach is composed of two steps [18]. First, design a sliding mode surface in such a way that the parameters of SMC satisfies the design specification of TORA system. Second, a control function is designed making the TORA surface attractive to the desired system in equilibrium state in the presence of uncertainties and external disturbance. In addition, the following contributions are required to be highlighted:

- The SMC and TORA modelling is derived into algorithmic parameter to show the decoupling strategy.
- The control input hence formed, is improved with reference to the stability, by using the algorithmic approach

These approaches are satisfactory obtained by many authors in different fields with different circuits and in their respective papers. However, there may be few studies focussed on the sliding mode control for TORA system. In [39], author proposed decoupled self-tuning signed distance fuzzy sliding mode controller (DSSFSMC) for TORA system and the use of both fuzzy controller and sliding mode controller is done for his implementation. In [2] the author globally stabilized TORA system by transferring the dynamic model to the cascaded form and applied sliding mode control approach.

Different from the above two approaches, a hierarchical sliding mode controllers for TORA system are designed in two different ways in [40]. In order to show the performances, the close loop control system is simulated with Simulink but the Simulink diagram is not well shown by the author in paper. Instead, the waves of sliding surface is shown in this paper. In addition to this, the control input to the TORA system is shown in this dissertation. The observed control input is also improved to be used for further circuit also.

This dissertation comprises of mainly 6 topics which are concluded into 5 chapters overall. In the present ongoing chapter, a brief introduction with literature review on different approaches proposed by several authors into the journals and other conferences is given. Many authors used SMC as controllers of several non-linear devices while other used SMC as an observer of states. More or less they all used SMC to make a non-linear device work properly and obtain the most possible linear characteristics. Since, most practical system in the world are non-linear in behaviour, which makes SMC a very useful and understandable device/controller to be used in the non-linear devices. In the present dissertation, we will be doing the same thing only. We are using SMC as a controller on a non-linear (TORA) system in order to make it work without rigidly on a sliding surface without the interfere of external disturbance.

Chapter 2, will represent the benchmark system to evaluate the performance of a general non-linear TORA system, as shown in [28]. Here equations of TORA system are studied transcribed into the algorithm.

In chapter 3, the design of sliding mode controller is represented and the different equations used in maintaining the required approach for the successful implementation of SMC equations into the proposed algorithm are also discussed.

The simulations of equations obtained in chapter 2 and chapter 3 will be entertained in chapter 4 and hence all the algorithmic results are shown in this chapter with their improved advanced analysis.

Chapter 5 is all about the conclusion and future scope of this technique in the world of control system. Last but not the least, two main algorithms which are used in obtaining the results are also shown as reference point of view.

2. TORA DESIGN

Horizontal motion in one dimensional field is referred to the Translational Oscillator, while rotatory motion refers to the torque. A system which is continuously oscillating in a one dimensional space with the effect of torque (which is being produced by the rotators) is referred as TORA system. The torque factor introduces rotation, which merely produce precise torque which either causes rotation or balanced by some opposing torque. Rotatory actuators have a vast range of applications. They can range from air core gauges used in display devices to valve gauges in petrochemical industries.

Talking of TORA system, a super special problem was given by Robert T. Bupp [28], which is accepted as the benchmark problem for TORA systems in the field of research of control engineering. We must use the mathematical equations of this benchmark system according to the parameter being used in the controller. The SMC controller must satisfy the parameter variations of this general system of TORA.

In this chapter, the focus is to use the mathematical equations of the general TORA system and then implement those in the parameters of the controller with the help of coding in MATLAB.

2.1 System Description

A model [28] of Translational Oscillator Rotational Actuator (TORA) system is shown in Figure 1. The mass of cart in the oscillator is M and connected to a fixed wall by a spring of stiffness k. The cart is restricted to have one dimensional travel. The actuator attached to cart has mass m and moment of inertia I about of center of mass which is located at a distance of L, from the point about which the mass m rotates. We will not consider any gravitational force in this dissertation because the motion is occurred in horizontal plane completely.

In control system, this is considered as the universal TORA system. If someone wish to develop any controller for any non-linear translational oscillatory system with rotatory actuator, this is

the system one must follow. System is famously known as the benchmark system for the performance evaluation of TORA system proposed in [28] and well discussed in [27].
In Figure 1. u denotes the control torque applied to the mass m and x is the disturbance force on the cart and θ denotes the angular position of the rotational actuator of mass m.

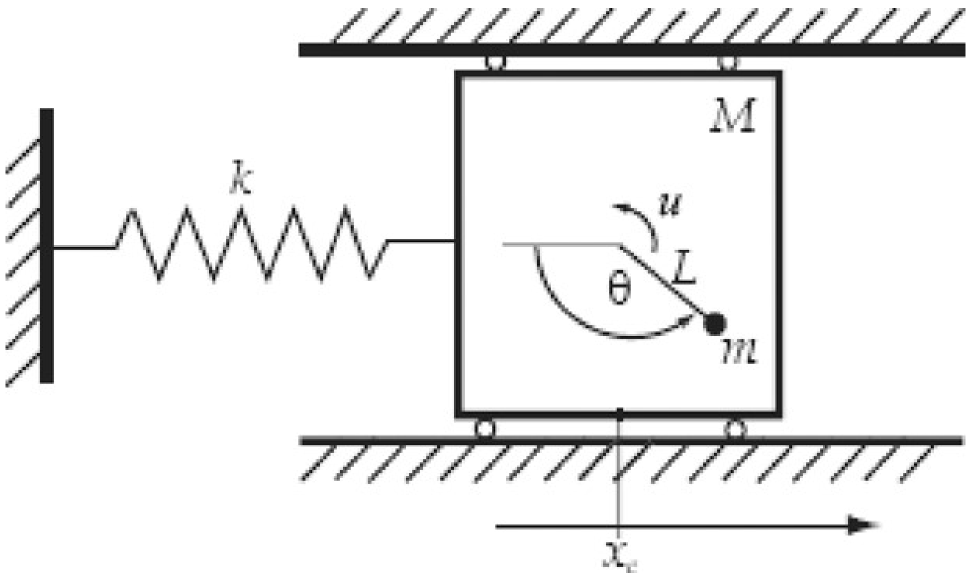

Figure 1. Model of a TORA system. [28]

According to the international journal papers, researchers in control system engineering have to consider the following equations as the general dynamics of TORA system:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = \frac{-z_1 + \varepsilon\theta_2^2 sin\theta_1}{1 - \varepsilon^2 cos^2\theta_1} - \frac{\varepsilon cos\theta_1}{1 - \varepsilon^2 cos^2\theta_1}\, v$$

$$\dot{\theta}_1 = \theta_2 \tag{2.1}$$

$$\dot{\theta}_2 = \frac{\varepsilon cos\theta_1 (z_1 - \varepsilon\theta_2^2 sin\theta_1)}{1 - \varepsilon^2 cos^2\theta_1} + \frac{1}{1 - \varepsilon^2 cos^2\theta_1}\, v$$

Where,

z_1 is normalized displacement of the platform from the equilibrium position $z_2 = \dot{z}_1$ and v is control input. The control goals are:-

$$z_1,\ \dot{z}_1,\ \theta_1,\ \dot{\theta}_1 \rightarrow 0\ ,\ \text{for } t \rightarrow \infty \qquad (2.2)$$

i.e. to achieve the stability in the system, the normalized displacement and angular position of the system must be near to zero at infinite time.

2.2 To decouple a coupled underactuated system

To attach any system with a controller, it must be decoupled with the controller. Which means, the system and the controller must have similar parameter variations and must not have time as a performance variation factor.

In control system, decoupling between two states can be explained as a condition in which the time variation dx_i/dt of i^{th} state is independent of states other than this i^{th} state. Decoupling helps in designing and modelling, since it allows the model to set the controller for every state independent of any other state(s).

Hence, it is clear that if we need to use our TORA system with a SMC controller then TORA system must be first decoupled in order to make it independent of other state(s). The procedure of converting a coupled system into decoupled system is shown here in this section of this dissertation.

Now, for two matrix p & q.
with:

$$q = [q_1 \quad q_2]$$

$$p = [p_1 \quad p_2]$$

The coupled under actuated system would be:-

$$\dot{q_1} = p_1$$

$$\dot{p_1} = f_1(q, p) + g_1(q_2)\, u$$

$$\dot{q_2} = p_2 \tag{2.3}$$

$$\dot{p_2} = f_2(q, p) + g_2(q_2)\, u$$

Where,

f_1, f_2, g_1, g_2 are all smooth functions. Then the global change of co-ordinates, as designed in [29], [30] should be taken as following:

$$x_1 = q_1 - \int_0^{q_2} \frac{g_1(s)}{g_2(s)} ds$$

$$x_2 = p_1$$

$$x_3 = q_2 - \frac{g_1(q_2)}{g_2(q_2)} p_2 \tag{2.4}$$

$$x_4 = p_2$$

If we can successfully eliminate the control factor from the coupled system, then our actuated system would be ready to work as a decoupled system.

Hence, our decoupling algorithm would be of any use if we can remove u in $\dot{x_2}$.

From Eq. (2.3) we may get:

$$\dot{x_2} = \dot{p_1} - \frac{d}{dt}\left(\frac{g_1}{g_2}\right) p_2 - \left(\frac{g_1}{g_2}\right) \dot{p_2}$$

$$\dot{x_2} = f_1 + g_1\text{u} - \frac{d}{dt}\left(\frac{g_1}{g_2}\right)p_2 - \frac{g_1}{g_2}(f_2 + g_2\text{u})$$

$$\dot{x_2} = f_1 - \frac{d}{dt}\left(\frac{g_1}{g_2}\right)p_2 - \frac{g_1}{g_2}f_2 \tag{2.5}$$

Then Eq. (2.3) can be decoupled as:

$$\dot{x_1} = x_2$$
$$\dot{x_2} = f_1 - \frac{d}{dt}\left(\frac{g_1}{g_2}\right)p_2 - \frac{g_1}{g_2}f_2$$
$$\dot{x_3} = x_4$$
$$\dot{x_4} = f_2 + g_2\text{u} \tag{2.6}$$

In equation (2.6), the state at $\dot{x_2}$ is not a factor of control signal u. Which makes Equation (2.6) as a decoupled system of system in equation (2.3). By doing so, we have a general technique to decouple a coupled system in control engineering.

2.3 Decoupling of TORA System

A decoupling algorithm is explained from equation (2.3), on applying the decoupling algorithm in Eq. (2.3) to Eq. (2.1), we may observe the decoupling system of TORA.

We have the TORA equation in (2.1), as:-

$$\dot{z_1} = z_2, \qquad \dot{z_2} = \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1} - \frac{\varepsilon cos\theta_1}{1-\varepsilon^2 cos^2\theta_1}\, v$$

$$\dot{\theta_1} = \theta_2, \quad \dot{\theta_2} = \frac{\varepsilon cos\theta_1(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} + \frac{1}{1-\varepsilon^2 cos^2\theta_1}\, v$$

After observing these equations, for further analysis, let us assume:-

$$f_1 = \frac{-z_1 + \varepsilon\theta_2^2 sin\theta_1}{1 - \varepsilon^2 cos^2\theta_1} \tag{2.7}$$

$$g_1 = \frac{-\ \varepsilon cos\theta_1}{1 - \varepsilon^2 cos^2\theta_1} \tag{2.8}$$

$$f_2 = \frac{\varepsilon cos\theta_1(z_1 - \varepsilon\theta_2^2 sin\theta_1)}{1 - \varepsilon^2 cos^2\theta_1} \tag{2.9}$$

$$g_2 = \frac{1}{1 - \varepsilon^2 cos^2\theta_1} \tag{2.10}$$

and:-

$$x_1 = z_1 + \varepsilon sin\theta_1\ ,$$

$$x_2 = z_2 + \varepsilon\theta_2 cos\theta_1$$

$$x_3 = \theta_1$$

$$x_4 = \theta_2 \tag{2.11}$$

Than we have:-

$$\frac{g_1}{g_2} = \varepsilon cos\theta_1 \qquad (2.12)$$

For the system to work under stability, the condition given in equation (2.2) must be followed. Hence, from Eq. (2.11), the control goals $z_1,\ \dot{z}_{1,}\ \theta_1,\ \dot{\theta_1} \rightarrow 0$ are equivalent to $x_i \rightarrow 0$, i=1,2,3,4.

Since, from equation (2.11), by using differentiation technique, we have:-

$$\dot{x_2} = \dot{z_2} + \varepsilon\dot{\theta}_2 cos\theta_1 - \varepsilon\theta_2^2 sin\theta_1$$

$$= \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1} - \frac{\varepsilon cos\theta_1}{1-\varepsilon^2 cos^2\theta_1} v + \varepsilon\left(\frac{\varepsilon cos\theta_1(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} + \frac{1}{1-\varepsilon^2 cos^2\theta_1} v\right) cos\theta_1 - \varepsilon\theta_2^2 sin\theta_1$$

$$= \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1} + \frac{\varepsilon^2\, cos\theta_1\,(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} - \varepsilon\theta_2^2 sin\theta_1$$

$$= \frac{-z_1(1-\varepsilon^2\, cos\theta_1)+\varepsilon\theta_2^2 sin\theta_1(1-\varepsilon^2\, cos\theta_1)}{1-\varepsilon^2 cos^2\theta_1} - \varepsilon\theta_2^2 sin\theta_1$$

$$= (-z_1 + \varepsilon\theta_2^2 sin\theta_1)\frac{1-\varepsilon^2 cos^2\theta_1}{1-\varepsilon^2 cos^2\theta_1} - \varepsilon\theta_2^2 sin\theta_1$$

$$= -z_1 + \varepsilon\theta_2^2 sin\theta_1 - \varepsilon\theta_2^2 sin\theta_1$$

$$= -z_1$$

Which gives:-

$$\dot{x_2} = -z_1 \qquad (2.13)$$

From the same equation (2.11), we also have:-

$$-z_1 = -x_1 + \varepsilon sin\, x_3 \qquad (2.14)$$

Since, $-z_1$ is a function of $x_1$andx_3. Which makes $\dot{x_2}$ a function of $x_1$andx_3.

From equation (2.13) & (2.14):-

$$\dot{x_2} = f_1(x_1, x_3) \qquad (2.15)$$

Let,

$$u = \frac{\varepsilon cos\theta_1(z_1 - \varepsilon\theta_2^2 sin\theta_1)}{1 - \varepsilon^2 cos^2\theta_1} + \frac{1}{1 - \varepsilon^2 cos^2\theta_1} v, \qquad (2.15)$$

Rewriting the above equation we get:-

$$v = \varepsilon cos\theta_1(z_1 - \varepsilon\theta_2^2 sin\theta_1) - (1 - \varepsilon^2 cos^2\theta_1) \qquad (2.16)$$

(This equation is to be used in decoupling algorithm for the determination of parameters)

also, from Eq. (2.11):-

$$-z_1 = -x_1 + \varepsilon sin\, x_3 \qquad \& \qquad \theta_1 = x_3 \qquad (2.17)$$

Combining Eq. (2.16) & (2.17):-

$$v = \varepsilon cos x_3(x_1 - (1 + x_4^2)\varepsilon sin x_3) - (1 - \varepsilon^2 cos^2 x_3) \qquad (2.18)$$

Using equations (2.1) & (2.11):

$$\dot{x_1} = \dot{z_1} + (\dot{\varepsilon sin\theta_1})$$

$$= z_2 + \varepsilon\theta_2 cos\theta_1$$

$$= x_2 \quad (2.19)$$

From equation (2.1):

$$\dot{\theta_1} = \theta_2$$

From equation (2.11):

$$x_3 = \theta_1 \quad \& \quad x_4 = \theta_2$$

Which gives:

$$\dot{x_3} = x_4 \quad (2.20)$$

Combining equation (2.1) & equation (2.15), we have:

$$\dot{x_4} = \mathrm{u} \quad (2.21)$$

Now,

From this above analysis for equations i.e. (2.15),(2.19),(2.20),(2.21), the Eq. (2.1) can be decoupled as:-

$$\dot{x_1} = x_2 \quad (2.22)$$

$$\dot{x_2} = f_1(x_1, x_3) \quad (2.23)$$

$$\dot{x_3} = x_4 \tag{2.24}$$

$$\dot{x_4} = \text{u} \tag{2.25}$$

Where,

$f_1(x_1, x_3) = -\, x_1 + \; \varepsilon sin\, x_3.$

i.e. $\dot{x_2} = \; -x_1 + \; \varepsilon sin\, x_3$

Eq. (2.22) to Eq. (2.25) shows the decoupled equations for a TORA system.

But,

In paper [2], a SMC is designed for one of the kind of underactuated system, in which $f_1(x_1, x_3)$ must be satisfied as three assumptions. If any underactuated system is failed to satisfy any one of these assumptions then the system should not be considered as a stable system for all the states.

For a system which does not stabilize all the states, it will not be possible to design a SMC controller. If we do so, out controller will not give us the stabilized wave for at t →∞ or we will not get any waveform at all.

So our system must satisfy all these three assumptions in order to stabilize the output. These three assumptions are:

Assumption 1:

$$f_1(0{,}0) \rightarrow 0;$$

Assumption 2:

$$\frac{df_1}{dx_3}\; is\; invertible;$$

Assumption 3:

$$\text{If, } f_1(0, x_3) \to 0, \qquad \text{then } x_3 \to 0$$

In our calculations, Eq. (2.23) is not satisfied by Assumption 2.

In this case, we can redefine $\dot{x_2}$ as :

$$\dot{x_2} = -x_1 + \varepsilon sin\, x_3 + 11\varepsilon x_3 - 11\varepsilon x_3$$

Where, $+11\varepsilon x_3$ is disturbance and hence $f_1(x_1, x_3)$ is redefined as:-

$$f_1(x_1, x_3) = -x_1 + \varepsilon sin\, x_3 + 11\varepsilon x_3.$$

Then,

$\frac{df_1}{dx_3} = \varepsilon cos\, x_3 + 11\varepsilon,$ Which is invertible and assumption 2 is satisfied.

$f_1(0,0) = 0.$ Which satisfies assumption 1.

For $x_3 \to 0$, $f_1(0, x_3) = 0$ Which satisfies assumption 3.

Hence,

We will have stabilize state for this function now.

So, Eq. (2.22) to (2.25) can be rewritten as:-

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= f_1(x_1, x_3) - 11\varepsilon x_3 \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= u \end{aligned} \qquad (2.26)$$

These are the actual required decoupled equations for our TORA system.

Now, all we need to do is to implement all these equations in the algorithms in correct order and by using the correct programming just implement them into a .m file and then make them work into the Simulink model shown in next chapters to obtain the stable graphs.

3. SLIDING MODE CONTROLLER (SMC) DESIGN

A sliding mode control (SMC) is a robust control algorithm which can change the control structure initiatively to obtain the desired output. Because of its no-sensitive to the variation of parameters and external disturbances, it is being widely used to design the controller for underactuated systems.

In [41], authors presented a decoupled fuzzy sliding mode controller (FSMC) for the single and double inverted pendulums system. Authors achieved a good performance in the simulation results, and analysed the stability of sliding mode surface. Wang in [42] developed a cascade sliding mode control (CSMC) approach. Some controller parameters are needed to be switched frequently for assuring the system stability. Yi in [43] also presented a cascade sliding mode controller (CSMC) for largescale underactuated systems, whose sliding mode surface was stable asymptotically. Hao in [44] presented a robust controller using sliding mode control method for a class of mechanical underactuated systems. The total control can assure every state variable to follow its own sliding mode surface to minimize (or zero) by choosing proper parameters of the given controller.

In all these above given papers, authors not only just designed the SMC controller for the required system but also established the stability either mathematically or graphically. This is the most important part with the design of controllers. One should justify the stability of one's approach.

In this chapter not only the SMC design procedure is followed by in the second section of chapter the convergence analysis is also performed on the SMC controller in order to verify and justify the stability of the SMC controller with the non-linear system. This is done with the help of Hurwitz criteria and Lyupnov function.

3.1 SMC Design

The most essential feature of this SMC technique is to choose switching surface of the state space according to the desired dynamical specifications. The control laws are designed in such a way so that all the state trajectories may reach on the switching surface and will remain on it.

Key feature of SMC design is:-

- Robustness against the large class of uncertainty.
- Compared with classical control technique it needs less amount of information.
- Can stabilize those non-linear systems which are no generally stabilize by continuous state techniques.

To realize $x_i \rightarrow 0$, for i=0,1,2,3,…….. as t $\rightarrow\infty$. We may define the error equation of sliding mode controller as:

$$e_1 = x_1$$

$$e_2 = \dot{e_1} = x_2$$

$$e_3 = f_1(x_1, x_3)$$

$$e_4 = \dot{e_3} = \dot{f_1}(x_1, x_3) \tag{3.1}$$

Where,

$$\dot{f_1}(x_1, x_3) = \frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4$$

If, there occurs $e_3 = \dot{e_2}$in above equations then it will enter into singularity. So, we must not let $e_3 = \dot{e_2}$.

The general Sliding Mode Function for a sliding surface is:-

$$s=c_1e_1 + c_2e_2+c_3e_3 + e_4 \quad (3.2)$$

Where,

c_1, c_2, c_3 are positive constant.

Differentiating equation (3.2) w.r.t. time, we get:

$$\dot{s}=c_1\dot{e_1} + c_2\dot{e_2}+c_3\dot{e_3} + \dot{e_4} \quad (3.3)$$

from eq. (3.1) we have:

$$\dot{e_1} = x_2 \qquad \& \qquad e_4 = \dot{e_3} \quad (3.4)$$

from eq. (2.26) & (3.1), we have:

$$\dot{e_2} = \dot{x_2} = f_1(x_1, x_3) - 11\varepsilon x_3 \quad (3.5)$$

And

$$\dot{e_4} = \dot{\left(\frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4\right)} \quad (3.6)$$

Putting equations (3.4),(3.5),(3.6) into (3.3), we get:

$$\dot{s} = c_1x_2 + c_2(f_1 - 11\varepsilon x_3)+c_3e_4 + \frac{d}{dt}\left(\frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4\right) \quad (3.7)$$

Where,

$$\frac{d}{dt}\left(\frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4\right) = \frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3)+\frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right)+\frac{df_1}{dx_3}u \quad (3.8)$$

Substituting Eq. (3.8) into Eq. (3.7):-

We get,

$$\dot{s}=c_1x_2 + c_2(f_1 - 11\varepsilon x_3)+c_3e_4+\frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) + \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) + \frac{df_1}{dx_3}u \tag{3.9}$$

Let,

$$M= c_1x_2 + c_2(f_1 - 11\varepsilon x_3)+c_3e_4+\frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) + \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) \tag{3.10}$$

Rewriting Eq. (3.9), we get:

$$\dot{s} - c_1x_2 - c_2(f_1 - 11\varepsilon x_3) - c_3e_4 - \frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) - \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) = \frac{df_1}{dx_3}u \tag{3.11}$$

Putting Eq. (3.10) in Eq. (3.11):-

$$\dot{s} - M = \frac{df_1}{dx_3}u \tag{3.12}$$

Then,

Design of the sliding mode controller will be:-

$$u=\left[\frac{df_1}{df_3}\right]^{-1}(-M - nsgns - ks) \tag{3.13}$$

Where,

n and k are positive constant. (say n=k=1)

Eq. (3.13) gives the design for sliding mode controller which is to be used for the implementation of SMC on TORA system. The SMC hence formed must follow the asymptotic stability, which was described earlier in the section 1.7 of this dissertation.

To get this Lyapunov function, substitute Eq. (3.13) into Eq. (3.9):-

$$\dot{s} = -nsgn(s) - ks \tag{3.14}$$

The Lyapunov function:

$$V = \frac{1}{2}s^2$$

then,

$$\dot{V} = s\dot{s} = -n|s| - ks^2. \tag{3.15}$$

For the Lyapunov function to provide stability within the states, Eq. (3.15) must be negative i.e. $\dot{V} < 0$. Since, n & k are positive in nature so equation (3.15) will always be negative in nature. Which shows that the Lyapunov function is sensing stability in the system.

This Lyapunov function is used to decide the stability of the SMC. This method is very useful in describing the convergence of the controller, which is further described in the next section of this dissertation.

3.2 Convergence Analysis

So far, we have discussed the model parameters of both TORA system and SMC controller. In order to get the stable and distortion less performance, the TORA system and SMC controller must obey the similar parameter functions in algorithm. This is made sure by their convergence analysis in mathematical form.

From Eq. (3.1), we have error equations as:

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = e_3 - 11\varepsilon x_3$$

$$\dot{e}_3 = e_4 \tag{3.16}$$

From Eq. (3.15), if:

$$s\dot{s} < 0$$

Then, the system will reach and stay at the sliding surface s=0.

On the sliding surface s = 0, from Eq. (3.2) we have:

$$e_4 = -c_1 e_1 - c_2 e_2 - c_3 e_3 \tag{3.17}$$

Let,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c_1 & -c_2 & -c_3 \end{bmatrix}$$

Define c_1, c_2, c_3 in such a way that A becomes Hurwitz i.e. the real part of the characteristic value of A must be negative.

Then, we get the error state eq. as:-

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = e_3 - 11\varepsilon x_3$$

$$\dot{e}_3 = -c_1 e_1 - c_2 e_2 - c_3 e_3 \tag{3.18}$$

Defining,

$E = [e_1 \quad e_2 \quad e_3]^T$, disturbance $d_2 = -11\varepsilon x_3$

Eq. (3.18) can be rewritten as:

$$\dot{E} = AE + D \tag{3.19}$$

Where,

$D = [0 \quad d_2 \quad 0]^T$

To guarantee E → 0, A must be Hurwitz, i.e. the real part of characteristic equation of A must be negative:

By solving:

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -c_1 & -c_2 & -c_3 - \lambda \end{vmatrix} = 0$$

i.e. $$\lambda^3 + c_3\lambda^2 + c_2\,\lambda + c_1 = 0 \tag{3.20}$$

for,

$c_1 = a^3$,

$c_2 = 3a^2$,

$c_3 = 3a$

And d > 0, (3.21)

A will be a Hurwitz.

Now,

Defining Q = Q^T, there exist a Lyapunov equation as:

$$A^TP + PA = -Q \tag{3.22}$$

Now we have, V(e) = E^TPE, a Lyapunov function for the given system.

Then we have,

$$\dot{V(e)} = \dot{E}^T PE + E^T P\dot{E}$$

$$= [AE + D]^T PE + E^T P[AE + D]$$

$$= E^T A^T PE + D^T PE + E^T PAE + E^T PD$$

$$= E^T [A^T P + PA]E + D^T PE + E^T PD$$

$$= -E^T QE + D^T PE + E^T PD$$

Where,

$$D^T PE + E^T PD <= 2\lambda_{max}(P)$$

We have,

$\lambda_{min}(Q)$ is the minimum eigen value of positive definite matrix Q.

$\lambda_{max}(P)$is the maximum eigen value of positive definite matrix P.

Then,

$$\dot{V(e)} < 0 \tag{3.23}$$

If, we assume Q = Identity matrix, we will have:

$$\lambda_{min}(Q) = 1$$

$$\lambda_{max}(P) = \frac{1}{2\lambda_{left}(-A)} \tag{3.24}$$

Which can be guarantee by design of A matrix.

Since,

A is Hurwitz, then E = $[e_1 \quad e_2 \quad e_3]^T \rightarrow 0$ will also be satisfied.

i.e.

$$e_1 \rightarrow 0, \; e_2 \rightarrow 0 \text{ and } e_3 \rightarrow 0 \tag{3.25}$$

Also from, s →0, we have $e_4 \rightarrow 0$ (3.26)

From,

$$e_1 \rightarrow 0,\ e_2 \rightarrow 0 \text{, we may have } x_1 \rightarrow 0 \text{ and } x_2 \rightarrow 0 \qquad (3.27)$$

From, Assumption 3:

$$e_3 = f_1(0,0,x_3) \rightarrow 0 \qquad (3.28)$$

Using Eq. (3.1) now, we get:

$$x_3 \rightarrow 0 \ and \ x_4 \rightarrow 0. \qquad (3.29)$$

Eq. (3.25) to Eq. (3.29) shows that all the required goal of Eq. (3.1) have been achieved and hence the convergence is achieved by the SMC controller.

Eq. (3.23) shows that the $\dot{V}(e)$ is negative i.e. stability is achieved by the controller in the convergence analysis.

Now,

All the equations formed in chapter 2 and chapter 3 will be transcript into the coding in MATLAB and then be used in the SIMULINK to get the desired control input waveform.

4. SIMULATION AND DISCUSSION

The MATLAB platform is optimized for determining the scientific and engineering issues. The world's most naturally categorised machine arithmetic is the matrix based MATLAB language. Simple to see and gaining insight knowledge is the key feature of its inherent graphics. Such MATLAB tools are tested and designed to figure along [45].

MATLAB is being used in heavy body satellites, automobile safety systems, health observance devices, LTE cellular networks, power grids etc, which makes it a very important and useful platform in engineering field. The same is used in this dissertation for the purpose of analysing the equations of system and controller into coding and observing the performance in the form of waves.

Simulink is totally based on block diagram environment, which is a model based design and used for multipurpose design. It provides editors, processors and modelling of dynamic systems. Engineers in every field uses this facility every day to raise their ideas to new levels in their research.

4.1 Implementation of Algorithm in MATLAB

From earlier explanation of modelling of both controller and system. We have a clear idea of what we need to implement in our algorithm to make it work like a SMC controller on a TORA system.

Now, by using MATLAB algorithm skills, we need to implement these above mentioned equations into the MATLAB .m files. These .m files will definitely be implemented into the blocks of SIMULATION file and then we will get different outputs for different parameters.

Of course, only one stable output will be formed, that too when the system and controller parameters will match to each other. In this dissertation, two different analysis of MATLAB simulation and the suitable algorithm for each is applied and finally the result is recorded and shown as:

4.1.1 Conventional Analysis

In Fig. 2, the response of z_1& $\dot{z}_1$ is shown. Since, the response is showing z_1, $\dot{z}_{1,}$ → 0, for t →∞. Hence, Eq. (2.2) is verified.

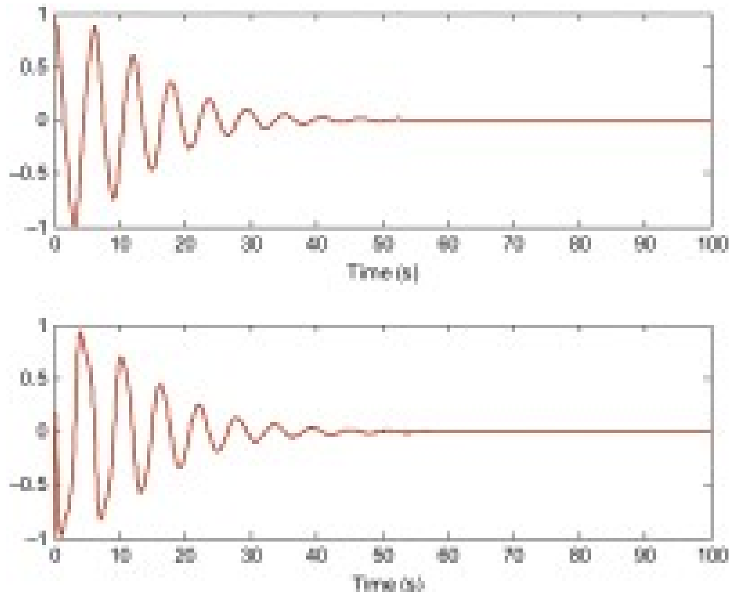

Fig. 2: The speed tracking response of z_1& $\dot{z}_1$ in conventional analysis.

In Fig. 3, the response of θ_1 & $\dot{\theta}_1$ is shown. Since, the response is showing θ_1, $\dot{\theta}_1$ → 0 , for t →∞. Hence, Eq. (2.2) is verified.

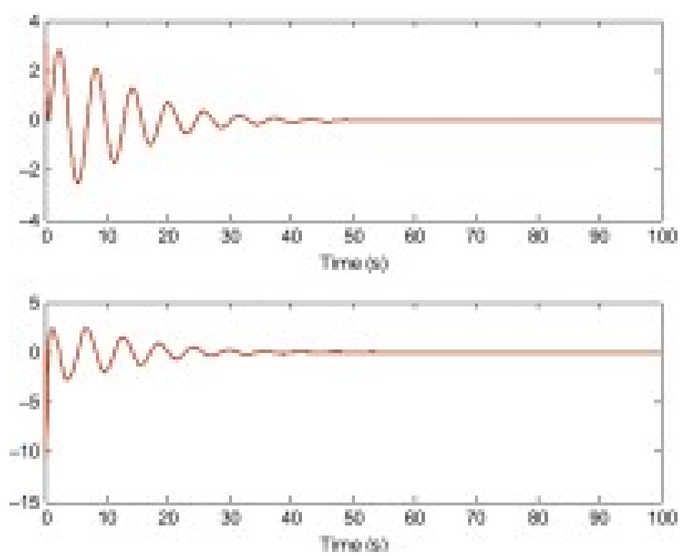

Fig. 3: The position tracking response of θ_1 & $\dot{\theta}_1$in conventional analysis.

These two figures (Fig. 2 & Fig. 3), are showing the variation in different input parameters to the SMC controller i.e. the output from the TORA plant, as shown in Fig. 4, which is actually the SIMULATION form of the required MATLAB model.

In Fig. 4, the .mdl mode of SIMULINK file is shown. The SMC block is encrypted with the algorithm explained in Eq. (3.1) and from Eq. (3.22) to Eq. (3.29). The TORA block consist of the algorithm encrypted with the Eq. (2.11) to Eq. (2.26). The Control is Input is actually the plot of the controlled output given by the SMC controller i.e. input to the TORA system.

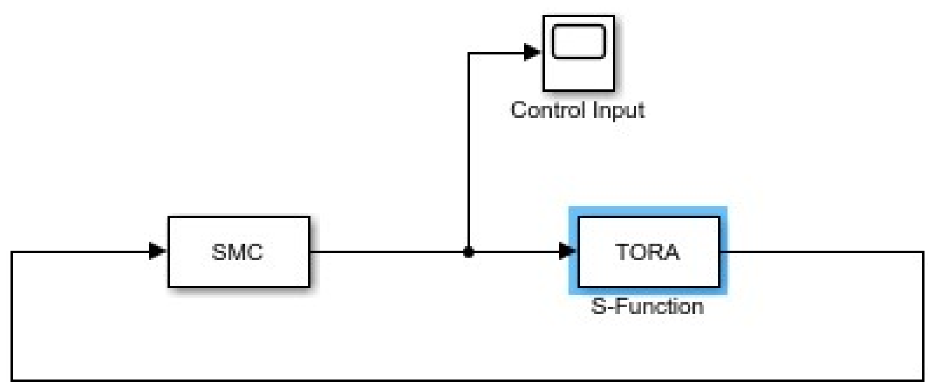

Fig. 4: The SIMULINK model of .mdl file in conventional analysis.

In Fig. 5, the conventional output of TORA system is shwon, which we obtained from the simulation block. Looking closely to it, we can see that this is the result of the combination of Fig. 2 and Fig. 3. This output of TORA system is automatically the input of our controller i.e. SMC controller

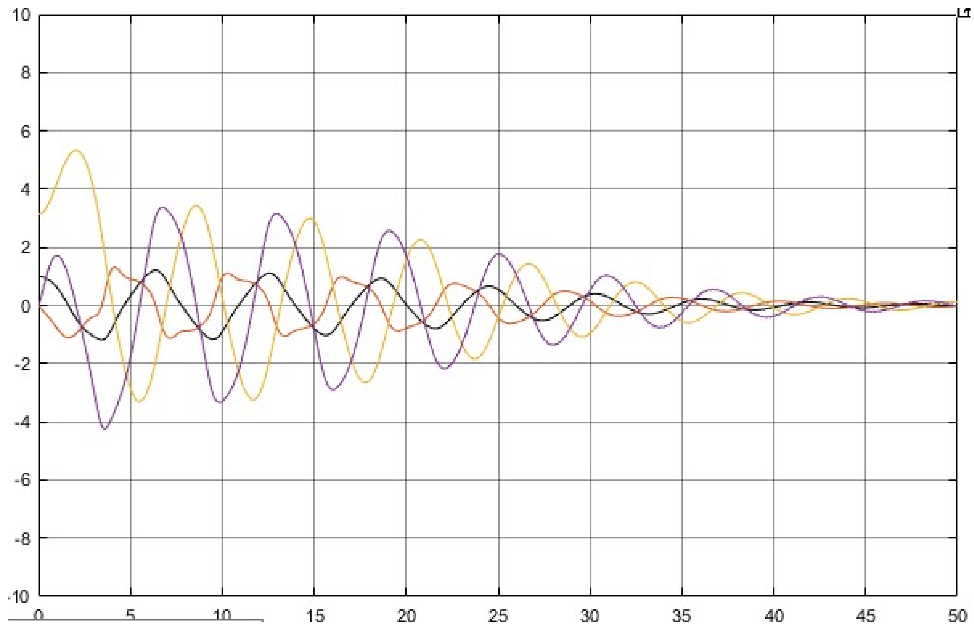

Fig.5: The response of $z_1,\ \dot{z}_1,\ \theta_1,\ \dot{\theta}_1$ obtained from the Simulink block.

In Fig. 6, the Output waves of the control input to the TORA system is shown according to the conventional approach of SMC controller:

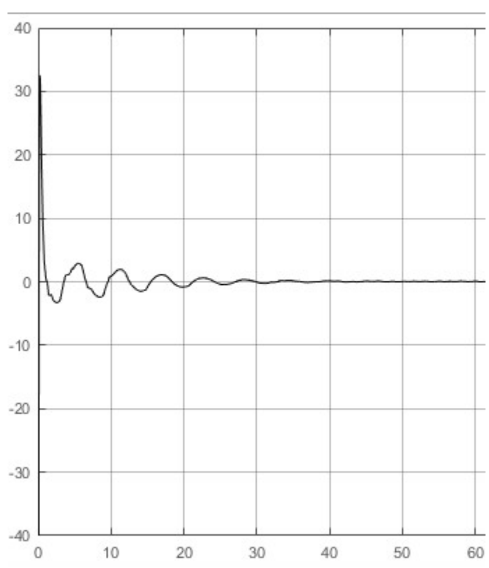

Fig. 6: The response of control input in conventional analysis.

4.1.2 Advanced Analysis

This section shows the advance analysis of same problem. In this advance analysis, the type of the system control input is increased in order to reduce the error and improve the stable behavior of the SMC controller. By doing so, the system is open to other applications of underactuated type with even less peak in control input. Let us see what kind of results are obtained by doings this method.

For advanced analysis also we must satisfy the Eq. (2.2)

i.e.

$$z_1,\ \dot{z}_{1,}\ \theta_1,\ \dot{\theta}_1 \rightarrow 0\ ,\ \text{for t} \rightarrow \infty\text{, should be satisfied.}$$

To make this satisfied the response of $z_1,\ \dot{z}_{1,}\ \theta_1$ & $\dot{\theta}_1$must be same for both conventional and advanced analysis i.e. must be zero for t →∞.

This makes few point pretty clear;

- Input to the SMC controller is actually the output of the TORA system from the block diagram
- Control input is actually the controlled input of the TORA system not the input of SMC controller.
- Simulation diagram will show the controlled output of the SMC controller as the control input in Fig. 6 & Fig. 9.

Now,

in Fig. 7, the response of z_1& $\dot{z}_1$ is shown. Since, the response is showing:

$$z_1,\ \dot{z}_{1,} \rightarrow 0\ ,\ \text{for t} \rightarrow \infty.$$

Once again, Eq. (2.2) is verified.

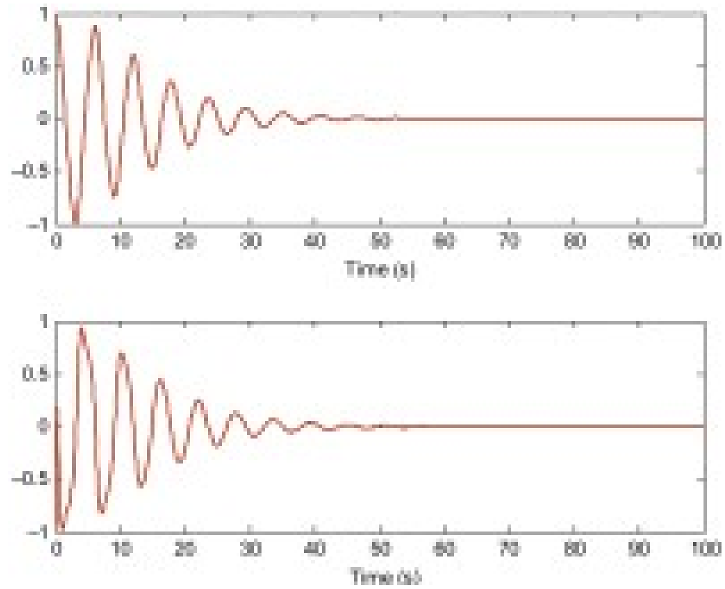

Fig. 7: The speed tracking response of z_1& $\dot{z_1}$ in advanced analysis.

In Fig. 8, the response of θ_1 & $\dot{\theta}_1$ is shown. Since, the response is showing θ_1, $\dot{\theta}_1 \rightarrow 0$, for t →∞.

Again, Eq. (2.2) is verified for advanced analysis also.

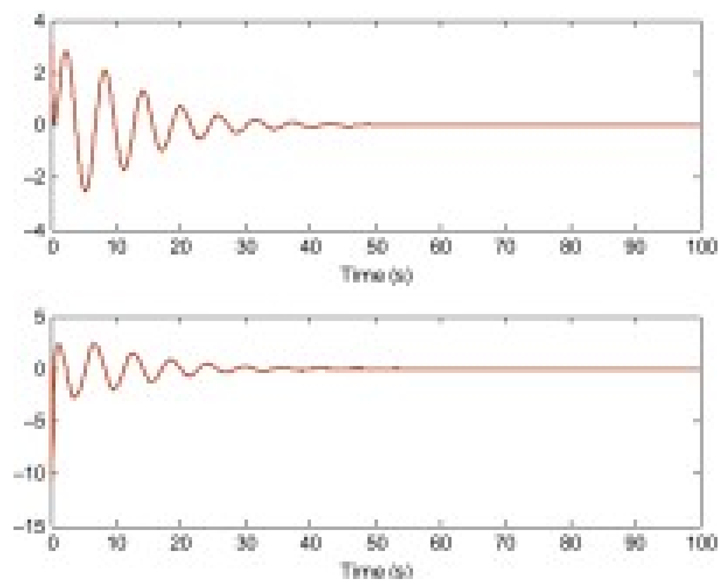

Fig. 8: The position tracking response of θ_1 & $\dot{\theta}_1$in advanced analysis.

Fig. 7 & Fig. 8 are similar to Fig. 2 & Fig. 3, which are compulsory to make our advanced analysis proved right.

From the figures it is quite clear that the required parameters are satisfied.
i.e.

$$z_1,\ \dot{z}_{1,}\ \theta_1\ \&\ \dot{\theta}_1 \rightarrow 0\ ,\ \text{for } t \rightarrow \infty.$$

Now, by implementing a type incremental block into the control input, we will get a wide range of system stability. This will help in making the elimination of disturbance easy.

We also need to vary the algorithm parameters a bit according to the requirements to make the fluctuations form the control input eliminated. The new formed .mdl MATLAB file is shown in Fig. 9. Where, the SMC_controller block is implemented by the improved algorithm of parameters of SMC controller and the (In1 to Out1) block is increasing to type of signal with appropriate specifications and as the result we will get an improved control input.

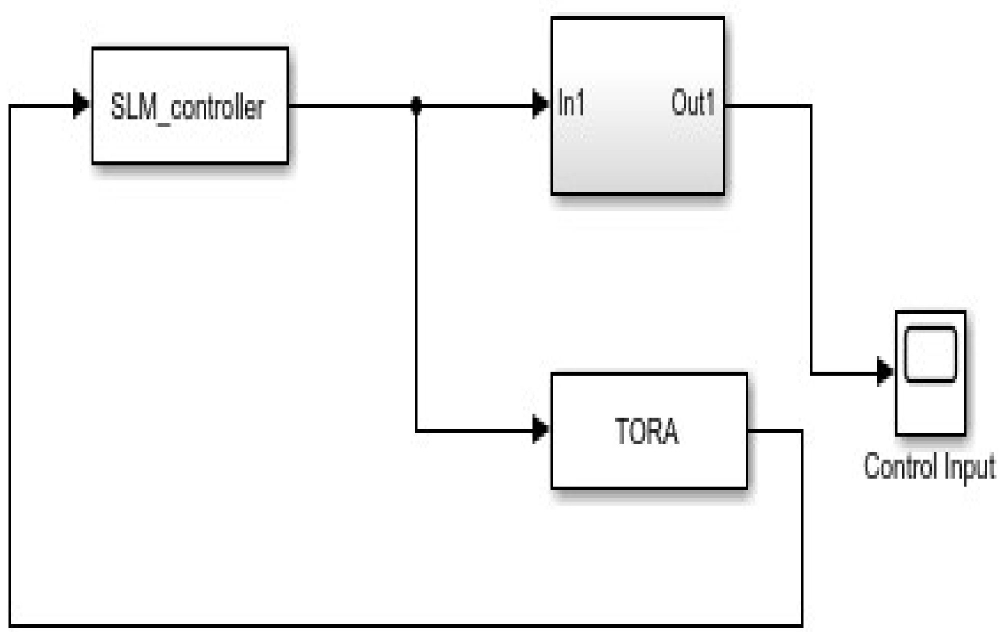

Fig. 9: The SIMULINK model of .mdl file in advanced analysis

In Fig. 10, the advanced analysis output of TORA system is shwon, which we obtained from the simulation block. Looking closely to it, we can see that this is the result of the combination of Fig. 7 and Fig. 8. This output of TORA system is automatically the input of our controller i.e. SMC controller

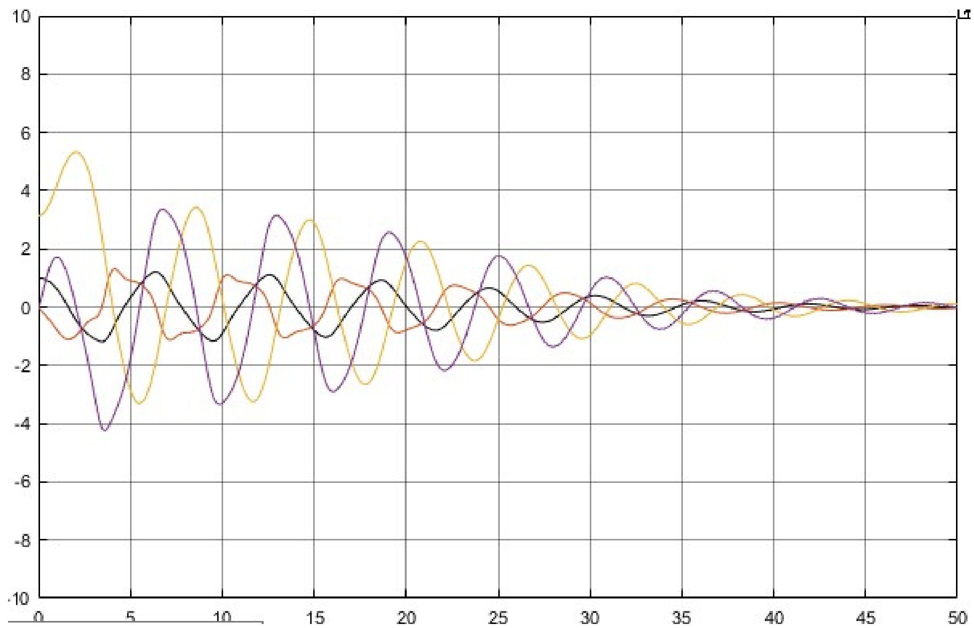

Fig.10: The response of $z_1,\ \dot{z}_{1,}\ \theta_1,\ \dot{\theta}_1$ obtained from the Simulink block.

In Fig. 11, the graph observed at the control input of .mdl model is shown. In this new graph the change in the parameters of the plot is clearly observed:

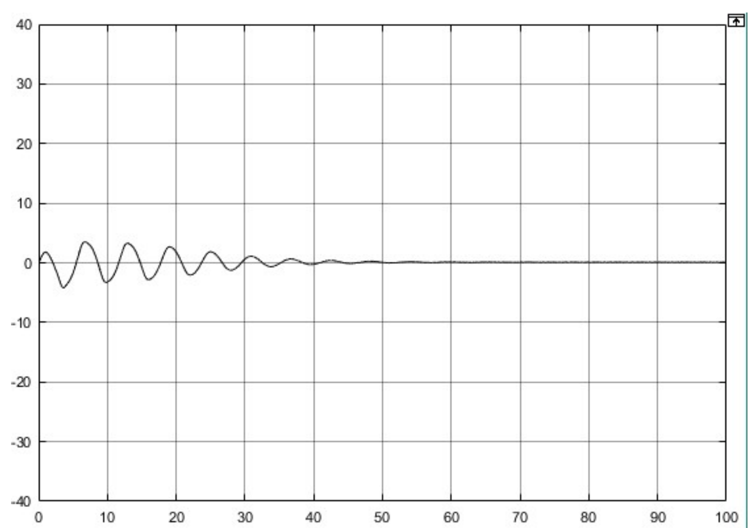

Fig. 11: The response of control input in advanced analysis.

Fig. 6 & Fig.11 are showing the exact control input for both the type of analysis shown in this paper. From the analysis of these two figures, the benefits observed are shown in Table I.

TABLE I. DIFFERENCE BETWEEN BOTH APPROACHES

	Conventional analysis	Advanced analysis
First Peak	More	Less
Stability	Less	More
Disturbance rejection	Less	More
Error	More	Less
Reliable	Less	More

THE ABOVE RESULTS ARE OBSERVED ON THE WAVEFORM GRAPHS AND THE PARAMETER ANALYSIS IS DONE AND THE RECORDED PARAMETERS ARE SHOWN HERE IN THE TABLE II.

TABLE II. COMPARISION BETWEEN THE PARAMETERS OF BOTH APPROACHES

	Conventional analysis	Advanced analysis
Steady state error	0 degree	0 degree
Peak Undershoot	2.2 degree	1.6 degree
Peak time	1 second	0.5 seconds
Peak overshoot	32 degree	2 degree

5. CONCLUSION AND FURTHER SCOPE

By using the algorithmic approach on a TORA system, we are successfully able to conclude these points:

- Controller and non-linear equations used in the mathematical modelling are derived into algorithmic parameter forms.
- Decoupling strategy is used in algorithmic approach to make controller and non-linear devices work together successfully.
- The control input to the non-linear (TORA) system is derived by using the Simulink block to make it possible to use it in other circuit also.
- Controller and system parameters are improved in the algorithmic approach due to the better precision of this approach
- Algorithm (after implementation) makes the working of a model less complex.
- The model parameters are improved in the algorithmic as well as in the simulation block.

After using above approach in this dissertation, the following benefit with respect to the results in the advanced analysis are obtained:

- First peak of the control input is reduced in order to get a reliable range of disturbance for operation.
- System becomes more stable due to this reduced peak.
- Ability of the system against disturbance rejection is improved in order to use it in any device for further operations.
- Chances of error into the results/operations are very low.
- Having the less error, the system becomes more and more reliable as compared to the earlier analysis performed on non-linear devices by other authors.

As far as we talk about the future scopes of this programming, then we should come to a final decision, that till we have non-linear systems to deal with, we will be dealing with controllers like SMC. This technique is being used for many decades and will be using for the understanding of controlling and observer technique for many coming years also.

Author in [31], show that how a SMC can be used in homogeneity analysis and several application based on the same on the other hand in [31], author used a 2^{nd} order SMC system in a switch time based adaption. Such studies of respected journals shows that SMC has a wide range of operations and is being used for many large number of problems. Since, homogeneity and switch time based problems are vast in number of applications, it is quite clear to say that the SMC has a vast future scope on control system engineering.

Except for the TORA system used in this dissertation, the similar controller can be used on other systems like:

- Inverted pendulum.
- Underactuated systems of higher order.
- SMC with fuzzy networking
- SMC with neural networking.
- SMC in discrete time.

Hence, it can be concluded that the SMC implementation on different systems in control system has applications like the boundaries of a sea. Which will never end and goes on.

APPENDICES

1. CODING USED IN .m FILE FOR “slm_controller”

```
function[sys,x0,str,ts]=s_function(t,x,u,flag)
switch flag,
case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;
case 3,
    sys=mdlOutputs(t,x,u);
case {2,4,9}
    sys=[];
otherwise
    error(['unhandled flag=',num2str(flag)]);
end
function [sys,x0,str,ts]=mdlInitializeSizes


sizes=simsizes;
sizes.NumContStates=0;
sizes.NumDiscStates=0;
sizes.NumOutputs=1;
sizes.NumInputs=4;
sizes.DirFeedthrough=1;
```

```
sizes.NumSampleTimes=1;
sys=simsizes(sizes);


x0=[];
str=[];
ts=[0 0];
function sys=mdlOutputs(t,x,u)


z1=u(1);
z2=u(2);
theta1=u(3);
theta2=u(4);


epc=0.1;


x1=z1+epc*sin(theta1);
x2=z2+epc*theta2*cos(theta1);
x3=theta1;
x4=theta2;


f1=-x1+epc*sin(x3)+11*epc*x3;
f1_x1=-1;
f1_x3=epc*(cos(x3)+11);
```

```
dt_f1_x1=0;
dt_f1_x3=-epc*sin(x3)*x4;

e1=x1;
e2=x2;
e3=f1;
e4=f1_x1*x2+f1_x3*x4;

a=3.0;
c1=a^3;c2=3*a^2;c3=3*a;
s=c1*e1+c2*e2+c3*e3+e4;

M=c1*x2+c2*(f1-11*epc*x3)+c3*e4+f1_x1*(f1-11*epc*x3)+dt_f1_x3*x4;

k=0.5;xite=0.5;
fai=0.010;
if abs(s)<=fai
    sat=s/fai;
else
    sat=sign(s);
end
```

```
ut=1/f1_x3*(-M-k*s-xite*sat);

v=(1-epc^2*(cos(theta1))^2)*ut-epc*cos(theta1)*(z1-epc*theta2^2*sin(theta1));


sys(1)=v;
```

2. CODING USED IN .m FILE FOR "tora"

```
function[sys,x0,str,ts]=s_function(t,x,u,flag)

switch flag,

   case 0,

         [sys,x0,str,ts]=mdlInitializeSizes;

   case 1,

         sys=mdlDerivatives(t,x,u);

   case 3,

         sys=mdlOutputs(t,x,u);

case {2,4,9}

         sys=[];

otherwise

         error(['unhandled flag=',num2str(flag)]);

end


function [sys,x0,str,ts]=mdlInitializeSizes

sizes=simsizes;

sizes.NumContStates=4;

sizes.NumDiscStates=0;

sizes.NumOutputs=4;

sizes.NumInputs=1;

sizes.DirFeedthrough=0;

sizes.NumSampleTimes=0;
```

```
sys=simsizes(sizes);

x0=[1;0;pi;0];

str=[];

ts=[];



function sys=mdlDerivatives(t,x,u)


z1=x(1);


theta1=x(3);

theta2=x(4);


epc=0.1;

v=u(1);


den=1-epc^2*(cos(theta1))^2;

dz2=(-z1+epc*theta2^2*sin(theta1)-epc*cos(theta1)*v)/den;

dth2=(epc*cos(theta1)*(z1-epc*theta2^2*sin(theta1))+v)/den;


sys(1)=x(2);

sys(2)=dz2;

sys(3)=x(4);

sys(4)=dth2;
```

```
function sys=mdlOutputs(t,x,u)

sys(1)=x(1);

sys(2)=x(2);

sys(3)=x(3);

sys(4)=x(4);
```

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