

A COMPARITIVE STUDY OF AUXILIARY SIGNALS OF STATIC VAR SYSTEM FOR DAMPING SUBSYNCHRONOUS RESONANCE

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By

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CERTIFICATE

This is to certify that **ANKITA BANSAL (2K11/PSY/02)** has carried out her major project work in partial fulfillment for the degree of **MASTER OF TECHNOLOGY IN POWER SYSTEM** on the topic “**A COMPARATIVE STUDY OF DIFFERENT AUXILIARY SIGNALS OF STATIC VAR SYSTEM FOR DAMPING SUBSYNCHRONOUS RESONANCE**” during the period under my supervision and guidance and has completed the project to my satisfaction.

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I express sincere and heartfelt gratitude to my parents, brother and in laws.

(Ankita Bansal)

ABSTRACT

Transmission systems are being restructured to provide increased power transfer capability to accommodate a much wider range of possible generation patterns, due to the present pace of power system. The power transfer in the integrated power system is impeded by transient stability, voltage stability, and small signal stability. To enhance the power transfer capability, Series compensation has been widely used. The fast acting power electronic converters, Flexible AC Transmission System (FACTS) devices with their equally fast and efficient controlling capabilities are becoming the pillars of support of such a highly integrated power systems. FACTS devices are very effective and capable for increasing loadability, reducing system loss, improved stability of network and reduced cost of production.

However, presence of series compensation gives rise to the problems of dynamic instability and sub synchronous resonance (SSR). SSR is basically an electrical power system condition where the electrical network exchanges energy with the turbine generator at one more natural frequencies of the combined system below the synchronous frequency of the system. A number of power electronic devices have been proposed for dynamic compensation, improving system stability, directing power flows, etc. One of the power electronic devices used for reactive power compensation is a Static VAR Compensator (SVC). Such a system when connected in shunt with a power system is referred to as a static VAR System.

An idea of the thesis is to damp SSR by adding static var compensator (SVC). This thesis shows that by using different auxiliary controllers of SVC, damping performance is enhanced. The major objective is to compare various auxiliary controllers of SVC. The results are obtained by modelling a linearized system in MATLAB.

A detailed system model has been developed. The study system consists of a generator supplying power to an infinite bus over a long transmission line. IEEE type-1 excitation system is considered for the generator. The SVC of switched capacitor-thyristor controller reactor (SC-TCR) type provides dynamic voltage support at the midpoint of line. A lumped parameter T-circuit represents the network. The SVC and series compensation are located at the centre of transmission line in order to optimize the performance of these devices.

Different SVC auxiliary controllers are compared and evaluated for enhancement of dynamic stability of a series compensated power system by computing the eigenvalues of linearized system model.

The performance of combined reactive power frequency controller (CRPF) and combined voltage angle and reactive power controller (VARP) has been compared and it is found that the combined voltage angle and reactive power (VARP), SVC auxiliary controller is the most effective for system damping at a given operating point.

Concluding remarks of the work has been presented.

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LIST OF SYMBOLS

| | |
|--------|---|
| B | Susceptance |
| D | Damping Constant |
| E, e | Source Voltage |
| f_0 | Nominal system frequency |
| G | Conductance |
| I | Current |
| K | Spring Constant |
| K_P | Proportional gain of SVS voltage controller |
| K_I | Integral gain of SVS voltage controller |
| K_B | Gain of SVS auxiliary controller |
| K_D | Slope of SVS control characteristics |
| L | Inductance |
| M | Inertia constant |
| P | Real power |
| Q | Reactive power |
| R, r | Resistance |
| S | Complex power |
| T_e | Electrical torque |
| T_m | Mechanical torque |
| T_s | Firing delay time constant |

| | |
|-------|------------------------------|
| T_D | Thyristor dead time constant |
| T_M | Measurement time constant |
| t | Time in seconds |
| U | Input variable |
| V | Voltage |
| X | Reactance |
| Y | Admittance |
| Z | Impedance |

Greek Symbols

| | |
|----------|----------------------------|
| δ | Generator rotor angle |
| ψ | Flux linkage |
| θ | angle |
| ω | Angular velocity (rad/sec) |

Prefix

| | |
|----------|--------------------|
| Δ | Incremental Change |
|----------|--------------------|

Superscripts

| | |
|-----|-----------|
| t | Transpose |
|-----|-----------|

Subscripts

| | |
|--------------------|---|
| R,M,E,N | Rotor circuit, Mechanical, Excitation System, Network |
| C | SVS auxiliary controller |
| G,g | Generator |
| $\alpha, \beta, 0$ | α - β - 0 axis variables |
| d,q,0 | d-q-o axis variables |
| D, Q, 0 | D- Q_0 axis variables |
| 0 | Initial value |
| f,g,h,k | field and other rotor coils |

A dot over a symbol indicates differentiation with respect to time.

CHAPTER-1

INTRODUCTION

1.1GENERAL

Due to the overgrowing demand of electrical energy, the size and complexity of power systems is growing equally. The main components of an electrical power system includes generating stations, transmission lines and distribution systems apart from various other controlling equipment. The generating stations comprising of turbines and generators convert the mechanical energy in the form of water pressure or steam pressure to electrical energy. Generated electrical energy is then transmitted to the distribution stations through the transmission network. The transmission lines also help in connecting one power system to the other .

Fixed capacitors have been used to increase the steady state power transfer capabilities of a transmission lines. This is due to partially compensating the reactance of transmission lines. A major concern associated with fixed series capacitors is the sub synchronous resonance (SSR), which arises as a result of the interaction between the compensated transmission line and turbine generator shaft.

1.2 SUBSYNCHRONOUS RESONANCE (SSR)

When a massive turbo-generator unit is connected to a power system, system switching events such as fault occurrence, fault clearance, incorrect synchronising and reclosing and live switching which generally occurs independent of subsynchronous resonance conditions will have the effect on turbo-generator shafts. Such events consequently decay the fatigue life of the shaft [10].

Subsynchronous resonance is defined as an electric power system condition where the electric network exchanges energy with a turbine generator at one or more of the natural frequencies

of the combined system below the synchronous frequency of the system [10]. SSR results in excessively oscillatory torque on the machine shafts causing their fatigue and damage.

- $f_{SSR} = f_0 - f_{er}$
- $f_{er} \approx f_0 [X_C / (X_E + X_T + X'')]^{1/2}$

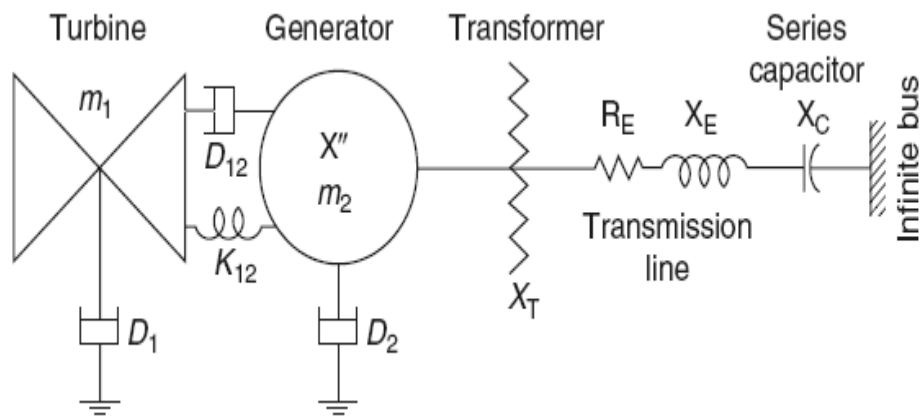


Fig.1.1 Turbine-generator with series compensated transmission line (From IEEE Committee Report)

Where,

f_{er} = electrical system resonant frequency

f_0 = synchronous frequency

X_C = capacitive reactance

X'' = sub transient reactance of the generator

X_E = inductive reactance of transmission line

X_t = leakage reactance of transformer

f_r = induced rotor current frequency

1.3 SELF EXCITATION

In a power system employing series capacitor compensated transmission line, the electrical sub-synchronous currents entering the generator terminals produces sub-synchronous terminal voltage components. These voltage components may sustain the currents to produce the effect that is called self excitation. There are two types of self excitation: one involving electrical dynamics and the other involving both electrical and mechanical turbine generator dynamics, accordingly to resonance phenomena can be classified into two categories:

1. Induction generator effect
2. Torsional interaction

a. Induction generator effect:

Self excitation of the electrical system alone is caused by induction generator effect. Because the rotor circuits are turning more rapidly than the rotating mmf caused by the subsynchronous armature currents, the rotor resistance to sub-synchronous currents viewed from the armature terminals is negative. When this negative resistance exceeds the sum of the armature and network resistance at a resonant frequency, there will be self excitation [10].

b. Torsional interaction:

Generator rotor oscillations at a torsional mode frequency (f_n) induce armature voltage component of subsynchronous and super-synchronous frequency (f_{en}). The armature voltage frequency component are related to the torsional made frequency by the equation

$$f_{en} = f_0 \pm f_n$$

When f_{en} is close to f_{er} , the subsynchronous frequency voltage is phased to sustain the subsynchornous torque. If the component of sub-synchronous torque in phase with rotor velocity deviation equals or exceeds the inherent mechanical damping torque of the rotating system, the system will become self excited. This interplay between the electrical and mechanical system is called torsional interaction [10].

1.4 COUNTERMEASURES TO SUBSYNCHRONOUS RESONANCE PROBLEMS:

As, series capacitor compensated transmission line will help to have the optimum/ economical power transfer. The cost of constructing another line for the transfer of same amount of additional power that the series capacitor compensated line can carry will be much higher as compared to the cost of additional equipments to be installed to counteract the subsynchronous resonance problem.

The analysis of subsynchronous resonance in series capacitor compensated power transmission system is a complex technical problem. When such an analysis reveals either unstable system operating condition or an unacceptable risk of equipment damage during system disturbance, measures to resolve the problems must be implemented. These measures may include modification to the transmission system, restrictive operating criteria and/or the addition of equipments designed to counteract the problems. For many long distance transmission systems series compensation including the required corrective measures is the most economical alternative [31].

1.5 STATIC VAR SYSTEM (SVS)

A static var compensator (or SVC) is an electrical device for providing fast-acting reactive power on high-voltage electricity transmission networks. By changing the firing angle can modulate the effective reactance of a SVS and hence through the use of this device SSR can be mitigated.

SVC's are used to control the voltage profile under load variations, increase power transfer capability and improve system stability. They can be used for damping power system oscillations incorporating some auxiliary signals.

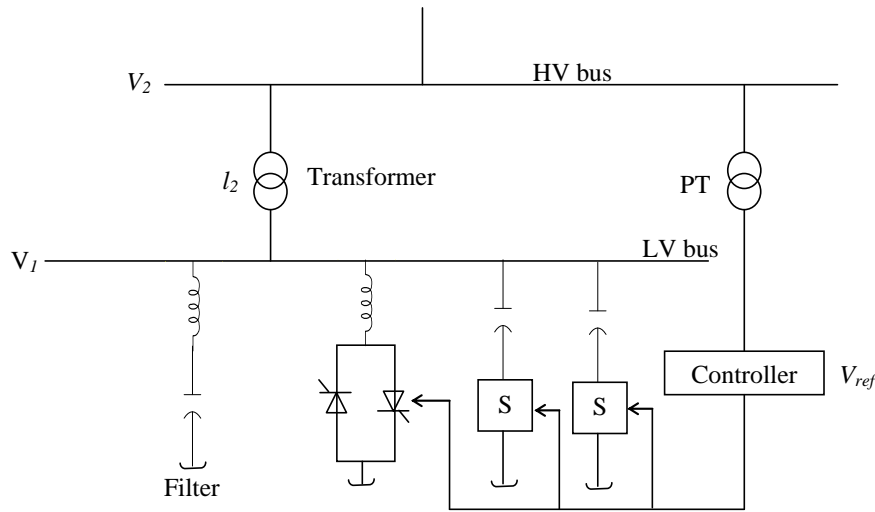


Fig.1.3 Schematic of (FC-TCR) type SVS

Some of the basic requirements of a power transmission system can be performed by static VAR systems:

- Maintaining the system voltage profile under dynamic load conditions such as line switching or load rejection.
- Improving the power transfer capability of the system, thereby increasing the dynamic and transient stability of the system.
- Suppression of power systems oscillations thereby improving the system damping.
- Suppression of voltage fluctuations caused by disturbing loads such as rolling mills, arc furnaces and single phase traction loads.
- Controlling the reactive power flow and thereby minimising the system losses.

1.6 SSR ANALYSIS TOOL

The SSR phenomenon involves exchange of energy between mechanical and electrical system. There are several methods available for the study of SSR and some methods are described below:

1.) Eigen Value Analysis

This is performed with the network and the generator modelled by a system of simultaneous differential equations. The results provide both the natural frequencies of oscillation as well as the damping of each frequency. This technique thus will be used for conducting small-signal analysis to provide a comprehensive understanding of the various aspects of the SSR phenomenon.

2.) Frequency Scanning

This technique computes the equivalent resistance and inductance seen from the stator winding of the generator to the network as function of frequency. If there is a frequency at which the inductance is zero and the resistance is negative, self sustaining oscillations would be expected due to the induction generator effect. This method is particularly suited for preliminary analysis of SSR problems [10].

CHAPTER-2

LITERATURE REVIEW

The chapter presents a comprehensive review of the development in the area of SVS application for improving the power system performance. The emphasis has been given on the recent advances that have taken place in various fields of SVS application. Recent advances in the field of emerging new technology, known as Flexible AC transmission System (FACTS) are also highlighted.

Narendra Kumar [1] developed a Narendra Kumar-subynchronous resonance (NK-SSR) damping scheme for damping torsional modes due to subynchronous resonance (SSR) in a series compensated power system. The scheme, utilized the effectiveness of combined reactive power and frequency (CRPF) SVS auxiliary controller in coordination with an induction machine-damping unit (IMDU) coupled to the T-G shaft. The studies were conducted on the first IEEE benchmark model. The scheme stabilized all torsional modes over a wide operating range of power transfer. The digital computer simulation study, using a nonlinear system model exhibited a remarkable improvement in the system transient performance.

Hongtao Liu, Zheng Xu Zhi Gao [3] presented the results of complex torque coefficient method realized by time domain simulation, SSR characteristics of power system with SVS. The studied model is modified from the first IEEE Subynchronous Resonance benchmark system added by a SVC, consisting of a Thyristor-controlled reactor (TCR) and a Thyristor –Switched Capacitor (TSC). A number of cases were presented by simulation through varying some key parameters. By comparing the electric damping in different cases, the subynchronous resonance characteristics of power system with SVCs were indentified.

Hitoki Sugimoto et al. [4] forwarded the comparative studies among the power electronics devices that can suppress the SSR in series compensated power transmission and distribution system. Computer simulations were made to evaluate the SSR damping effect with three different countermeasures, Narain G. Hingorani (NGH), TCSC, Dynamic Stabilizer (DS). The results of this study shows that the DS scheme can decrease the

cost of equipment more than the other schemes because of the lowest thyristor rated capacitor. However, the damping effect of SSR in case of applying DS scheme is the weakest among all the schemes.

Zhang Zhi-Qiang, Xiao Xiang-Ning [16] developed a technique to mitigate the SSR in transmission system using SVC. The complex torque coefficient approach based on time domain simulation had been used to analysis the influence of the SSR under the condition with SVC or without SVC. Finally, deviations between using SVC and without SVC had been analyzed by using time domain simulation, and point that for SSR in power system the SVC is available and effective.

Qun Gu et al. [18] presented a method of achieving damping of single angle oscillations and improving transient stability of an interconnected power system using fuzzy logic control SVC. Different input measurement and their combination were tested. Also the damping effectiveness of fuzzy controlled SVS and conventional power system stabilizer was compared.

R.K. Varma and S. Auddy [19] proposed the concept of using remote signals acquired through phasor measurement units (PMU) to damp SSR. An auxiliary subsynchronous damping controller (SSDC) for a static VAR compensator (SVC) using the remote generator speed as the stabilizing signal was designed to damp subsynchronous oscillations. The IEEE first benchmark model was used to show the effectiveness of the controller. Extensive simulation results in EMTDC/PSCAD showed that an SVC already installed in a transmission system with the primary objective of improving power transfer capability can also damp SSR with the auxiliary controller using remote generator speed.

R. N. Sharma et al. [20] carried out a study on the application of SVC in improving the voltage profile of long distance transmission lines. A suitable mathematical model of SVC was developed. On the basis of study, they recommended that the operation of the line was not possible without an intermediate switching station and SVC.

Rivera Salamanca et al. [21] presented a comprehensive analytical methodology for the analysis and study of subsynchronous torsional interactions (SSTI) between turbine-generators and static Var compensators (SVCs) embedded in a series capacitive-compensated AC power network. The methodology was based on a flexible approach to perform state-space and frequency domain analyses of multi-machine power systems.

IEEE papers [22],[23],[24] presents proposed terms, definitions and symbols in pursuit of Electric Utility Industry uniformity and common understanding in the analysis of subsynchronous resonance for series compensated transmission systems. These definitions are recommended, where applicable, in other unique areas encompassing subsynchronous oscillations. The work presented is a product of the Subsynchronous Resonance Working Group as part of the activity of the IEEE System Dynamic Performance Subcommittee.

E. Hammad, M. El-Sadek [25] presented Static VAr compensators (SVC) are extensively used, in power systems, for controlling voltage as well as damping machine rotor (mode 0) oscillations, It appeared, therefore, natural to extend the role of the extremely fast controlled static VAr compensators to include damping and other subsynchronous modes of oscillations.

S.T Nagrajan and N. Kumar [26] designed the Genetic Algorithm (GA) based control strategy. This design is more effective and robust auxiliary control of Static VAR System (SVS) for Sub Synchronous Resonance (SSR) damping improvement. The problem of tuning all the control parameters for effective stabilization of multiple torsional modes over various system conditions is formulated into a standardized optimization problem and optimized using GA for improving SSR damping. This is carried out on First IEEE bench mark model. With the proposed GA, the problem is efficiently solved and the auxiliary controller of SVS is optimized over a wide operating range. Eigenvalue analysis using a linearized system model has been conducted on the test system to validate the approach and the designed SVS based auxiliary controller.

Narendra Kumar, S.K. Agarwal [32] developed control strategies for damping of torsional stresses due to SSR using SVS To damp out torsional oscillations, a proportional integral (PI) controller in coordination with auxiliary control signal was employed to modulate the reactive power output of SVS in accordance to locally measurable signals such as combined active power & reactive power, combined voltage angle & active power and combined derivative of active power & derivative of reactive power. Eigenvalue analysis and digital

simulation using non-linear system model was performed in order to demonstrate the effectiveness of the proposed controller under disturbance conditions. The time domain simulation study demonstrates that the CARP/CVARP/DRDAP auxiliary controllers improve the damping of the torsional electromechanical oscillations due to sub synchronous resonance (SSR) in the series compensated power system.

S.H. Hosseini and O. Mirshekar [32] tried to damp SSR by optimal control of SVC in typical power system. The system was linearized and its state space was obtained. To demonstrate the effectiveness of the proposed optimal control, time domain simulations were also performed on the first benchmark model with matlab simulink. The results showed that optimal control can better damp SSR than the PID controller and, because of the small control signal; system saturation is prevented with optimal control.

Sanjiv Kumar, Narendra Kumar, Vipin Jain [37] investigated damping subsynchronous oscillation using static VAR compensators (SVC). TCR-FC (Thyristor Controlled Rectifier with Fixed Capacitor) is a well known combination to improve voltage stability. Supplementary signals such as variation in reactive power, variation in frequency, variation in active power, variation in current can be used to enhance the dynamic response of the system. Their derivatives also can be used for better performance. Signals are compared and it was shown that Deviation in Reactive power gives best performance.

Narendra Kumar and P.R. Sharma [38] developed combined reactive power and frequency auxiliary controller and incorporated in the SVS control system located in the middle of a series compensated transmission line in coordination with controlled series compensation (CSC) along with induction machine damping unit (IMDU) coupled to T-G shaft.

CHAPTER-3

MODELLING OF POWER SYSTEMS

3.1 INTRODUCTION

Dynamic stability has been a major problem in power system following the introduction of excitation system and long transmissions lines. The dynamic performance of the synchronous machine and the transmission network which are described as the set of differential and algebraic equations are non linear. From the stability point, these equations may be linearized by assuming that a disturbance is considered to be small. Small-signal analysis using linear techniques provides valuable information about the inherent dynamic characteristics of the power system and assists in its design.

This chapter presents an analytical method useful in the study of small-signal analysis of subsynchronous resonance (SSR) establishes a linearized model for the power system and performs the analysis of the SSR using eigen value analysis.

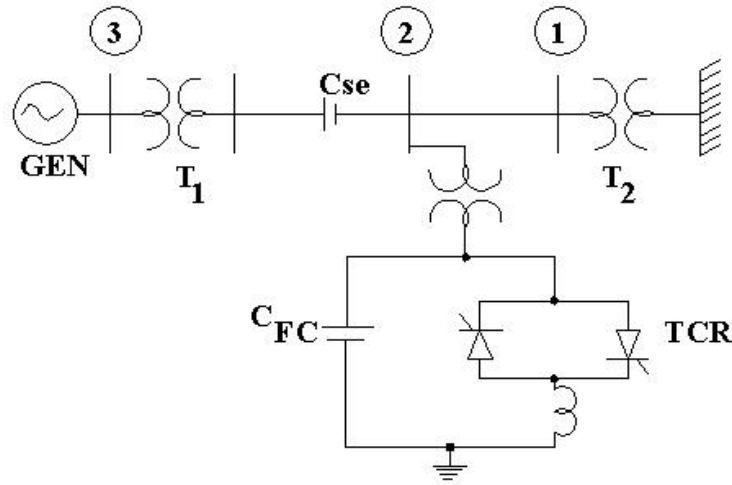
The stability of power system had been adversely affected by high rating of machines along transmission lines. The FACTS devices can reduce the flow of power in heavily loaded lines, resulting in increase loadability, low system loss, and improved stability of network. The installation of static VAR system at the intermediate point of transmission line can maintain the voltage deviations within close tolerance and causes enhancement of dynamic stability of system.

3.2 SYSTEM MODEL [32]

3.2.1 System Description

The study system consists of a generating station having two synchronous generators each of 555 MVA capacity supplying bulk power to a large power system over a long distance transmission line. SVS is located at the system of transmission line. An equivalent synchronous generator, of 1110 MVA capacity, represents the generating station and the large power system is represented as an infinite bus. The single line diagram of the study system is shown in Fig. 3.1.

reactor type configuration
study system can be represented as



thyristor controlled
reactor type configuration. The

Fig. 3.1 Study system

3.2.2 Generator Model

The detailed mathematical model of synchronous generator used here is in such form, which can be directly utilized for dynamic stability analysis and system simulation. The synchronous generator is represented by three symmetrical placed armature winding a, b, c and one field winding, f and three damper winding h, g, k. the damper windings are considered to model the eddy current effects in solid rotor machine. The following assumptions are used in deriving the basic equations of the machines.

- The m.m.f distribution is sinusoidal in the air gap and harmonics are neglected.
- Sub Transient saliency is neglected, $x_d'' = x_q''$.

- iii. The effects of machine damping and prime mover dynamic are small and hence neglected for simplicity.
- iv. Magnetic saturation and hysteresis are neglected.
- v. The synchronous generator model can be divided in following subsections are:
 - a) Stator, Rotor, Mechanical System
 - b) Excitation System

(1) STATOR: In the detailed machine model the stator of synchronous generator is represented by a dependent current source I_s in parallel with inductance $[L''_s]$ as shown in fig3.2 . The above model simplifies the problem of interfacing the machine and network; this is required for solution of synchronous machine equations along with network equations. The dependent current replaces the time varying coupling between the stator winding and rotor windings, I_s is a (3*1) vector and L''_s is a (3*3) matrix. These are expressed as:

$$I_s = [I_a \ I_b \ I_c]^t = I_d C + I_q S \quad (1)$$

Where

$$C^T = \sqrt{2/3} [\cos\theta \ \cos(\theta - 2\pi/3) \ \cos(\theta + 2\pi/3)]; \ S^T = \sqrt{2/3} [\sin\theta \ \sin(\theta - 2\pi/3) \ \sin(\theta + 2\pi/3)]$$

$$I_d = c_1 \Psi_f + c_2 \Psi_h$$

$$I_q = c_3 \Psi_g + c_4 \Psi_k$$

I_d and I_q are the component of dependent current source along d and q axis respectively.

θ = rotor angle

$c_1 - c_4$ are constants

$$L''_s = L_0/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2L_d''/3 \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \quad (2)$$

Where, L_d'' = Sub – transient inductance of machine

L_0 = Zero sequence inductance of machine

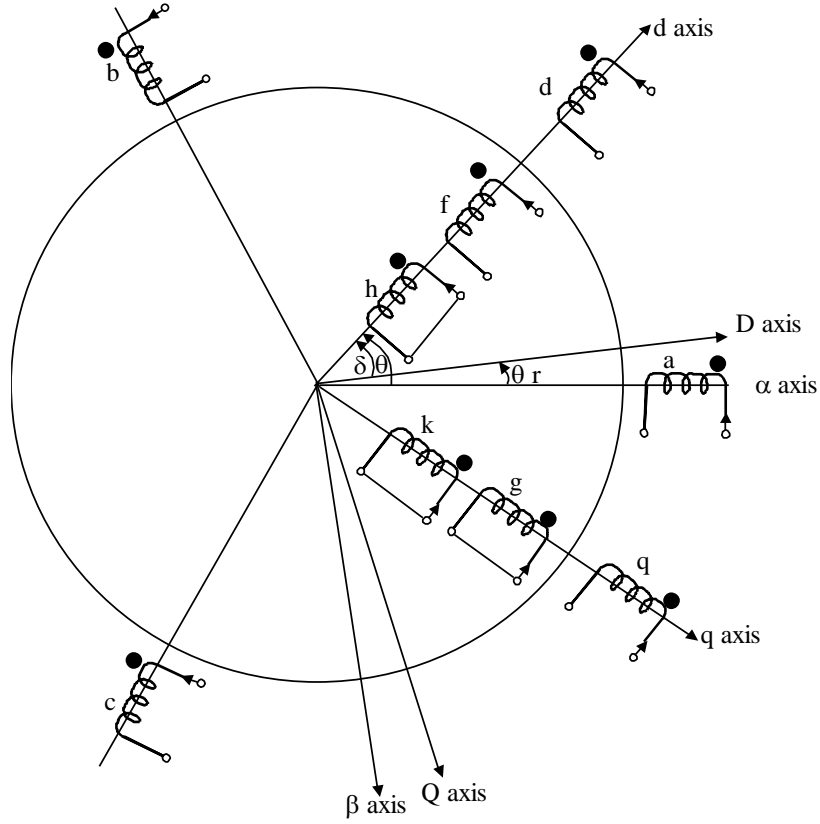


Fig.3.2 Schematic layout of windings of synchronous Machine and their two axis representation

If the external network connected to machine terminals is symmetrical, as considered in this case a, b, c components can be transformed to α , β , 0 components by using Clarke's transformation. R_a denotes the armature resistance. The components of armature current in the meshes correspond to α , β , 0 components of armature currents. The relationship between α , β , 0 components and phase current i_a , i_b , i_c is given by:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{3} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (3)$$

The dependent current sources in α , β , 0 frames to reference are defined as:

$$i_\alpha = I_d \cos\theta + I_q \sin\theta \quad (4)$$

$$i_\beta = -I_d \sin\theta + I_q \cos\theta \quad (5)$$

$$i_0 = 0$$

The α - axis equivalent representation of the machine stator current can be directly combined with α - network of the AC transmission system.

(2) ROTOR :

The rotor flux linkages are defined as follows:

$$\begin{aligned} \dot{\psi}_f &= a_1 \psi_f + a_2 \psi_h + b_1 V_f + b_2 I_d \\ \dot{\psi}_h &= a_3 \psi_f + a_4 \psi_h + b_3 I_d \\ \dot{\psi}_g &= a_5 \psi_g + a_6 \psi_k + b_5 I_q \\ \dot{\psi}_k &= a_7 \psi_g + a_8 \psi_h + b_6 I_q \end{aligned} \quad (6)$$

V_f is the field excitation voltage.

i_d, i_q are d, and q axis components of the machine terminal current respectively which are defined by:

$$i_d = \sqrt{2} / 3 [i_a \cos\theta + i_b \cos(\theta - 2\pi/3) + i_c \cos(\theta + 2\pi/3)] \quad (7)$$

$$i_q = \sqrt{2} / 3 [i_a \sin\theta + i_b \sin(\theta - 2\pi/3) + i_c \sin(\theta + 2\pi/3)] \quad (8)$$

Currents i_d and i_q are defined w.r.t machine reference frame. These currents are transformed to D-Q frame of reference to have common axis of representation with the AC network and SVS, which is rotating at synchronous speed, ω_0 . The following transformation is employed:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \delta & -\sin \delta \\ \sin \delta & \cos \delta \end{bmatrix} \begin{bmatrix} i_D \\ i_Q \end{bmatrix} \quad (9)$$

Where, i_D and i_Q are the components of the machine current along D-axis and Q-axis respectively. Substitute eqn. (9) in eqn. (6) and linearizing the resultant equations, we have the state equation of the rotor circuit [15] as follows:

$$\begin{bmatrix} \Delta\Psi_f \\ \Delta\Psi_h \\ \Delta\Psi_g \\ \Delta\Psi_k \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_5 & a_6 \\ 0 & 0 & a_7 & a_8 \end{bmatrix} \begin{bmatrix} \Delta\Psi_f \\ \Delta\Psi_h \\ \Delta\Psi_g \\ \Delta\Psi_k \end{bmatrix} + \begin{bmatrix} -b_2 i_{q0} & 0 \\ -b_3 i_{q0} & 0 \\ b_5 i_{q0} & 0 \\ b_6 i_{q0} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [\Delta v_f] + \begin{bmatrix} b_2 \cos \delta & -b_2 \sin \delta \\ b_3 \cos \delta & -b_3 \sin \delta \\ b_5 \cos \delta & b_5 \cos \delta \\ b_6 \cos \delta & b_6 \cos \delta \end{bmatrix} \begin{bmatrix} \Delta i_D \\ \Delta i_Q \end{bmatrix}$$

$$\text{Or } \dot{X}_R = [A_R] X_R + [B_{R1}] U_{R1} + [B_{R2}] U_{R2} + [B_{R3}] U_{R3} \quad (10)$$

Where, $X_R = [\Delta\Psi_f \ \Delta\Psi_h \ \Delta\Psi_g \ \Delta\Psi_k]^t$

$U_{R1} = [\Delta\delta \ \Delta\omega]^t$, $U_{R2} = [\Delta v_f]^t$, $U_{R3} = [\Delta i_D \ \Delta i_Q]^t$

Output equations of rotor circuit are developed by using the relationship between I_d , I_q and rotor linkages.

$$I_d = c_1 \Psi_f + c_2 \Psi_h$$

$$I_q = c_3 \Psi_g + c_4 \Psi_k$$

Where, I_d and I_q are the components of the dependent current source along d- and q- axis respectively.

By Kron's transformation

$$\begin{bmatrix} i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (11)$$

Linearizing equation (11), we get

$$\begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} = \begin{bmatrix} c_1 \cos \delta & c_2 \cos \delta & c_3 \sin \delta & c_4 \sin \delta \\ -c_1 \sin \delta & -c_2 \sin \delta & c_3 \cos \delta & c_4 \cos \delta \end{bmatrix} \begin{bmatrix} \Delta\Psi_f \\ \Delta\Psi_h \\ \Delta\Psi_g \\ \Delta\Psi_k \end{bmatrix} + \begin{bmatrix} I_{Q0} & 0 \\ -I_{D0} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \quad (12)$$

$$\text{Or } Y_{R1} = [C_{R1}] X_R + [D_{R1}] U_{R1} \quad (13)$$

3.3 MECHANICAL SYSTEM

The rotor of a turbine generator unit is a complex mechanical system made up of several rotors of a different size, each with the system mechanical shaft section and couplings.

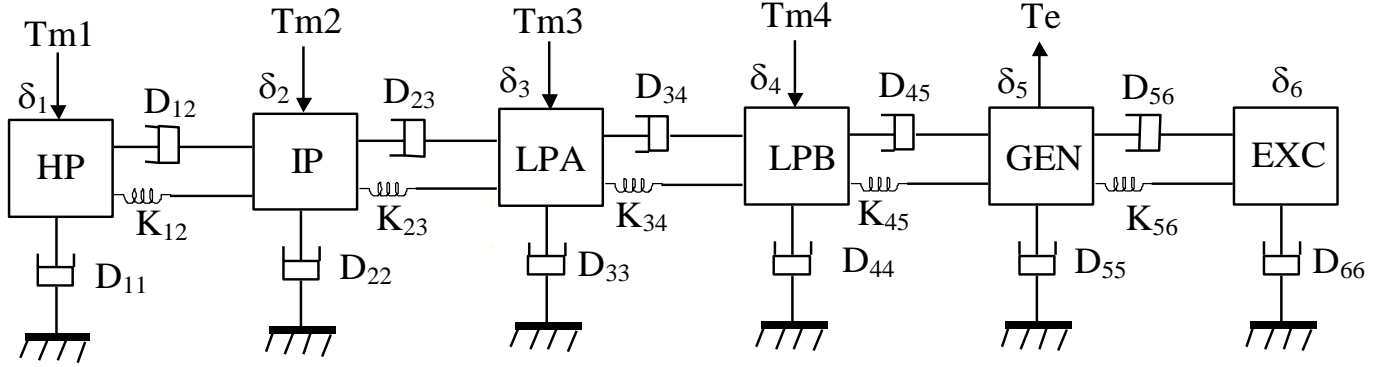


Fig 3.3 Six mass model of Turbogenerator Shaft System

$$\omega_3 = [(D_{23} \omega_2 - (D_{23} + D_{33} + D_{34}) \omega_3 + D_{34} \omega_4 - K_{23} (\delta_3 - \delta_2) - K_{34} (\delta_3 - \delta_4) + T_{M3})] / M_3$$

$$\omega_4 = [(D_{34} \omega_3 - (D_{34} + D_{44} + D_{45}) \omega_4 + D_{45} \omega_5 - K_{34} (\delta_4 - \delta_3) - K_{45} (\delta_4 - \delta_5) + T_{M4})] / M_4$$

$$\omega_5 = [(D_{45} \omega_4 - (D_{45} + D_{55} + D_{56}) \omega_5 + D_{56} \omega_6 - K_{45} (\delta_5 - \delta_4) - K_{56} (\delta_5 - \delta_6) + T_e)] / M_5$$

$$\omega_6 = [(D_{56} \omega_5 - (D_{56} + D_{66}) \omega_6 - K_{56} (\delta_6 - \delta_5) + T_e)] / M_6$$

$$\omega_1 = [-(D_{11} + D_{12}) \omega_1 + D_{12} \omega_2 - K_{12} (\delta_1 - \delta_2) + T_{m1}] / M_1$$

$$\omega_2 = [(D_{12} \omega_1 - (D_{12} + D_{22} + D_{23}) \omega_2 + D_{23} \omega_3 - K_{12} (\delta_2 - \delta_1) - K_{23} (\delta_2 - \delta_3) + T_{M2})] / M_2$$

$$T_e = X''_d (i_D I_Q - i_Q I_D)$$

Linearizing all above equations and writing in state space form, we get

$$\dot{X}_M = [A_M] X_M + [B_M] U_M \quad (15)$$

Where,

$$\dot{X}_M = [\Delta\delta_1 \ \Delta\delta_2 \ \Delta\delta_3 \ \Delta\delta_4 \ \Delta\delta_5 \ \Delta\delta_6 \ \Delta\omega_1 \ \Delta\omega_2 \ \Delta\omega_3 \ \Delta\omega_4 \ \Delta\omega_5 \ \Delta\omega_6]^t$$

$$U_M = [\Delta T_{M1} \ \Delta T_{M2} \ \Delta T_{M3} \ \Delta T_{M4} \ -\Delta T_e]^t$$

Since governor system involves large time constants, so

$$\Delta T_{M1} = \Delta T_{M2} = \Delta T_{M3} = \Delta T_{M4} = 0 \quad (16)$$

$$\text{Also, } \Delta T_e = -X''_d [\Delta i_D I_{Q0} + i_{D0} \Delta I_Q - \Delta i_Q I_{D0} - i_{Q0} \Delta I_D] \quad (17)$$

$$\Delta T_e = -X''_d \begin{bmatrix} -i_{Q0} & i_{D0} \end{bmatrix} \begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} - X''_d \begin{bmatrix} -I_{Q0} & I_{D0} \end{bmatrix} \begin{bmatrix} \Delta i_D \\ \Delta i_Q \end{bmatrix} \quad (18)$$

Substitute equation (16) in equation (18) yields

$$\dot{X}_M = [A_M] X_M + [B_{M1}] U_{M1} + [B_{M2}] U_{M2} \quad (19)$$

Where,

$$U_{M1} = [\Delta I_D \ \Delta I_Q]^t$$

$$U_{M2} = [\Delta i_D \ \Delta i_Q]^t$$

The output equation of the mechanical system is constituted by the angular position and velocity at the generator mass δ_5 and ω_5 . Hence, output equation of the mechanical system will be:

$$Y_M = [C_M] X_M \quad (20)$$

Where,

$$Y_M = [\Delta\delta_5 \ \Delta\omega_5]^t$$

$$\text{As, } \Delta\delta = \Delta\delta_5$$

$$\Delta\omega = \Delta\omega_5$$

Where, $\Delta\delta$ and $\Delta\omega$ are the incremental changes in the generator rotor angle and angular speed.

3.4 EXCITATION SYSTEM

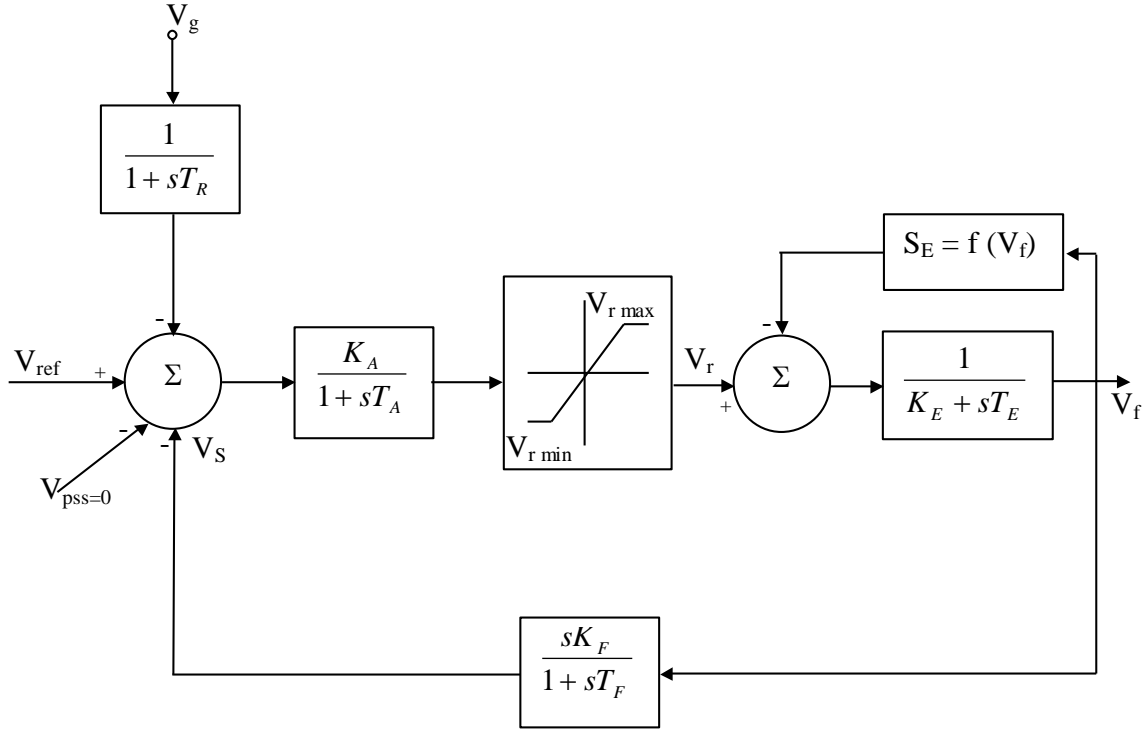


Fig.3.4 IEEE-type-I excitation system model

The excitation system is represented by the IEEE type-1 model as shown in fig3.4. V_g represents terminal voltage and S_E is the saturation function.

The excitation system is described by the following equations:

$$\begin{aligned}\dot{V}_f &= -\frac{(K_E + S_E)}{T_E} V_f + \frac{1}{T_E} V_r \\ \dot{V}_S &= -\frac{K_F(K_E + S_E)}{T_E T_F} V_f - \frac{1}{T_E} V_S + \frac{K_F}{T_E T_F} V_r \\ \dot{V}_r &= -\frac{K_A}{T_A} V_S - \frac{1}{T_A} V_r - \frac{K_A}{T_A} V_g + \frac{K_A}{T_A} V_{REF}\end{aligned}\tag{21}$$

The state and output equations of the linearized system are:

$$\begin{bmatrix} \dot{V}_f \\ \dot{V}_s \\ \dot{V}_r \end{bmatrix} = \begin{bmatrix} \frac{-(K_E + S_E)}{T_E} & 0 & \frac{1}{T_E} \\ \frac{-K_F(K_E + S_E)}{T_F T_E} & \frac{-1}{T_F} & \frac{K_F}{T_F T_E} \\ 0 & \frac{-K_A}{T_A} & \frac{-1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta V_f \\ \Delta V_s \\ \Delta V_r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{-K_A}{T_A} \end{bmatrix} [V_g]$$

$$\text{Or } \dot{X}_E = [A_E]X_E + [B_E]U_E \quad (22)$$

The output equation is given by:

$$[\Delta V_f] = [1 \quad 0 \quad 0] \begin{bmatrix} \Delta V_f \\ \Delta V_s \\ \Delta V_r \end{bmatrix}$$

$$\text{Or } Y_E = [C_E]X_E \quad (23)$$

Where, $X_E = [\Delta V_f \ \Delta V_s \ \Delta V_r]^t$, $U_E = \Delta V_g$, $Y_E = \Delta V_f$

3.5 Network Model

The transmission line is represented by a single lumped T circuit as shown in fig.3.5. The charging capacitor of the line is combined with fixed capacitor of the SVS. The network has been represented by its α - axis equivalent circuit, which is identical with the positive sequence network. The generator, transformer and receiving end transformer are represented by their leakage inductances. The shunt capacitance of the line and magnetizing current are neglected.

I_α is the α -axis component of dependent current source as described in the stator circuit model of synchronous machine. Series compensation is provided at the centre of the line towards the line side of transformer. The current entering the infinite bus, SVS and generator is represented by $I_{1\alpha}$, $I_{2\alpha}$, and $I_{3\alpha}$ respectively. The corresponding terminal voltages are indicated by $V_{1\alpha}$, $V_{2\alpha}$ and $V_{g\alpha}$. R, L represents the series resistance and

inductance for half line section L_{T1} and L_{T2} are the leakage inductances of the transformers at the sending and receiving ends. At the SVS bus, c_n represents the sum of line charging capacitor of the two lines sections and SVS fixed capacitor.

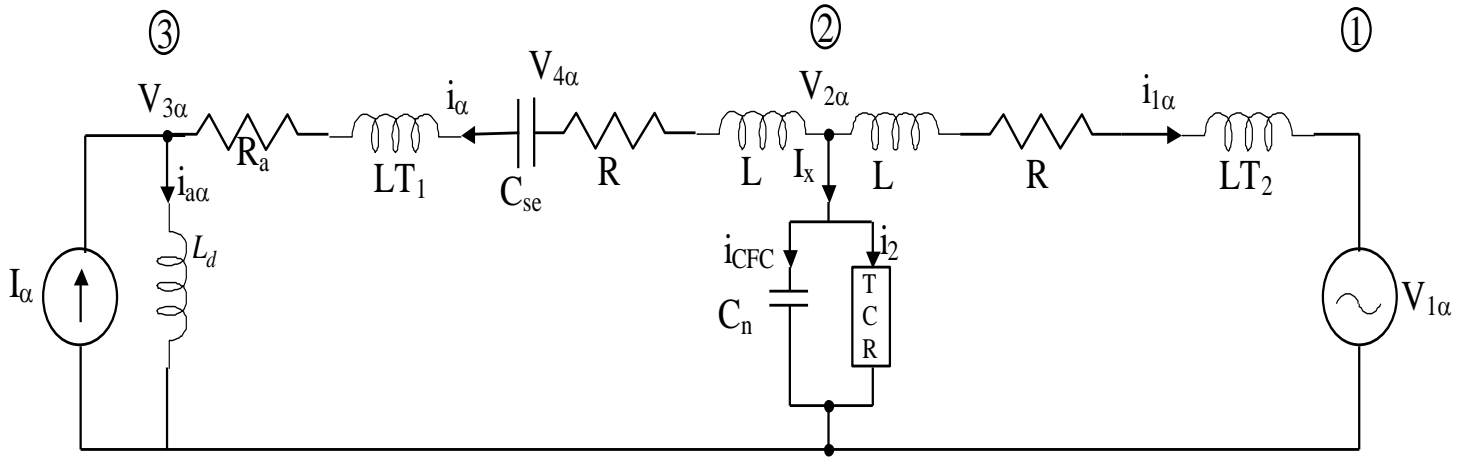


Fig. 3.5 Representation of Series Compensated Network

The following differential equations are derived for the T-circuit represented of the α -network.

$$\frac{di_{1\alpha}}{dt} = \frac{-R}{L_2} i_{1\alpha} + \frac{1}{L_2} V_{2\alpha} - \frac{1}{L_2} V_{1\alpha} \quad (24)$$

$$\frac{di_{\alpha}}{dt} = \frac{1}{L_1} V_{1\alpha} - \frac{R_1}{L_1} i_{\alpha} - \frac{L_d''}{L_1} \frac{di_{\alpha}}{dt} - \frac{1}{4} V_4 \alpha \quad (25)$$

$$\frac{dV_2}{dt} = -\frac{1}{c_n} i_{\alpha} - \frac{1}{c_n} i_{2\alpha} - \frac{1}{c_n} i_{1\alpha} \quad (26)$$

$$\frac{dV_{4\alpha}}{dt} = \frac{1}{C_{se}} i_{\alpha}$$

When,

$$C_n = C_T + C_{FC} \quad L_A = L_{T1} + L_n'' \quad L_1 = L + L_A$$

$$L_2 = L + L_{T2} \text{ and } R_1 = R + R_a$$

Similarly the equations can be derived for β network.

The equations of α - β network are transformed to synchronously rotating D-Q frame of reference using Kron's transformation and are subsequently linearized. Since the infinite bus voltage is constant,

$$\Delta V_{1D} = \Delta V_{1Q} = 0$$

The linearized state equation of the network model is finally obtained as follows:

$$\begin{bmatrix} \Delta \dot{i}_{1D} \\ \Delta \dot{i}_D \\ \Delta \dot{V}_{2D} \\ \Delta \dot{V}_{4D} \\ \Delta \dot{i}_{1Q} \\ \Delta \dot{i}_Q \\ \Delta \dot{V}_{2Q} \\ \Delta \dot{V}_{4Q} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_2} & 0 & \frac{1}{L_2} & 0 & -\omega_0 & 0 & 0 & 0 \\ 0 & -\frac{R_1}{L_1} & \frac{1}{L_1} & \frac{-1}{L_1} & 0 & -\omega_0 & 0 & 0 \\ -\frac{1}{c_n} & -\frac{1}{c_n} & 0 & 0 & 0 & 0 & -\omega_0 & 0 \\ 0 & \frac{1}{c_{se}} & 0 & 0 & 0 & 0 & 0 & -\omega_0 \\ \omega_0 & 0 & 0 & 0 & \frac{-R}{L_2} & 0 & \frac{1}{L_2} & 0 \\ 0 & \omega_0 & 0 & 0 & 0 & \frac{-R_1}{L_1} & \frac{1}{L_1} & \frac{-1}{L_1} \\ 0 & 0 & \omega_0 & 0 & -\frac{1}{cn} & -\frac{1}{cn} & 0 & 0 \\ 0 & 0 & 0 & \omega_0 & 0 & \frac{1}{c_{se}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{1D} \\ \Delta i_D \\ \Delta V_{2D} \\ \Delta V_{4D} \\ \Delta i_{1Q} \\ \Delta i_Q \\ \Delta V_{2Q} \\ \Delta V_{4Q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-1}{cn} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{cn} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{2D} \\ i_{2Q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{-\omega_0 L_d''}{L_1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{\omega_0 L_d}{L_1} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{L_d}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{-L_d''}{L_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} \quad (27)$$

$$\text{Or, } \dot{X}_N = [A_N] X_N + [B_{N1}] u_{N1} + [B_{N2}] u_{N2} + [B_{N3}] u_{N3} \quad (28)$$

The output variables of the network model are the voltage and current at the generator terminal are the SVS bus voltage. The generator terminal voltage for the α -network is given by

$$V_{g\alpha} = R_a i_\alpha + L_d'' (i_\alpha + I_\alpha) \quad (29)$$

Similarly the equation can be written for the β network as

$$V_{g\beta} = R_a i_\beta + L_d'' (i_\beta + I_\beta) \quad (30)$$

Transforming the above equations to the synchronously rotating D-Q frame of reference

$$V_{gD} = R_a i_D + L_d'' (i_D + I_D) + \omega_0 L_d'' (i_D + I_D). \quad (31)$$

$$V_{gQ} = R_a i_Q + L_d'' (i_Q + I_Q) - \omega_0 L_d'' (i_D + I_D). \quad (32)$$

Linearizing equations (31) and (32) the network output equations can be obtained as:

$$\begin{aligned} \begin{bmatrix} \Delta \dot{V}_{gD} \\ \Delta \dot{V}_{gQ} \end{bmatrix} &= \begin{bmatrix} -(R_a - \frac{R_1 L_d''}{L_1}) & 0 & \frac{L_d''}{L_1} & -\frac{L_d''}{L_1} & 0 & 0 & \omega_0 c_n (R_a - \frac{R_1 L_d''}{L_1}) & 0 \\ 0 & 0 & -\omega_0 c_n (R_a - \frac{R_1 L_d''}{L_1}) & 0 & -(R_a - \frac{R_1 L_d''}{L_1}) & 0 & \frac{L_d''}{L_1} & -\frac{L_d''}{L_1} \end{bmatrix} (X_N) \\ &+ \begin{bmatrix} -(R_a - \frac{R_1 L_d''}{L_1}) & 0 \\ 0 & -(R_a - \frac{R_1 L_d''}{L_1}) \end{bmatrix} \begin{bmatrix} \Delta i_{2D} \\ \Delta i_{2Q} \end{bmatrix} \\ &+ \begin{bmatrix} 0 & \omega_0 L_d'' (1 - \frac{L_d''}{L_1}) \\ -\omega_0 L_d'' (1 - \frac{L_d''}{L_1}) & 0 \end{bmatrix} \begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} \end{aligned}$$

$$\text{Or, } Y_{N1} = [C_{N1}] X_N + [D_{N1}] U_{N1} + [D_{N2}] U_{N2} + [D_{N3}] U_{N3} \quad (33)$$

The output equation in terms of generator terminal currents will be as follows: -

$$\begin{bmatrix} \Delta I_D \\ \Delta I_Q \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} X_N$$

$$\text{Or, } Y_{N2} = [C_{N2}] X_N \quad (34)$$

The output equation in terms of the SVS voltage is given by

$$\begin{bmatrix} \Delta \dot{V}_{2D} \\ \Delta \dot{V}_{2Q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} [X_N]$$

$$\text{Or, } Y_{N3} = [C_{N3}] X_N \quad (35)$$

Where

$$Y_{N1} = [\Delta V_{gd} \ \Delta V_{gq}]^t, \ Y_{N2} = [\Delta I_D \ \Delta I_Q]^t, \ Y_{N3} = [\Delta V_{2D} \ \Delta V_{2Q}]^t$$

$$X_N = [\Delta I_D \ \Delta I_Q \ \Delta V_{2D} \ \Delta V_{2Q} \ \Delta I_g \ \Delta V_{2Q} \ \Delta V_{4Q}]^t$$

$$Y_{N1} = [\Delta I_{2D} \ \Delta I_{2Q}]^t, \ U_{N2} = [\Delta I_D \ \Delta I_a]^t, \ U_{N3} = [\Delta I_D \ \Delta I_Q]^t$$

3. 6. Satic VAR system Model

The linearized model of SVS control system is considered. The voltage regulator is assumed to be proportional plus integral (PI) controller. The salient feature of this representation is the modelling of TCR transients. The delay associated with the SVS controller and measurement unit are also incorporated.

The α , β axis currents entering TCR from network are expressed as:

$$L_s \frac{di_{3\alpha}}{dt} + R_s i_{3\alpha} = V_{3\alpha} \quad (36)$$

$$L_s \frac{di_{3\beta}}{dt} + R_s i_{3\beta} = V_{3\beta} \quad (37)$$

Where,

L_s and R_s , represents inductance and resistance of TCR respectively

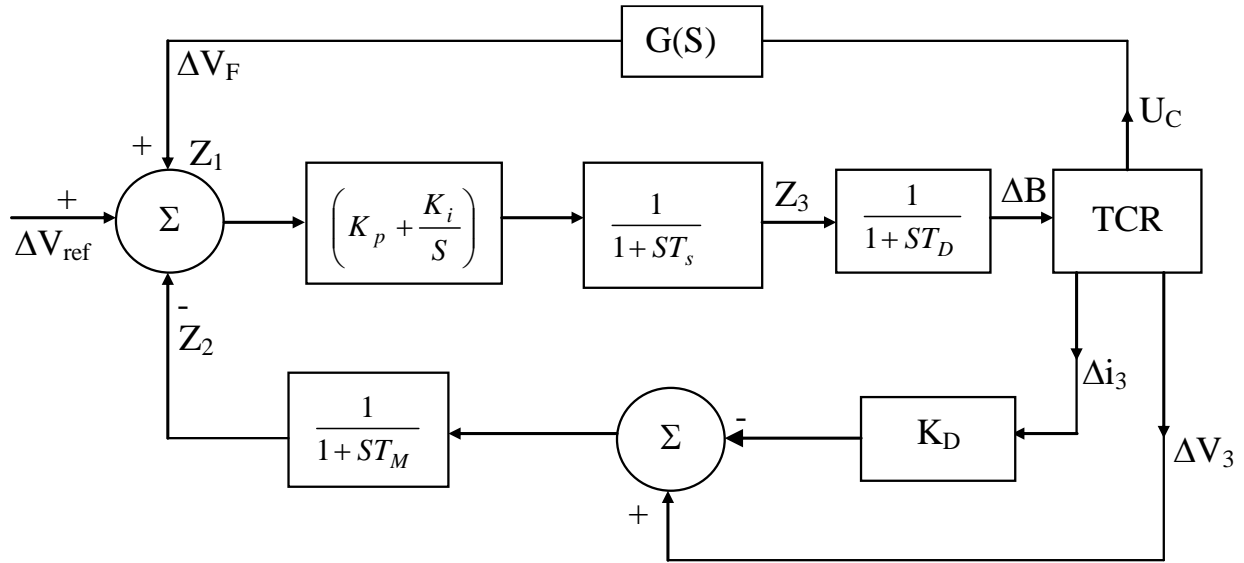


Fig3.6 SVS Control System

Transforming equation (36) and (37) to D-Q frame of references gives:

$$i_{3D} = \frac{1}{L_s} V_{3D} - \frac{R_s}{L_s} i_{3D} - \omega_0 i_{3Q} \quad (38)$$

$$i_{3Q} = \frac{1}{L_s} V_{3Q} - \frac{R_s}{L_s} i_{3Q} - \omega_0 i_{3D} \quad (39)$$

$$\begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \end{bmatrix} = \omega_0 \begin{bmatrix} -\frac{1}{Q} & -1 \\ 1 & -\frac{1}{Q} \end{bmatrix} \begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \end{bmatrix} + \omega_0 B_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta V_{3D} \\ \Delta V_{3Q} \end{bmatrix} + \omega_0 \begin{bmatrix} \Delta V_{3D0} \\ \Delta V_{3Q0} \end{bmatrix} \Delta B \quad (40)$$

Where, Q = Quality Factor = $\frac{\omega_0 L_s}{R}$

And B = $\frac{1}{\omega_0 L_s}$

The state and output equation of the SVS model can be written as:

$$\begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \\ Z_1 \\ Z_2 \\ Z_3 \\ \Delta B \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & -\omega_0 & 0 & 0 & 0 & \omega_0 V_{3D0} \\ \omega_0 & -\frac{R_s}{L_s} & 0 & 0 & 0 & \omega_0 V_{3Q0} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -\frac{K_D K_{iD}}{T_M} & -\frac{K_D K_{iQ}}{T_M} & 0 & -\frac{1}{T_M} & 0 & 0 \\ 0 & 0 & -\frac{K_I}{T_s} & \frac{K_P}{T_s} & -\frac{1}{T_s} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_D} & -\frac{1}{T_D} \end{bmatrix} \begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \\ Z_1 \\ Z_2 \\ Z_3 \\ \Delta B \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ \frac{K_{VD}}{T_M} & \frac{K_{VQ}}{T_M} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{3D} \\ \Delta V_{3Q} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -\frac{K_P}{T_s} \\ 0 \end{bmatrix} \begin{bmatrix} \Delta V_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -\frac{K_P}{T_s} \\ 0 \end{bmatrix} \begin{bmatrix} \Delta V_F \end{bmatrix}$$

or, $\dot{X}_S = [A_S]X_S + [B_{S1}]U_{S1} + [B_{S2}]U_{S2} + [B_{S3}]U_{S3}$ (41)

The output equation of the SVS is constituted by TCR current and is given by:

$$\begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{3D} \\ \Delta i_{3Q} \\ Z_1 \\ Z_2 \\ Z_3 \\ \Delta B \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{3D} \\ \Delta V_{3Q} \end{bmatrix}$$

Or,

$$Y_S = [C_S] X_S + [D_S] U_{S1} \quad (42)$$

3.7. Development of System Model

The state and output equations of the different constituent subsystem will be combined resulting in the state equation of the overall system. The various subsystem are so derived that inputs to any subsystem are directly obtained as output of the other subsystem. Fig 3.7 shows the interconnections of different subsystem.

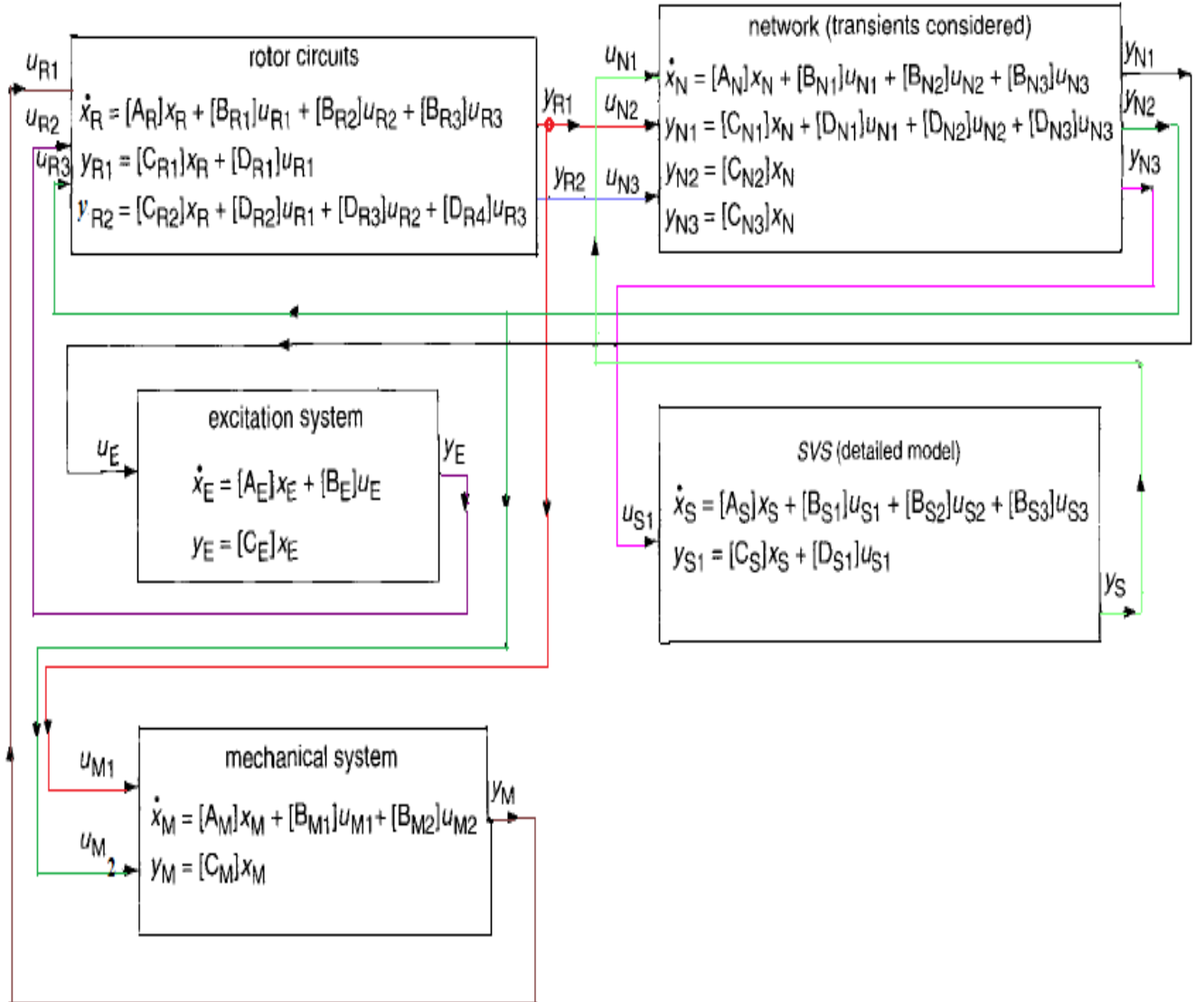


Fig 3.7 Interconnections of various subsystem in overall system Model

The overall system can be developed as follows

$$\dot{X} = [A] X + [B] \Delta V_{ref}$$

$$X = [X_R \ X_M \ X_E \ X_N \ X_S]^t, B = [0 \ 0 \ 0 \ 0 \ B_{S2}]^t$$

The system Matrix is given by:

| | | | | |
|---|---|------------------------------------|--|-------------|
| A_R | $B_{R1}C_M$ | $B_{R2}C_E$ | $B_{R3}C_{N2}$ | 0 |
| $B_{M1}C_{R1}$ | $A_M+B_{M1}D_{R1}C_M$ | 0 | $B_{M2}C_{N2}$ | 0 |
| $B_ED_{N2}C_{R1}$ + $B_ED_{N3}C_{R2}$ | $B_ED_{N2}D_{R1}C_M$ + $B_ED_{N3}D_{R2}C_M$ | A_E + $B_ED_{N3}D_{R3}C_E$ | B_EC_{N1} + $B_ED_{N1}D_S C_{N3}$ + $B_ED_{N3}D_{R4}C_{N2}$ | 0 |
| $B_{N2}C_{R1}$ + $B_{N3}C_{R2}$ | $B_{N2}D_{R1}C_M$ + $B_{N3}D_{R2}C_M$ | $B_{N3}D_{R3}$ | $A_N+B_{N1}D_S C_{N3}$ + $B_{N3}D_{R4}C_{N2}$ | $B_{N1}C_S$ |
| 0 | 0 | 0 | $B C_{N3}$ | A_S |

The overall dimension of the system matrix is 21 with network having lumped T-circuit representation.

3.8 AUXILIARY CONTROLLER DESIGN

It is assumed that the auxiliary controller is a first order transfer function $G(S)$ as shown in figure 3.8. The effect of any general first order auxiliary controller can be implemented through $G(S)$.

$$G(S) = \frac{\Delta V_F}{\Delta f_s} = K_B \left[\frac{1 + ST_1}{1 + ST_2} \right]$$

The controller can be made equal to

$$G(S) = K_B \frac{T_1}{T_2} + K_B \left[\frac{1 - T_1/T_2}{1 + ST_2} \right]$$

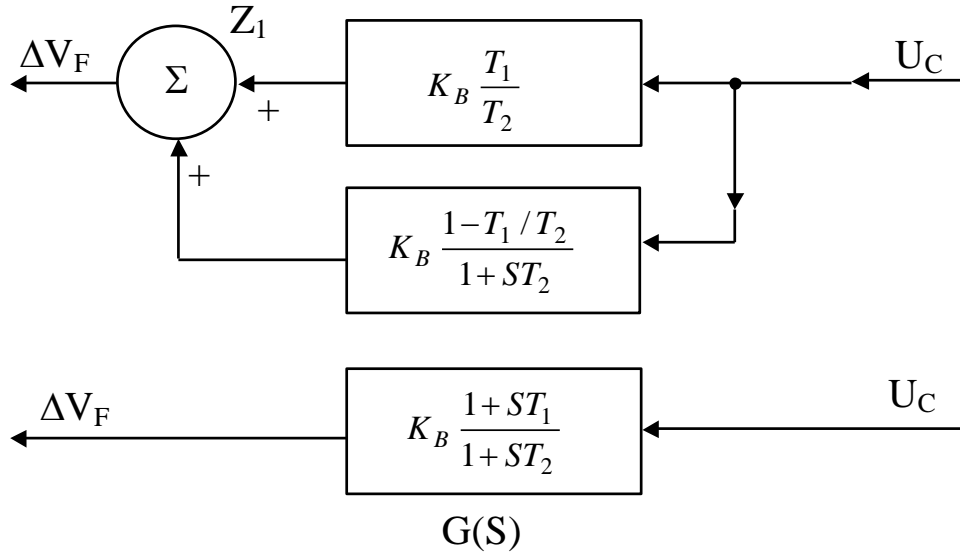


Fig. 3.8 General First Order Auxiliary Controller

$$\frac{Z_C}{U_C} = K_B \left[\frac{1 - T_1/T_2}{1 + ST_2} \right]$$

$$\text{Or } \dot{Z}_C = -\frac{1}{T_2} Z_C + \left[\frac{K_B}{T_2} \left(1 - \frac{T_1}{T_2} \right) \right] U_C$$

$$\text{Or } \dot{\mathbf{X}}_C = [\mathbf{A}]\mathbf{X}_C + [\mathbf{B}]U_C$$

The output equation of an auxiliary controller is given by

$$\Delta V_f = K_B \left[\frac{T_1}{T_2} \right] U_C + Z_C$$

Where

$$X_c = Z_c, A_c = -\frac{1}{T_2}, B_c = \frac{K_B}{2} \left[1 - \frac{T_1}{T_2} \right]$$

$$Y_c = \Delta V_f, C_c = 1, D_c = K_B \frac{T_1}{T_2}$$

The eigenvalues analysis carried out for the system model developed without inclusion of any auxiliary control signal are listed in table 3.1

Table 3.1 Eigenvalues of the study system without auxiliary controller

| |
|--------------------------|
| $P_g = 800\text{MW}$ |
| $.1491 \pm j4.5325$ |
| $-.8944 \pm j0.9597$ |
| -2.8996 |
| $6.3367 \pm j312.3499$ |
| $12.6525 \pm j1247.5740$ |
| $-16.3533 \pm j620.1469$ |
| $-23.3340 \pm j313.7184$ |
| $-25.9166 \pm j24.1572$ |
| $-32.6218 \pm j0.7570$ |
| $-53.555 \pm j89.0395$ |
| $-547.3129 \pm j82.738$ |

3.9 CONCLUSION

From the analysis it is evident that with the addition of SVC without auxiliary controller does not provide damping characteristics. Mechanical modes are not stable until the inclusion of auxiliary control signal. So, application of this concept to the IEEE first benchmark model proved that SVC without auxiliary controller provided at the mid-point of the transmission line cannot damp the torsional oscillations effectively. But, by adding an auxiliary controller to it we can damp the subsynchronous resonance (SSR) upto some extent.

CHAPTER-4

DEVELOPMENT OF THE COMBINED REACTIVE POWER AND FREQUENCY (CRPF) SVS AUXILIARY CONTROLLER

4.1 Introduction

Due to the capability of SVS to work as VAR generation and absorption system it is employed in an increasing extent in today's power system. On a simple one-machine power system it is found that an SVS can dynamically alter the system transfer characteristics by changing its reactive output. SVS in co-ordination with auxiliary controlled signal [1, 2, 35] not only improves voltage profile and transmission capability but also can be used for damping power system oscillations.

In the present chapter, effectiveness of the SVS auxiliary controller has been studied. The method of eigenvalue analysis has been employed to assess the dynamic behavior of system and unstable modes are investigated. For enhancement of dynamic stability various auxiliary signals reactive power and bus frequency and their effective combination can be incorporated with SVC. Based upon the performance of these auxiliary signals the most effective controller can stabilize all the torsional modes at a given operating point. In this chapter, an auxiliary signal have been studied and incorporated with SVS control scheme.

4.2 SVS AUXILIARY CONTROLLER MODEL

To evaluate the comparative effectiveness of SVS auxiliary controllers and to enhance the dynamic performance of a long distance series compensated transmission line, the following auxiliary controllers are considered.

- Line reactive power auxiliary controller.
- Bus frequency auxiliary controller.
- Voltage angle auxiliary controller.
- Combined Reactive power and frequency (CRPF) auxiliary controller.
- Combined voltage angle and reactive power (CVARP) auxiliary controller.

Reactive power signal [32]

The control scheme for single signal is depicted in Fig.4.1

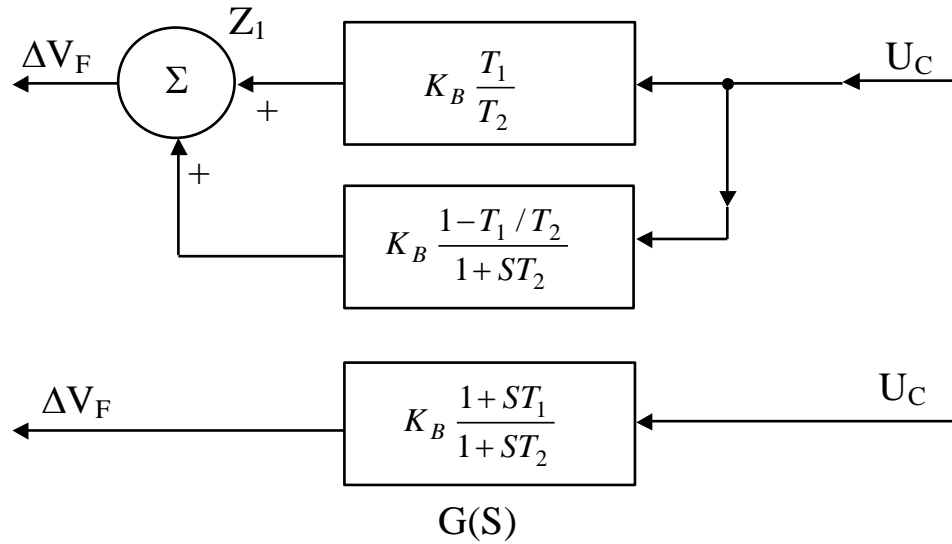


Fig.4.1 General First order Auxiliary Controller

The line reactive power existing SVS bus in given as

$$Q_2 = V_{2D}i_Q - V_{2Q}i_D \quad (4.1)$$

Where, V_{2D} and V_{2Q} are direct and quadrature axis SVS bus voltages & i_D and i_Q are direct and quadrature axis currents.

$$\Delta Q_2 = V_{2D0}\Delta i_Q + i_{Q0}\Delta V_{2D} - V_{2Q0}\Delta i_D - i_{D0}\Delta V_{2Q}$$

$$\Delta Q_2 = [F_5]X_T$$

Bus frequency Auxiliary Controller [35]

The angle of SVS bus in given by

$$Q_2 = \tan^{-1} \frac{V_{2Q}}{V_{2D}} \quad (4.2)$$

Where, V_{2D} and V_{2Q} are direct and quadrature axis SVS bus voltages as per Fig.3.5 so the bus frequency will be

$$f_{SVS} = f_{SVS} = \frac{1}{V_{20}^2} [V_{2D} \dot{V}_{2Q} - V_{2Q} \dot{V}_{2D}]$$

After linearizing equation, Δf_{SVS} is selected as auxiliary control signal

$$\Delta f_{SVS} = \frac{V_{2D0}}{V_{20}^2} \Delta \dot{V}_{2Q} - \frac{V_{2Q0}}{V_{20}^2} \Delta \dot{V}_{2D}$$

Putting the values of ΔV_{2D} and ΔV_{2Q}

$$\begin{aligned} \Delta f_{SVS} = \frac{K_3}{C_n} & [-\Delta I_Q - \Delta I_{2Q} - \Delta I_{1Q} + \omega_0 C_n \Delta V_{2D}] \\ & - \frac{K_4}{C_n} [-\Delta I_D - \Delta I_{2D} - \Delta I_{1D} - \omega_0 C_n \Delta V_{2Q}] \end{aligned} \quad (4.3)$$

The equation (4.8) can be written in matrix form as

$$U_c = [F_{CR}]X_R + [F_{CM}]X_M + [F_{CE}]X_E + [F_{CN}]X_N + [F_{CS}]X_S$$

Where $U_c = \Delta f_{SVS}$, $F_{CR} = F_{CM} = F_{CE} = 0$

Or

$$U_c = [F_{CN}]X_M + (F_{CS})X_S \quad (4.4)$$

Where,

$$F_{CN} = \left[\frac{K_4}{C_n} \quad \frac{-k_4}{C_n} k_3 \omega_0 \quad \frac{-k_3}{C_n} \quad \frac{-k_2}{C_n} k_4 \omega_0 \right]$$

$$F_{CS} = \left[\frac{K_4}{C_n} \quad \frac{-K_3}{C_n} \quad 0 \quad 0 \quad 0 \quad 0 \right]$$

Where

$$K_3 = \frac{V_{2D0}}{V_{20}^2}$$

$$K_4 = \frac{V_{2Q0}}{V_{20}^2}$$

4.3 Combined Reactive Power and Frequency (CRPF) Auxiliary Controller [35]

The auxiliary controller in the case comprises of a combination of bus frequency and line reactive power signals the control scheme is depicted in Fig. 4.1. The auxiliary control signals U_{C1} and U_{C2} corresponds, respectively, to the line reactive power and the bus frequency deviations which are derived at the SVS bus.

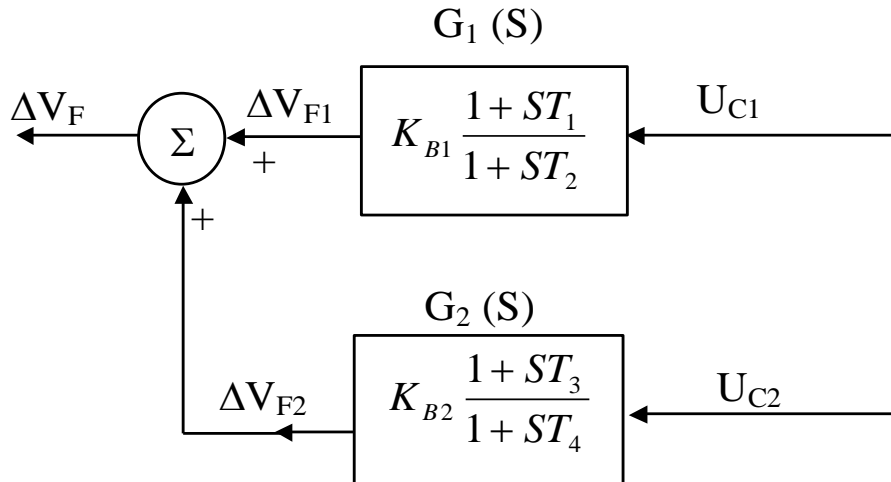


Figure 4.2 Control Scheme for Combined Auxiliary Controller

4.4 DEVELOPMENT OF SYSTEM MODEL

The state and output equations of the constituent subsystems are combined to get the overall system model as described in chapter 3. The state equations of overall system in given by

$$\dot{X}_T = [A] X_T$$

Where $[A]$ is given as

| | | | | | |
|--|--|---------------------------------|--|------------------------------|-------------|
| A_R | $B_{R1}C_M$ | $B_{R2}C_F$ | $B_{R3}C_{N2}$ | 0 | 0 |
| $B_{M1}C_{R1}$ | $A_M+B_{M1}D_{R1}C_M$ | 0 | $B_{M2}C_{N2}$ | 0 | 0 |
| $B_ED_{N2}C_{R1} +$ $B_ED_{N3}C_{R2}$ | $B_ED_{N2}D_{R1}C_M +$ $B_ED_{N3}D_{R2}C_M$ | $A_E +$ $B_ED_{N3}D_{R3}C_E$ | $B_EC_{N1}+$ $B_ED_{N1}D_SC_{N3}+$ $B_ED_{N3}D_{R4}C_{N2}$ | 0 | 0 |
| $B_{N2}C_{R1} +$ $B_{N3}C_{R2}$ | $B_{N2}D_{R1}C_M +$ $B_{N3}D_{R2}C_M$ | $B_{N3}D_{R3}$ | $A_N+B_{N1}D_SC_{N3}$ $+B_{N3}D_{R4}C_{N2}$ | $B_{N1}C_S$ | 0 |
| $B_{S3}D_C F_{CR}$ | $B_{S3}D_C F_{CM}$ | 0 | $B_{S1}C_{N3}+$ $B_{S3}D_C F_{CN}$ | A_S+ $B_{S3}D_C F_{CS}$ | $B_{S3}C_C$ |
| $B_C F_{CR}$ | $B_C F_{CM}$ | 0 | $B_C F_{CN}$ | $B_C F_{CS}$ | A_C |

And, $X_T = [X_R, X_M, X_E, X_M, X_S, X_C]^t$

The dimension of system matrix after incorporating the SVS auxiliary controller becomes 25 for T circuit.

4.5 CASE STUDY

The study system consists of two 555 MVA synchronous generators represented by an equivalent 1110 MVA machine, supplying power to an infinite bus over a long transmission line. The midpoint connected SVS is rated at 300 MVA leading to 200 MVA lagging.

The synchronous machine as described in chapter 3. The data corresponding to generator AVR and SVS including all control system delays is given in Appendix.

The objective of this study is to evaluate the comparative effectiveness of various auxiliary signals of SVS for enhancing the dynamic performance of series compensated transmission line in order to determine the most effective auxiliary control signal.

4.6 DYNAMIC PERFORMANCE

The eigenvalues have been computed for the system with and without auxiliary controller incorporated in the SVS control system using T circuit model of AC network.

The following SVS auxiliary controllers have been used in respect of their ability to stabilize the unstable system modes.

- Line reactive power auxiliary controller
- Bus frequency auxiliary controller
- Voltage angle auxiliary controller
- Combined Reactive power and bus frequency (CRPF) auxiliary controller.

Table 4.1 Eigenvalues of the study system without auxiliary controller

| |
|--------------------------|
| $P_g = 800\text{MW}$ |
| $.1491 \pm j4.5325$ |
| $-.8944 \pm j0.9597$ |
| -2.8996 |
| $6.3367 \pm j312.3499$ |
| $12.6525 \pm j1247.5740$ |
| $-16.3533 \pm j620.1469$ |
| $-23.3340 \pm j313.7184$ |
| $-25.9166 \pm j24.1572$ |
| $-32.6218 \pm j0.7570$ |
| $-53.555 \pm j89.0395$ |
| $-547.3129 \pm j82.738$ |

Table 4.2 Eigenvalues of the study system with auxiliary controller CRPF

| |
|--------------------------|
| $-.4985 \pm j634.2980$ |
| $-.5984 \pm j0.8587$ |
| $-1.4732 \pm j325.45941$ |
| $-2.1684 \pm j7.6865$ |
| -2.8231 |
| -3.9739 |
| -8.6178 |
| $-16.9648 \pm j1270.680$ |
| $-23.9654 \pm j40.8559$ |
| $-25.0910 \pm j312.2842$ |
| $-25.4680 \pm j22.5597$ |
| -31.9726 |
| -118.6970 |
| $-548.5105 \pm j89.9851$ |

4.7 CONCLUSION

From the analysis it is evident that with the addition of SVC with auxiliary controller provides superior performance characteristics. All mechanical modes are stable with the inclusion of auxiliary control signal CRPF. So, application of this concept to the IEEE first benchmark model proved that SVC with auxiliary controller provided at the mid-point of the transmission line can damp the torsional oscillations effectively.

CHAPTER-5

DEVELOPMENT OF THE VOLTAGE ANGLE AND REACTIVE POWER (VARP) SVS AUXILIARY CONTROLLER

5.1 Introduction

Dynamic voltage support and reactive power compensation have been long recognized as a very significant measure to improve the performance of electric power systems. The primary purpose of SVS application is to maintain bus voltage at or near a constant level. In addition, SVS is known to extend the stability limit and improve system damping when connected at the midpoint of a long transmission line.

The application of auxiliary controlled static VAR systems is the novel concept for the dynamic performance enhancement of the long distance AC transmission systems. In the present chapter, a SVS auxiliary controller for improving dynamic performance of the long distance transmission lines has been presented. The following studies have been performed for a 400 kV, 600 km single circuit line.

The method of Eigen-value analysis has been employed to assess the dynamic behaviour of system and the unstable system modes are investigated a given operating point and the combination of line reactive power and voltage angle auxiliary controller is used. As a result, the unstable system modes are stabilized at a given operating point.

5.2 System Model

5.2.1 System Description

The study system consists of a steam turbine driven synchronous generator supplying power to an infinite bus over a long transmission line as shown in Fig. 3.1. An SVS is connected at the midpoint of the transmission line, which provides continuously controllable reactive

power at the terminals in response to bus voltage and other auxiliary control signals. The SVS is assumed to be of SC-TCR configuration.

The system model is derived from the models of constituent subsystems:

- (i) Synchronous generator
 - (a) Stator Circuit
 - (b) Rotor Circuit
 - (c) Mechanical system
 - (d) Excitation System
- (ii) Network
- (iii) SVS
 - (a) Voltage controller
 - (b) Auxiliary feedback controller

The component models of the stator and rotor circuit of a sixth order representation of synchronous generator along with IEEE type-1 excitation system have already been derived in chapter 3. The model of SVS auxiliary controller utilizing various locally available signals has been studied in chapter 3. An auxiliary controller namely voltage angle and reactive power (VARP) auxiliary controller has been developed in the present chapter.

5.2.2 Network Model

The AC transmission network in this chapter is represented by lumped parameter T-circuit. The network has been represented by its α -axis equivalent circuits, which are identical with the positive sequence networks. respectively. The differential equations, state, and output equations for the network have been derived in chapter 3.

5.2.3 SVS Auxiliary Controller Model

A auxiliary control signal designated as voltage angle and reactive power (VARP) has been developed in this chapter. It is emphasized that all auxiliary control signals considered here are available, locally, at the SVS bus terminal.

5.2.3.1 Voltage Angle and Reactive Power (VARP) Auxiliary Controller [32]

The auxiliary controller signal in this case is the combination of line reactive power and the voltage angle signals with the objective of utilizing the beneficial contribution of both signals towards improving the dynamic performance of system. The control scheme for the composite controller is illustrated in Fig 5.1. The control signals UC1 and UC2 correspond, respectively, to the line reactive power and the voltage angle deviation, which are derived at the SVS bus. The state and output equations for the VARP auxiliary controller are obtained as follows:

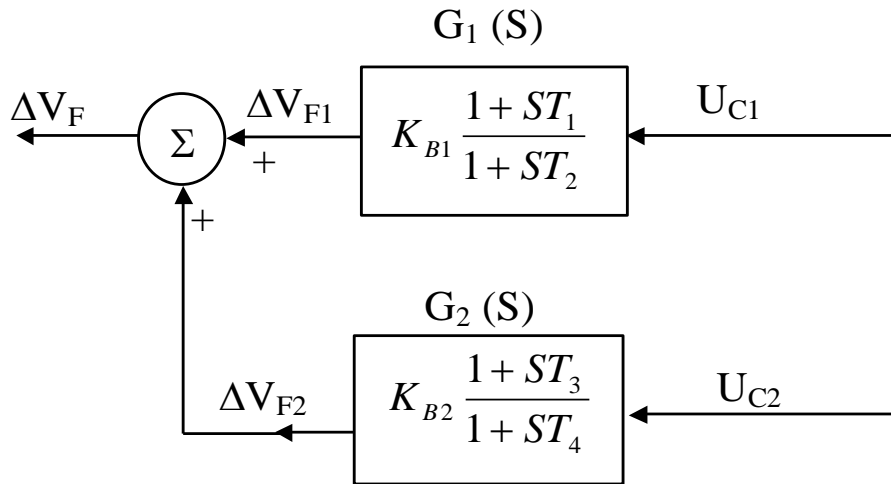


Figure 5.1 Control Scheme for Combined Auxiliary Controller[35]

Line reactive power auxiliary signal

$$\begin{aligned} \dot{X}_{C1} &= [A_{C1}] X_{C1} + [B_{C1}] U_{C1} \\ Y_{C1} &= [C_{C1}] X_{C1} + [D_{C1}] U_{C1} \end{aligned} \quad (5.1)$$

Where,

$$X_{c1} = [Z_{c1}], Y_{c1} = \Delta V_{F1} \quad (5.2)$$

$$U_{c1} = \Delta Q_3 \quad (5.3)$$

Voltage angle auxiliary signal

$$\begin{aligned} \dot{X}_{c2} &= [A_{c2}] X_{c2} + [B_{c2}] U_{c2} \\ Y_{c2} &= [C_{c2}] X_{c2} + [D_{c2}] U_{c2} \end{aligned} \quad (5.4)$$

$$X_{c2} = [Z_{c2}], Y_{c2} = \Delta V_{F2} \quad (5.5)$$

$$U_{c2} = \Delta \theta_3 \quad (5.6)$$

The state and output equation for the VARP auxiliary controller can be obtained by combining eqn. (5.1) to (5.6)

$$\begin{bmatrix} \dot{X}_{c1} \\ \dot{X}_{c2} \end{bmatrix} = \begin{bmatrix} A_{c1} & 0 \\ 0 & A_{c2} \end{bmatrix} \begin{bmatrix} X_{c1} \\ X_{c2} \end{bmatrix} + \begin{bmatrix} B_{c1} & 0 \\ 0 & B_{c2} \end{bmatrix} \begin{bmatrix} U_{c1} \\ U_{c2} \end{bmatrix}$$

$$[Y_c] = [C_{c1} \ C_{c2}] \begin{bmatrix} X_{c1} \\ X_{c2} \end{bmatrix} + [D_{c1} \ D_{c2}] \begin{bmatrix} U_{c1} \\ U_{c2} \end{bmatrix}$$

Where AC1, BC1, CC1 and DC1 are the matrices of the reactive power auxiliary controller and AC2, BC2, CC2 and DC2 are the matrices of the voltage angle auxiliary controller.

5.3 Development of the System Model

The state and output equations of the different constituent subsystems along with the auxiliary controller state and output equations, are combined to result in the linearized state equation of overall system as

$$\dot{X}_T = [A] X_T$$

Where,

$$X_T = [X_R \ X_M \ X_E \ X_N \ X_S \ X_C]^t$$

and $[A]$ is given in chapter 4.

The dimension of system matrix after incorporating the SVS auxiliary controller becomes 25 for T circuit and 21 without the damping scheme for the network.

5.4 Case Study

The study system consists of two synchronous generators of 555 MVA, which are represented by a single equivalent unit of 1110 MVA, at 22 kV. The electrical power is supplied to an infinite bus over 400 kV, 600 km long transmission line. The SVS rating for the line has been chosen to be 200 MVAR inductive to 350 MVAR capacitive. About 40% series compensation is used at the sending end of the transmission line.

Its detailed sixth order model represents the synchronous machine with automatic voltage regulator as described in chapter 3. The data corresponding to generator, AVR and SVS voltage controller including all control system delays is given in Appendix. The objective of this study is to evaluate the effectiveness of the auxiliary controlled SVS for enhancing the dynamic performance of long distance AC transmission lines.

The eigenvalues have been computed for the system without and with VARP SVS damping scheme for a operating point of power transfer. Table 5.1 presents the eigenvalues for the system at generator power $P_G = 800$ MW without any auxiliary controller and Table 5.2 presents the eigenvalues for the system at generator power $P_G = 800$ MW with the proposed scheme. When the damping scheme is not applied, some mechanical modes are found to be unstable. When the VARP SVS damping scheme is applied, the unstable mechanical modes are effectively stabilized.

Table 5.1 Eigenvalues of the study system without auxiliary controller

| $P_g = 800\text{MW}$ |
|----------------------|
| .1491±j4.5325 |
| -.8944±j0.9597 |
| -2.8996 |
| 6.3367±j312.3499 |
| 12.6525±j1247.5740 |
| -16.3533±j620.1469 |
| -23.3340±j313.7184 |
| -25.9166±j24.1572 |
| -32.6218±j0.7570 |
| -53.555±j89.0395 |
| -547.3129±j82.738 |

Table 5.2 Eigenvalues of the study system with VARP auxiliary controller

| $P_g = 800\text{MW}$ |
|----------------------|
| -2.355±j5.402 |
| -0.5154±j0.8327 |
| -26.588 |
| -7.2366 |
| -2.9699 |
| -25.4511±j24.2633 |
| -0.5154-j.8327 |
| -3.2739±j3499.111 |
| -3.2750±j2871.112 |
| -13.2268±j2495.518 |
| -14.9156±j1867.49 |
| -12.0135±j1147.234 |
| -8.52275±j516.2119 |
| -11.9609±j448.205 |
| -6.8055±j309.8051 |
| -7.2912±j200.1505 |
| -539.9058±j106.081 |
| -7.9522±j21.6182 |
| -182.1672 |
| -30.5191 |

Table 5.3 Auxiliary controller parameters

| SVS Auxiliary Signal | K_B | $T_1(\text{sec})$ | $T_2(\text{sec})$ |
|----------------------|-------|-------------------|-------------------|
| Voltage Angle | -.01 | .046 | .39 |
| Reactive Power | -.54 | .05 | .2 |

5.5 CONCLUSION

In this chapter, a SVS control strategy for damping torsional oscillations due to subsynchronous resonance (SSR) in a series compensated power system has been developed and demonstrated using detailed system model developed by lumped T-circuit. Auxiliary controller namely, VARP is investigated and employed in SVS control system in order to stabilize the unstable torsional modes over a operating point of generated power. The proposed SVS control strategy utilizes the effectiveness of voltage angle and reactive power (VARP) SVS auxiliary control signals. The SVS controller can be easily implemented as it utilizes the signals derivable from the SVS bus itself. The SVS is considered located at the middle of the transmission line.

CHAPTER-6

RESULTS AND CONCLUSIONS

6.1 General

Eigenvalue study conducted on IEEE first benchmark model shows the damping benefits on SSR of SVC with auxiliary controller. This thesis presents an application of the SVC auxiliary controllers for enhancing the dynamic performance of the long transmission lines over a given operating point. The effect of various auxiliary signals incorporated in the SVC control system has been examined for enhancement of performance of a series compensated power system. The most effective SVS auxiliary controller is studied by evaluating the comparative performance of various SVS auxiliary controllers for damping torsional modes of a series compensated power system. A review of the studies carried out and major contributions made in the thesis are presented below.

In this, we have studied the detailed system model, which consists of the stator represented by dependent current source in parallel with the inductance. The rotor flux linkages are expressed in terms of currents that are defined with respect to machine reference frame. To have a common axis of representation with the network and SVS, these flux linkages are transformed to synchronously rotating D-Q frame of reference. The generator model includes the field winding and a damper winding along d-axis and two damper windings along q-axis. A lumped parameter T- circuit represents the network. The network transients have been modelled by differential equations. The series compensation is located at the sending end of transmission line and SVS is located at the middle of transmission line. An auxiliary control signal is also incorporated in the system model to study the damping effect of various auxiliary control signals in SVS control system.

Two SVS auxiliary controllers combined line reactive power and bus frequency (CRPF) and voltage angle and reactive power (VARP) controllers are incorporated in the SVC control system and they are studied. The effect of auxiliary controllers on the system dynamic performance has been examined over a given operating point. The application of SVS

auxiliary controllers significantly increases the system damping. Eigenvalue tables (in chapters 4 and 5) show that the application of SVS auxiliary controllers significantly increases the system damping. The voltage angle and reactive power SVS auxiliary controller achieves the highest damping for torsional modes. The unstable rotor mechanical modes become stable at a given operating point.

The application of this scheme, which utilizes the combined effect of voltage angle and reactive power SVS auxiliary controller for damping subsynchronous resonance has been demonstrated for a series compensated power system. The proposed scheme provides an effective damping to the low frequency as well as high frequency voltage and power oscillations. Thus increase of power transfer capability of transmission line using series compensation becomes feasible without the fear of SSR. This justifies their use in flexible AC transmission systems.

Figure 6.1 and 6.2 depict root loci of subsystem without and with voltage angle and reactive power (VARP) auxiliary controller for $P_g = 800$ MW. All the roots of the sub system lie on left half of the 'S' plane and it is evident that that all the electrical and electromechanical modes with voltage angle and reactive power (VARP) auxiliary controlled signals are found to be more stable. Damping of SSR is better with VARP auxiliary signal as evident from the tables of eigenvalues (in chapter 4 and 5).

6.2 Suggestions for Further Work

The following are some of the interesting problems which could be investigated further based upon the work reported in the present thesis:

1. The present thesis has examined the effect various auxiliary signals incorporated with SVS control system for dynamic performance enhancement of series compensated power system. Other auxiliary signals can be developed and incorporated in the control schemes of other FACTS devices i.e. UPFC, TCSC, SSSC etc. for enhancement of dynamic and transient performance of the power system.
2. For GUPFC, IPFC (FACTS Devices) Auxiliary signals can be developed and incorporated in the control schemes for enhancement of dynamic and transient performance of the power system.
3. The present thesis has examined the effect of SSR on IEEE-1 based study system, as it can be studied for IEEE-2 based study system also.
4. It can be carried out by applying the proposed scheme to the real power system for choice of most suitable parameter for study.
5. Damping of SSR using SVC can be done on SIMULINK also.
6. Effect of coordinated applications of many FACTS controllers can be investigated for dynamic transient and voltage stability enhancement and damping of power system oscillations of further work.
7. AI, ANN and fuzzy logic based adaptive controllers for SVS can be developed.
8. Economic comparative study can be made for effective FACTS controllers for enhancement of power transfer capability enhancement and damping of sub synchronous resonance and to find out the most economic FACT controller.

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APPENDIX

System Data

Data of Different System Components

System Base Quantities

Voltage = 400 KV

MVA = 100

Frequency = 50 Hz

Synchronous Generator Data (on its own base)

| | | | |
|-------|-------------|------------|-------------|
| S_n | = 1110 MVA | T'_{do} | = 6.66 sec |
| V_n | = 22 KV | T'_{qo} | = 0.44 sec |
| p_f | = 0.9 | T''_{do} | = 0.032 sec |
| f_n | = 50 Hz | T''_{qo} | = 0.057 sec |
| R_a | = 0.0036 pu | X_d | = 1.933 pu |
| X_l | = 0.21 pu | X_q | = 1.743 pu |
| R_0 | = 0 pu | X'_q | = 0.467 pu |
| X_0 | = 0.195 pu | X''_q | = 1.144 pu |
| H | = 3.22 sec | X''_d | = 0.312 pu |
| D | = 0 | X''_q | = 0.312 pu |

This corresponds to a single generator mass

Data for sending and Receiving End Transformers (on generator base)

R_T = 0.00 pu

X_T = 0.15 pu

IEEE Type I Excitation System Data

| | |
|---------------------|------------------|
| $V_g = 1$ pu | $T_R = 0$ sec |
| $V_f = 1$ pu | $K_A = 400$ pu |
| $V_{pss} = 0$ | $T_A = 0.02$ sec |
| $V_{rms} = 9.75$ pu | $K_E = 1.0$ pu |
| $V_{min} = -7$ pu | $T_E = 1.0$ sec |
| $V_{fmax} = 5$ pu | $K_f = 0.06$ pu |
| $V_{fmin} = -7$ pu | $T_F = 1.0$ sec |
| $S_{Emax} = 0.95$ | $S_{Emin} = 0$ |

Transmission Line Data

| | |
|-------------------------------|-----------------------------------|
| Voltage | 400 KW |
| No. of conductors | 2 |
| Area of conductors | 0.4 inch ² |
| Resistance (R) | 0.034 Ω per phase per Km. |
| Reactance (X_L) | 0.325 Ω per phase per Km. |
| Susceptance ($B_C = 1/X_C$) | 3.68×10^{-6} mho per Km. |
| Charging current | 0.845 A per phase per km. |
| Surge impedance | 296 Ω |
| Natural Load | 540 MW |
| X_L / R ratio | 9.5 |

Static VAR System (SVS) Data

Configuration: Switched Capacitor – Thyristor Controlled Reactor (SC-TCR) to be switched in step of 1.0 p.u.

SYNCHRONOUS MACHINE MODEL PARAMETERS

The constants $a_1 - a_8$ are described as

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \frac{-\omega_0}{X_f X_h - X_{fh}^2} \begin{bmatrix} R_f X_h & -R_f X_{fh} \\ -R_h X_{fh} & R_h X_f \end{bmatrix}$$

$$\begin{bmatrix} a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} = \frac{-\omega_0}{X_g X_k - X_{gk}^2} \begin{bmatrix} R_g X_k & -R_g X_{gk} \\ R_k X_{gk} & R_k X_g \end{bmatrix}$$

The constant $b_1 - b_6$ are given as

$$b_1 = \frac{\omega_0 R_f}{X_{df}}, \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \frac{-\omega_0}{X_f X_h - X_{fh}^2} \begin{bmatrix} R_f (X_{df} X_h - X_{dh} X_{fh}) \\ R_h (X_f X_{dh} - X_{fh} X_{df}) \end{bmatrix}$$

$$b_4 = \frac{\omega_0 R_g}{X_{qg}}, \begin{bmatrix} b_5 \\ b_6 \end{bmatrix} = \frac{\omega_0}{X_g X_k - X_{gk}^2} \begin{bmatrix} R_f (X_{qg} X_k - X_{qk} X_{gk}) \\ R_k (X_g X_{qk} - X_{gk} X_{qg}) \end{bmatrix}$$

The constant $c_1 - c_4$ are given by

$$C_1 = \frac{X_{df} X_h - X_{dh} X_{fh}}{X_d'' (X_f X_h' - X_{fh}^2)}, C_2 = \frac{X_{dh} X_f - X_{fh} X_{df}}{X_d'' (X_f X_h' - X_{fh}^2)}$$

$$C_3 = \frac{X_{qf} X_k - X_{qk} X_{gk}}{X_d'' (X_g X_k - X_{qk}^2)}, C_4 = \frac{X_{qk} X_g - X_{gk} X_{qg}}{X_d'' (X_g X_k - X_{qk}^2)}$$

Where, X_f, X_h, X_g, X_k are reactances of the rotor coils specified by the subscripts.

R_f, R_h, R_g, R_k are resistances of the rotor coils specified by the subscripts.

$x_{df}, x_{dh}, x_{fh}, x_{gk}, x_{qg}, x_{qk}$ are mutual reactances between rotor coils specified by the subscripts.

The resistances and reactances of various rotor coils are defined as follows:

$$x_{df} = x_{dh} = x_{fh} = x_d - X_l, x_{qg} = x_{qk} = x_{gk} = x_q - X_l$$

$$X_{hl} = \frac{(X_d'' - X_l)(X_d' - X_l)}{(X_d' - X_d'')}, X_{fl} = \frac{(X_d' - X_l)X_{df}}{(X_d - X_d')}$$

$$X_{gl} = \frac{(X_q'' - X_l)(X_q' - X_l)}{(X_q' - X_q'')}, X_{kl} = \frac{(X_q' - X_l)X_{qk}}{(X_q - X_q')}$$

$$X_f = X_{df} + X_{fl}, X_h = X_{df} + X_{hl}$$

$$X_g = X_{qk} + X_{gl}, X_k = X_{qk} + X_{kl}$$

$$R_f = -\frac{X_{df}^2}{\omega_0 T_{d0}''(X_d - X_d')}, R_h = -\frac{(X_d' - X_1)^2}{\omega_0 T_{d0}''(X_d - X_d'')}$$

$$R_g = -\frac{(X_q' - X_1)^2}{\omega_0 T_{d0}''(X_q - X_q')}, R_k = -\frac{X_{qk}^2}{\omega_0 T_{d0}''(X_q - X_q'')}$$

Where, x_1 is the stator leakage reactance.

x_d, x_d', x_d'' are the direct axis synchronous, transient and subtransient reactances, respectively.

x_q, x_q', x_q'' are the quadrature axis synchronous, transient and subtransient reactances, respectively.

T_{d0}', T_{d0}'' are the direct axis transient and subtransient open circuit time constants, respectively.

T_{q0}', T_{q0}'' are the quadrature axis transient and subtransient open circuit time constants, respectively.

MATRIX ELEMENTS OF MECHANICAL SYSTEM MODEL

The matrices, A_M , B_{M1} and B_{M2} used in Chapter 3 are given as below :

The matrix A_M (12 x 12) can be written as :

$$A_M = \begin{bmatrix} 0 & I \\ A_{M1} & A_{M2} \end{bmatrix}$$

Where I is an identity matrix of dimension (6 x 6)

The nonzero elements of A_{M1} (6 x 6) are defined by:

$$\begin{aligned} A_{M1}(1,1) &= -\frac{K_{12}}{M_1}, A_{M1}(1,2) = -\frac{K_{12}}{M_1} \\ A_{M1}(2,1) &= -\frac{K_{12}}{M_2}, A_{M1}(2,2) = -\frac{(K_{12} + K_{23})}{M_2}, A_{M1}(2,3) = -\frac{K_{23}}{M_2} \\ A_{M1}(3,2) &= -\frac{K_{23}}{M_3}, A_{M1}(3,3) = -\frac{(K_{23} + K_{34})}{M_3}, A_{M1}(3,4) = -\frac{K_{34}}{M_3} \\ A_{M1}(4,3) &= -\frac{K_{34}}{M_4}, A_{M1}(4,4) = -\frac{(K_{34} + K_{45})}{M_4}, A_{M1}(4,5) = -\frac{K_{45}}{M_4} \\ A_{M1}(5,4) &= \frac{K_{45}}{M_5}, A_{M1}(5,5) = -\frac{(K_{45} + K_{56})}{M_5}, A_{M1}(5,6) = \frac{K_{56}}{M_5} \\ A_{M1}(6,5) &= \frac{K_{56}}{M_6}, A_{M1}(6,6) = -\frac{K_{56}}{M_6} \end{aligned}$$

The nonzero elements of A_{M2} (6 x 6) are defined by

$$\begin{aligned} A_{M2}(1,1) &= -\frac{(D_{11} + D_{12})}{M_1}, A_{M2}(1,2) = \frac{D_{12}}{M_1} \\ A_{M2}(2,1) &= \frac{D_{12}}{M_2}, A_{M2}(2,2) = -\frac{(D_{12} + D_{22} + D_{23})}{M_2}, A_{M2}(2,3) = \frac{D_{23}}{M_2} \\ A_{M2}(3,1) &= \frac{D_{12}}{M_2}, A_{M2}(3,2) = -\frac{(D_{12} + D_{22} + D_{23})}{M_2}, A_{M2}(3,3) = \frac{D_{23}}{M_2} \\ A_{M2}(4,1) &= \frac{D_{12}}{M_2}, A_{M2}(4,2) = -\frac{(D_{12} + D_{22} + D_{23})}{M_2}, A_{M2}(4,3) = \frac{D_{23}}{M_2} \\ A_{M2}(5,4) &= \frac{D_{45}}{M_5}, A_{M2}(5,5) = -\frac{(D_{45} + D_{55} + D_{56})}{M_5}, A_{M2}(5,6) = \frac{D_{56}}{M_5} \end{aligned}$$

$$A_{M2} (2,1) = \frac{D_{12}}{M_2}, \quad A_{M2} (2,2) = -\frac{(D_{12} + D_{22} + D_{23})}{M_2}, \quad A_{M2} (2,3) = \frac{D_{23}}{M_2}$$

The nonzero elements of B_{M1} (12 x 2) and B_{M2} (12 x 2) are given as

$$B_{M1} (11,1) = -\frac{X_d'' i_{Q0}}{M_5}, \quad B_{M1} (11,2) = \frac{X_d'' i_{D0}}{M_5}$$

$$B_{M2} (11,1) = \frac{X_d'' I_{Q0}}{M_5}, \quad B_{M1} (11,2) = -\frac{X_d'' I_{D0}}{M_5}$$

The nonzero elements of matrix C_M (2 x 12) are given by $C_M (1,5) = 1.0$, $C_M (2,11) = 1.0$