A Major Project Report On

COMPARATIVE ANALYSIS OF SLOPE STABILITY PROBLEM USING

ARTIFICIAL INTELLIGENCE TECHNIQUE

Submitted in Partial Fulfilment for the Award of the Degree of

MASTER OF TECHNOLOGY

IN

CIVIL ENGINEERING

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GEOTECHNICAL ENGINEERING

 $\mathbf{B}\mathbf{y}$

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July 2014



DELHI TECHNOLOGICAL UNIVERSITY

CERTIFICATE

This is to certify that the project report entitled "Comparative Analysis of Slope Stability Problem using Artificial Intelligence Technique" is a bonafide record of work carried out by Jayanti Prabha Bharti (2K12/GTE/09) under my guidance and supervision, during the session 2012-14 in partial fulfillment of the requirement for the degree of Master of Technology (Geotechnical Engineering) from Delhi Technological University, Delhi.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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DECLARATION

I, hereby declare that the work embodied in the dissertation entitled "Comparative Analysis of Slope Stability Problem using Artificial Intelligence Technique" in partial fulfilment for the award of degree of MASTER of TECHNOLOGY in "GEOTECHNICAL ENGINEERING", is an original piece of work carried out by me under the supervision of Dr. K.C Tiwari, Department of Civil Engineering, Delhi Technological University. The matter of this work either full or in part have not been submitted to any other institution or University for the award of any other Diploma or Degree or any other purpose what so ever.

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Last but not least, I specially thank all people who are active in this field. Reference material (pictures, tables and forms) from various national and international reports and journals are included as per requirement and all these are quoted under the reference section at the last of this report.

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ABSTRACT

The main purpose of this research work is to explore the use of artificial intelligence techniques to solve the problem of slope stability and to carry out a comparative analysis of the results. This project work includes the brief study of all the slope stability analysis methods such as limit equilibrium methods and numerical analysis methods. Thereafter, use of artificial intelligence techniques to solve the problem of slope stability is proposed to be explored followed by a comparative analysis of the results. It is proposed to explore two AI techniques namely genetic algorithm and artificial neural networks. However, only one of them, whichever turns out to be more suitable will be considered for implementation. All the implementations have been done on MATLAB software which stands for matrix laboratory.

In my research work, I have solved two problems of slope stability both by conventional methods and by genetic algorithm optimization tool. In first problem, i have made selection by random search using tournament selection taking two variables in optimization tool. In 2nd problem, i have made the selection stochastic uniform that is proportional selection by taking 2 variables and 3 variables. Also in 2nd problem, I have observed the variation of results on taking 2 variables and 3 variables. Thereafter, I have compared the results obtained by conventional methods and artificial intelligence technique and finally made conclusion that artificial intelligence method is very powerful tool for solving slope stability method as it is very fast and quick method and solves the problem in 3 variables very easily.

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CHAPTER 1

INTRODUCTION

1.1 OBJECTIVES

The main objectives of this thesis are

- a) Review the literature related to slope stability analysis methods
- b) Learn artificial intelligence techniques
- c) Solve the problem of slope stability by artificial intelligence technique.
- d) Compare the results obtained by conventional methods and artificial techniques

1.2 OUTLINE OF THESIS

- a) 1st chapter includes the Introduction part of thesis. In this part, objectives and terms of slope stability such as slope, slope stability, type of slopes, type of failures, reasons of failures has been introduced.
- b) 2nd chapter includes the review of literature. In this part, slope failure definitions and all the methods of slope stability analysis is deeply studied.
- c) 3rd chapter includes the brief introduction and terms used in artificial intelligence technique that is genetic algorithm.
- d) 4th chapter includes all the numerical investigations and calculations.In this chapter, two problems have been solved by both genetic algorithm optimization tool and conventional methods.
- e) 5th chapter includes the analysis of results. In this part, result is analysed by obtaining variation in results.
- f) 6th chapter includes the final conclusion and future scope. In this part, final conclusion and future scopehas been made.

1.3 SLOPE DEFINITION

Slope is defined as supported or unsupported soil mass of inclined surface. Some slopes are found on above the ground level and some slopes are found below the ground level. In another words, we can say that slopes are nothing but inclined ground surface. Some natural slopes are hills, mountains etc. Some artificial slopes are also made for supporting highways, railways, embankments, earth dams and many more as slopes are more economical than constructing walls along highways, dams and many more.

1.4 SLOPE STABILITY

When external forces become much greater than resisting forces then slope becomes unstable. Natural slope failure results in landslide. Due to landslide, much destructionhappens. Hence, checking of stability of slopes is most important task for geotechnical engineers in now-a-days.

In natural slopes, increase in presence of water decreases the shear strength of soil slope. Decrease in shear strength increases the instability of soil slopes.

In manmade slopes, increase in weight of structures which is made above the soil slopes decreases the shear strength of soil slope and increases the instability of soil slopes.

1.5 REASONS OF FAILURE OF SOIL SLOPES

- a) Gravity Forces
- b) Seepage Forces Developed Due to Seeping of Water
- c) Sudden Lowering of Water Table
- d) Seismic Forces
- e) Human Activity
- a) Gravity Forces: Due to increase in loads on the toe of slope increases the gravitational forces. Due to increase in gravitational forces, slope failure occurs.
- **b) Seepage Forces**: Seepage forces in the direction of slopes increases the gravitational forces and hence slope stability decreases.
- c) Sudden Lowering of Water: Due to sudden lowering of water table ,direction of flow reverses and results in decrease of shear strength . Decrease in shear strength decreases stability.
- **d) Seismic Forces**: Seismic forces induce dynamic shear forces and it decreases shear strength of soil slope. Due to seismic forces, pore water pressure increases suddenly and it finally decreases the stability.
- e) Human Activity: Human activities also affect the stability of slopes. Increasing construction work on slopes decreases the stability.

1.6 TYPE OF SLOPE FAILURES

Generally three types of slope failures occur. They are[1]

- a) Base Failure
- b) Face Failure
- c) Toe Failure
- a) Base Failure: In this type of failure, failure surface passes below the toe surface. This type of failure occurs when soil below the toe is relatively weak.[1]
- **b)** Face Failure: In this type of failure, slope angle is extremely large. This type of failure occurs when toe portion is comparative strong.[1]
- c) Toe Failure: In this type of failure surface passes through toe. This type of failure occurs in the case of homogeneous and steep slope.[1]

1.7TYPE OF SLOPES

- a) Infinite Slopes
- b) Finite Slopes
- a) Infinite Slopes: those slopes which extend over large distances and soil mass is inclined to the horizontal surface are called infinite slopes. Generally hills, mountains that is natural slopes are infinite slopes.
- b) Finite Slopes: those slopes which have base and top surface of limited heights are called finite slopes. Generally embankments, earth dams are finite slopes that is manmade slopes are finite slopes.

1.8 SOME SLOPE FAILURE CASES



Figure 1-[2]



Figure2-[3]



Figure 3-[4]



Figure 4-[5]



Figure 5-[6]



Figure 6 [7]

CHAPTER 2

LITERATUREREVIEW

2.1 DEFINITION OF SLOPE FAILURE

"A slope failure is defined as the movement of soil,rock or any debris down a slope because of gravitational force.[8]

"Slope failure is the second most hazardous and destructive failure after landslide." [9]

"Due to climate change and modernization, slope failure cases have been increased. Global climate changes such as El Nino and unscientific human changes are the main reason of slope failure in recent years."[9]

"Most slope failures have been observed in young tectonic mountains such as Andes and Rocky in American continent.[10]

"In Japan and hills of south Asia, slope failure has been observed[11].

China is one of the countries which has suffered slope failure disaster several times. Since 1980, due to increase in constructional activies and change in climate condition, slope failure hazard has been observed in almost all part of China.

In China, from last 20 years, slope failure resulted in great damage to the infrastructure of China cities.[12].

According to China ministry of land resources, in the first half of 2010, no of geohazards detected are approximately 14,614 and 464 deaths are registered.

Hence to minimize the landslide hazards, we should learn and analyze the slope stability problems by using different techniques which is most accurate.

2.2 SLOPE STABILITY ANALYSIS METHODS

Slope stability is one of the most hazardous failures in the field of geotechnical engineering. Many of the geotechnical engineers have developed many techniques to analyzeslope stability problems. Earlier methods was generally based on limit equilibrium methods. In now days, many engineers have developed complicated soft wares but most accurate method by using powerful computers. [14]

In general, we find the factor of safety in slope stability analysis. We also find the critical failure surface along which slopes fail. If the factor of safety for slopes is greater than 1 then the slopes has been considered safe. If the factor of safety comes equal or less than 1, then slopes are considered to be unsafe. Further we find slope failure surfaces by dividing the slopes into number of slices.

Generally slope stability analysis methods are categorized into three parts-

2.2.1 Limit Equilibrium Methods

2.2.2Numerical Analysis Methods

2.2.3Artificial Intelligence Techniques

In limit equilibrium analysis, many of the authors have derived their own forumale for determining factor of safety which is based on method of slices. In method of slices, slope failure surface has been divided into numbers of slices. In limit equilibrium methods, some of the engineers has been derived by assuming the failure surface as circular and some has derived the formulae by assuming the failure surfaces as non-circular. It has been seen that formulae based on non-circular failure gives more accurate result than formulae based on circular failure surface but non-circular failure based formulae is more complicated than circular failure based surface methods. In now a day, slope/w software is developed which is based on limit equilibrium analysis methods. They give more accurate result than analytical methods.

Numerical analysis methods include finite element analysis methods, finite difference methods, boundary element methods and many more. In now a days, many softwares are based on limit equilibrium methods such as PLAXIS, GEO 5, ANSYS and many more methods.

Artificial intelligence techniques are most famous in now a days. It includes many of the techniques like artificial neural networking and genetic algorithm .in these techniques , programming are written that means coding are done using MATLAB language .MATLAB is a software in which coding are written in MATLAB language. MATLAB stands for matrix laboratory in which all the programmings are in the form of matrix.

Artificial neural networking method works like a human brain in which number of interconnected unit in ANN works like a neuron in human brain.

Genetic algorithm technique is based on population in which fitness value is determined on each individual.

I have used genetic algorithm technique to analyse the slope stability problem.

2.2.1 Limit Equilibrium Methods

Limit equilibrium methods are the earliest methods used to analyse the slope stability problems. In these methods, factor of safety is determined by comparing the two forces that is resisting forces and driving forces by assuming the trial wedge surfaces of soil slope. The main differences among the limit equilibrium methods are the assumption of failure surface that is either circular or non-circular (logarithmic ,plane etc) and the equilibrium equation (force equilibrium or moment equilibrium or both).[14]

Limit equilibrium methods

2.2.1.1Swedish Slip Circle Method

2.2.1.2 Log Spiral Method

2.2.1.3 Friction Circle Method

2.2.1.4 Slices Division Method

2.2.1.1 Swedish Slip Circle Method [14]

This method is also called method of slices. It assumes the failure surface as circular. This method is applicable for purely cohesive soil and also for both cohesion friction soil.[14]

For purely cohesive soil,

Factor of safety =
$$\frac{\text{Cu} \times \text{La} \times \text{r}}{\text{W} \times \text{x}}$$
[14]

For cohesion-friction soil,

$$Factor\ of\ safety = \frac{(c \sum \Delta L + tan \Phi \sum N)}{\sum T}$$

Where,

Cu = unit cohesion

La = length of slip arc

r = radius of slip arc

W = weight of wedge

x = distance of the line of the action of W from vertical line through centre of rotation.

2.2.1.2 Log Spiral Method[14]

In this method the failure surface is assumed to be spiral. In this method, statically determinacyconcept and logarithmic spiral surface is assumed.

In this method, spiral slip surface of radius about the centre of the spiral slip surface which differs with the rotational angle θ The formulae is

$$\mathbf{r} = \mathbf{r}_0 \, e^{\theta \tan \Phi d}$$

Where,

 r_0 = initial radius of spiral failure surface,

r = radial distance between centre of spiral slip surface and the point

 θ = angle between r_0 & r

 Φ_d = developed friction angle depending on safety factor for slopes and angle of friction for soil.

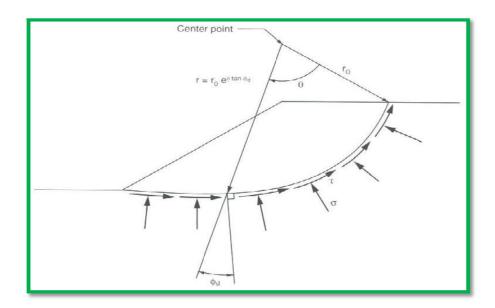


Figure 7 -Logarithmic spiral slip failure surface [15]

Along the failure surface, stresses are normal stress (σ) and shear stress (τ)which is explained by following equations

$$\tau = \frac{C}{F} + \sigma \frac{\tan \Phi}{F}$$

and, $\tau_d = c_d + \sigma \tan \Phi_d$

Where,

c& Φ are shear strength parameters

 $C_d \& \Phi_d$ are developed shear strength parameters

In this method, resultant of shear stresses and normal stresses act at the centre of the failure spiral surface. That's why no net moment produced, only forces are wt of slope and cohesive force and factor of safety is calculated by considering two forces weight and cohesion.[14]

2.2.1.3Friction Circle Method:

This method is valid for total and effective stress for which angle of friction is greater than zero. The resultant of shear stress and normal stress mobilizes along the circular surface. The tangent line of the circle of radius $R_f=R\sin\Phi_m$ which is called friction circle. R is the radius of failure circle & Φ_m is the failure surface of friction angle.

$$\Phi_{\rm m} = \tan^{-1} \tan \frac{\Phi}{F}$$

Where, F = factor of safety for failure surface of slopes

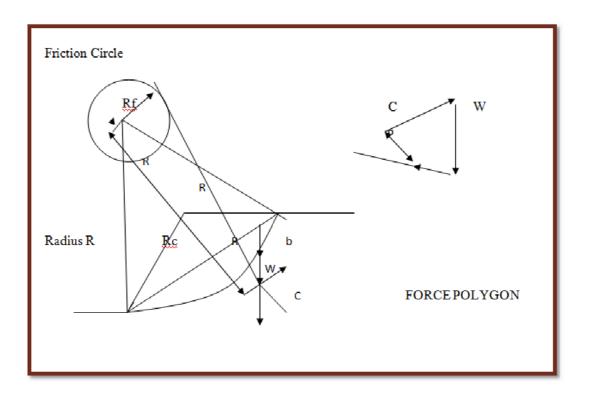


Figure -8Friction circle failure surface

The procedure for finding factor of safety is given by Abramson, is

- a) First calculate the slide weight W
- b) Then calculate the numerical value of pore water force and its direction
- c) Then find the value of distance which is perpendicular to the cohesion force.
- d) Then calculate resultant weight W.
- e) Finally calculate the factor of safety for slope failure surface.[14]

2.2.1.4 Method of Slice

In this method, soil mass above the slip surface is divided into number of slices, the slices which divided is vertical. In this method soil cohesion, friction angle and many other soil parameters is considered. In method of slices, factor of safety is calculated by ignoring all the unknown forces.[14]

Softwares for slope stability method are more commonly available which is mainly based on method of slices. There are many methods which come under the method of slices.[14]

- a) OrdinaryMethod of Slices
- b) Simplified Bishop's Method
- c) Janbu's Method
- d) Spencer Method
- e) Morgenstern and Price's Method
- f) Sarma's Method

a) Ordinary Method of Slices

This method is also called 'ordinary method of slices'. It is the simplest methos of slices. The main assumption of this method is that the intersliceforces of resultant is parallel to its base. Hence ,in this method, interslice forces are neglected (Fellenius ,1960). In this method, only moment equilibrium condition is satisfied. That's why it does not give the accurate value of safety factor. This method is applicable for those soil slopes which have both cohesion and friction.

Factor of safety =
$$\frac{\sum [c'\Delta l + (W\cos\alpha - u\Delta l\cos\alpha * \cos\alpha)\tan\theta]}{\sum W\sin\alpha}[14]$$

Where,

W = weight of slice

 α = inclination of the bottom of the slice

u = pore water pressure at the centre of the base of slice.

 Δl = length of the slice

 $\Sigma \Delta L$ = sum of lengths of slip arc,

 $\Sigma N=$ sum of all normal components,

 Σ **T**= sum of all tangential components

b) Bishop 's Method [14]

Bishop in 1955, took the side forces in factor of safety calculation. Side forces in Swedish method were neglected. The slip surface is assumed to be a circular and the factor of safety is defined as the ratio of actual shear strength to the limiting shear stress where limiting shear stress is named as mobilized shear strength.[14]

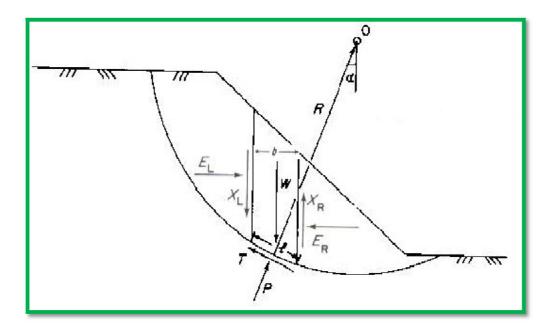


Figure 9 Bishop's method-[17]

Factor of safety
$$=\frac{\tau f}{\tau}$$

Where,

 τf = actual shear strength of soil slope

 τ = mobilized shear strength of soil slope

Assumptions taken in Bishop's method are

- a) The failure occurs is assumed by rotation of soil mass about its rotation point is circular. Hence bishop's method is not used for non-circular failure surfaces.
- b) The forces on the sides are assumed to be horizontal. Hence there are no shear stresses develop between slices.
- c) The total normal force is assumed to be occurring at the centre on its base of each slice and is derived by summing all the vertical forces.

c) Janbu's Method [14]

The janbu's simplified method is applicable for non circular slip failure surfaces. In this method, interslice forces are assumed to be horizontal and shear force is zero.

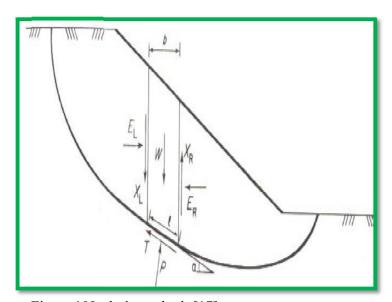


Figure 10Janbu's method -[17]

Factor of safety =
$$\frac{\sum (c'l + (P-ul)tan\Phi')sec\alpha}{\sum Wtan\alpha}$$

d) Spencer's Method[14]

This method is valid for both circular and non circular failure surfaces. In this method, centre of frictional rotation is assumed. In this method, interslice forces are assumed to be parallel and obviously the same angle of inclination.

$$\tan \theta = \frac{X_L}{E_L} = \frac{X_R}{E_R}$$

Where, θ is the angle of interslice force with horizontal.

Factor of safety,
$$F = \frac{\sum (c'l + (P-ul)tan\Phi')sec\alpha}{\sum (W - (X_R - X_L))tan \propto)}$$

e) Morgenstern & Price Method[14]

Morgenstern & price invented this method in 1965. This method considers not only normal and shear forces but also considers moment equilibrium condition. This method is valid for all slip surfaces. That means this method is valid for both circular and noncircular slip surfaces. This method assumes a relationship which is as follows

$$X = \lambda$$
. $f(x)$.

Where,

 λ = scaling factor

f(x) = Assumed function

E = horizontal interslice force

The unknowns in this method are factor of safety (F), λ (scaling factor), normal forces (p), the line of thrust which is solved by equating equilibrium equations.[14]

f) Sarma 's Method[14]

This method was developed by Sharma in 1975. This method is different from all othermethods. Because this method assumes seismic coefficient (k_c). Usually factor of safety in this method is kept 1 for determining seismic coefficient.

The Sarma method analyses slope stability by assuming a horizontal acceleration which is fraction of gravitational constant. The Sarma method was developed to analyzeearthen dam which fails due to earthquake.[14]

2.2.1.5 Comparison among the Methods of Limit Equilibrium of Slope Stability Analysis

Methods	Factor of safety		Interslice Force	Slip surface
	Force	Moment	Assumption	
	Equilibrium	Equilibrium	(H=horizontal,	
	1	•	V=vertical)	
(1) Ordinary (Swedish or USBR)		Yes	Ignore both H,	Circular
2Bishop's Simplified		Yes	V ignored, H considered	Circular
(3) Janbu's Simplified	Yes		V ignored, H considered	Circular
(4)Janbu'sGeneralised'	Yes		Both H, V considered	Non circular
(5) Spencer	Yes	Yes	Both H, V considered	Non circular
(6) Morgestern-Price	Yes	Yes	Both H, V considered	Non circular
(7) Lowe-Karafiath	Yes		Both H, V considered	Non circular
(8) Sarma's method	Yes		Both H, V considered	Non circular

2.2.2Numerical Analysis Methods[18]

These methods give more accurate values than limit equilibrium methods in determining factor of safety for stability of soil slopes. But numerical methods are more cumbersome and complex in compare to limit equilibrium methods.[18]

In these methods, strain that means displacement equations are taken into account, forces are not taken into account. Numerical methods are used in now a days is more and more due to computer power. Numerical methods has been classified in two parts

2.2.2.1 Continuum Method[18]

2.2.2.2 Discontinuum Method[18]

2.2.2.1In continuum method, three methods are used

- a) Finite Element Method
- b) Finite Difference Method
- c) Boundary Element Method

a) Finite Difference Method[18]

Finite difference method (FEM) is based on differential equations of elastic theory. In this method, differential equations are converted into linear equations and then problem is solved. It is the oldest method among all numerical analysis methods. In this method, grids are superimposed in the domain.[18]

b) Finite element method[18]

This is one of the powerful solving tool in engineering field. In this method, gravity increase method and strength increase method is used. In gravity increase method, gravitational forces increase until they fail. Hence, in this case, factor of safety is defined as the ratio of actual gravitational acceleration to the failure gravitational acceleration. In strength reduction method, shear strength parameters has been decreased until slope failure takes place. So in this case, factor of safety is defined as ratio of actual shear strength to critical shear strength. Gravity increase method is more suitable for analyzing the embankment stability because embankment fails due to increase in gravitational forces. [18].

c) Boundary element method[18]

In boundary element method, only the boundary of the continuum is discretized. In this method, solution is found approximately along boundary. The advantage of doing discritizing along boundary is the reduction of problem complexity. That means the 3 D problem can be reduced to 2D problem and 2 D problem can be reduced to single line problem. Due to this, slope stability problem is solved easily but it is not accurate.[18]

2.2.3 Artificial Intelligence Technique

Artificial intelligence technique means problem analysed by artificial intelligence using MATLABtools. MATLAB stands for matrix laboratory which is a numerical computing software. MATLAB allows matrix and vector manipulation, plotting functions, implementation of algorithms, interfacing with programmes written C, C+, C++, FORTRAN and many other languages.

In geotechnical engineering field, slope stability problem solved by two artificial intelligence techniques[18]

2.2.3.1 Artificial Neural Networking

2.2.3.2 Genetic Algorithm

2.2.3.1Artificial Neural Networking

This artificial intelligence technique works like a human brain. Like a human brain, it consists of several number of interconnected units called neurons. Each neuron receives the information, process the information and send the information to other neuron like human brain. In this technique, user is permitted to train the network and store results between interconnected units. After training stage, artificial neural network can be used to simulate the relationship between given / known inputs and dependent variables of output. [18]

First generation of artificial neural network started in the period of 1940-1960 but it came as an useful tool in 1980. There are several artificial neural network based on characteristics of each neuron, the learning and training scheme and network function.

Multilayer perception network (MPN) is generally used for solving the problem of slope stability in geotechnical engineering. Thisnetwork is composed of two or more layers.[18]

The information stored in each neuron is called state of neuron.

Artificial neural network works in number of steps. The steps are

- a) An array of input values is defined in the input layer.
- b) These values are transmitted to the second hidden layer following the protocol defined equation of Xi (k) in which the input is I and network k.
- c) Then the state of the neurons in second layer is transmitted into third layer where new calculations are done for obtained the state of neuron in third layer.
- d) The process is continued until the output has been reached.[18]

Training of the network is done by comparing the output provided by ANN and the actual result associated with input.

There are different strategies of minimizing error by changing the values of weights. Themost common method is back propagation algorithms in which derivatives of the error function with respect to weight is set to be zero.

In artificial neural networking, variables are taken according to the design of problems. The number of neuron and hidden layer is also dependent on the type and requirement of problem.

For slope stability problems, we need some experimental information and number of slopes with exact factor of safety. In ANN, we divide the problem in two sets first one is for training and other one is for validation.

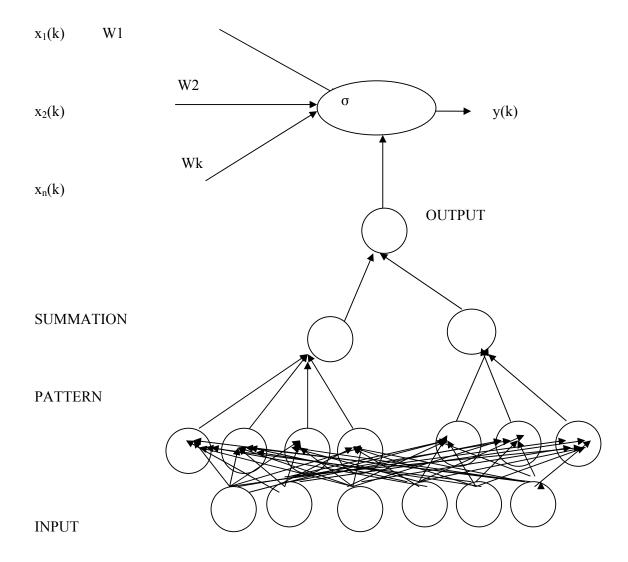


Figure 11 Four Layer Input Network

For solving slope stability problem, the ANN model has five input parameters – slope angle, slope height, angle of internal friction, cohesion and soil unitweight The output parameter for the ANN is the factor of safety. A tangent hyperbolic (tanh) transfer function was used for the internal layers with seven processing elements in the first hidden layer and four processing element in the second hidden layer. A linear bias transfer function was used for the output layer.

The ANN was trained using five runs with 2000 epoch for each run (Neurosolutions 2011). A total of 640 data sets, generated by the four methods Ordinary (Fellenius), Bishop, Jumbo and Spencer, with different input and output parameters were used to train and test the ANN models. The training and testing data were randomly chosen as 70% used as training data, 15% as validation data and 15% as testing data. The trained ANN model was tested using 95 randomly selected data sets as test data and ANN predicted factor of safety were then compared with the values of factor of safety calculated using the four methods based on certain performance criteria. [18]

2.2.3.2 Genetic Algorithm.

In the field of computer science of artificial intelligence, genetic algorithm is an optimization searching tool in which population is increased by the crossover and mutation. At first, an individual population size is initialized then crossover and mutation process starts until the best fitness function value reached.

2.2.3.2.1 Basic principle of Genetic Algorithm

Basic working principle of genetic algorithm is to find the generation of candidate population, finding objective function and finally get fitness factor.

They are categorized as follows

Formulate initial population
Randomly initialize population
Repeat
Evaluate objective function
77. 7 0
Find fitness function
Apply genetic operators
Reproduction
Crossover



Table 1 Generation of new population

2.2.3.2.2 Genetic Algorithm understanding example [19]

The best way to understand the genetic algorithm is to take a simple example of a mathematical equation like $Y = x^2$ in this equation value of x is taken from 1 to 30.

For solving this problem, genetic algorithm starts with randomly chooses population and reached the value by further crossover and mutation process. In genetic algorithm, function is terminated when maximum iterations possible is reached.

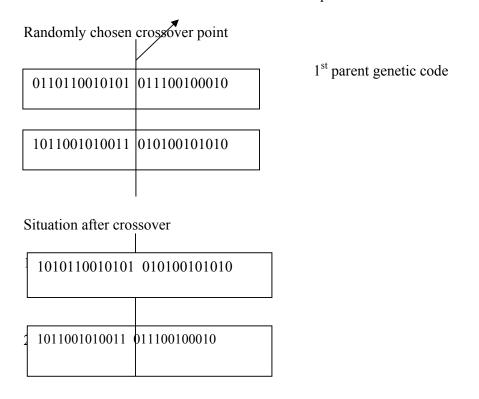


Figure 12 Genetic code of parents and crossover situation[19]

Optimization is being improved until it satisfies the function completely optimization is increased by increasing population through the process of crossover and mutation.

A series	Starting	Value of	$f(x) = x^2$	Offspring	Actual
of	Population	the		Number	offspring
numbers	(random)	Variable		fitness(fi/f _{avg})	number
		X			(rounded
					up)
1	01101	13	169	0.58	1
2	11000	24	576	1.97	2
3	01000	8	64	0.22	0
4	10011	19	361	1.23	1
Sum			1170	4.00	4
Average,			239	1.00	1
f_{avg}					
Maximum			576	1.97	2

Table 2 Starting of randomly chosen population in binary form[19]

First	Randomly	Cross-over	New	Value of	$F(x) = x^2$
generation	chosen	point(random)	population	the	fi/ f _{max}
offspring	cross-over			variable	(rounded
	partner				up)
0110/1	2	4	01100	12	144
1100/0	1	4	11001	25	625
11/000	4	2	11011	27	729
10/011	3	2	10000	16	256
Sum					1754
Average, f _{avg}					439
Maximum					729

Table -3First offspring generation by crossover[19]

After crossover process, parents generate new children. Like that, population is increased up to 1st generation through crossover.

By mutation process, we also develop new population which is randomly different from parent chromosomes that is initial population.

2.2.3.2.3 Advantages of Genetic Algorithm [19]

- a) It can solve every Optimisation problem which can be described with the chromosome encoding.
- b) It solves problems with multiple solutions.
- c) Since the genetic algorithm execution technique is not dependent on the error surface, we can solve multi-dimensional, non-differential, non-continuous, and even non-parametrical problems.
- d) Structural genetic algorithm gives us the possibility to solve the solution structure and solution parameter problems at the same time by means of genetic algorithm.
- e) Genetic algorithm is a method which is very easy to understand and it practically does not demand the knowledge of mathematics.
- f) Genetic algorithms are easily transferred to existing simulations and models.

2.2.3.2.4 Disadvantages of Genetic Algorithm [19]

- a) Certain optimization problems (they are called variant problems) cannot be solved by means of genetic algorithms. This occurs due to poorly known fitness functions which generate bad chromosome blocks in spite of the fact that only good chromosome blocks cross-over.
- b) There is no absolute assurance that a genetic algorithm will find a global optimum. It happens very often when the populations have a lot of subjects.
- c) Like other artificial intelligence techniques, the genetic algorithm cannot assure constant optimization response times. Even more, the difference between the shortest and the longest optimization response time is much larger than with conventional gradient methods. This unfortunate genetic algorithm property limits the genetic algorithms' use in real time applications.
- d) Genetic algorithm applications in controls which are performed in real time are limited because of random solutions and convergence, in other words this means that the entire population is improving, but this could not be said for an individual within this population. Therefore, it is unreasonable to use genetic algorithms for on-line controls in real systems without testing them first on a simulation model.

The main advantages of using artificial intelligence technique for solving stability problemis

No requirement of learning any software

No requirement to use limit equilibrium methods.

CHAPTER 3 GENETIC ALGORITHM

3.1 GENETIC ALGORITHM FUNDAMENTALS[19]

- a) Genetic algorithms are adoptable search algorithm which is done by evolution..
- b) Genetic algorithms are a part of search computation. It is a rapidly growing area of artificial intelligence field. GA's are followed by Darwin's theory about evolution –"survival of the fittest'.[19]
- c) GAs represents an intelligence technique of a random search used to solve optimization problems.
- d) GAs through randomized method to direct search for better results

3.2 INTRODUCTION TO GENETIC ALGORITHMS [19]

In computer science and artificial intelligence field, genetic algorithm is the process of search through the space of possible solutions. The set of solutions defines the algorithm problem for a given problem.[19]

In engineering and mathematics, genetic algorithm is called as a process of optimization. The problems are first modeled in mathematical forms which is shown in terms of functions and then to find best fitness factor by optimizing the function and give the best result.[19]

3.3 REQUIREMENTS OF GENETIC ALGORITHMS

In large state space or multi modal space solution, genetic algorithm technique is more beneficial than linear programming technique.

Genetic algorithm technique is more beneficial for finding three dimensional problems that is space search problems.[19]

3.4 PSEUDO CODE[19]

BEGIN

INITIALISE population with random candidate solution

EVALUATE each candidate

REPEAT until termination condition satisfy

- 1. SELECT parents;
- 2. RECOMBINE pairs of parents;
- 3. MUTATE the resulting off springs:
- 4. SELECT individuals or the next generation;

END[19]

3.5 SEARCH SPACE[19]

- a) Some useful solution exists under certain problem is called search space. [19]
- b) Each point in a search space exists one possible solution.
- c) Eachpossible solution can be represented by its fitness factor for the problem.
- d) The GAssearch for the best solution among some of possible solutions shown by one point in the search space.[19]
- e) Looking for a solution is then equal to looking for someextremum value (minimum or maximum) in the search space.
- f) In using GA, the process of finding solutions generates other points (possible solutions) because of revolution occur.

3.6 WORKING PRINCIPLES[19]

It is very necessary to know the fundamental principle of genetic algorithm.[19]

- a) Chromosome a set of genes -A chromosome contains the solution inform of genes.
- **b)** Gene a part of chromosome- A gene contains a part of solution. It determines the possible solution.
- c) Individual same as chromosome
- **d) Population** Number of individuals present with same length of chromosome.
- e) Fitness The value assigned to an individual based on how far or close a individual is from the solution, greater the fitness value better the solution it contains
- f) Fitness function A function that assigns fitness value to the individual. It is problem specific.
- g) Breeding Taking two fit individuals and then crossing there chromosome to create new individuals.
- h) Mutation Changing a random gene in an individual
- i) Selection Selecting individuals for creating the next generation

3.7 OUTLINE OF BASIC GENETIC ALGORITHM[19]

- a) [Start] Generate random population of n chromosomes (i.e. suitablesolutions for the problem).[19]
- **b)** Fitness] Evaluate the fitness f(x) of each chromosome x in the population.
- c) [New population] Create

A new population by repeating followingsteps until the new population is complete.

[Selection] Select two parent chromosomes from a population[19]

[Accepting] Place new offspring in the new population[19]

- d) [Replace] Use new generated population for a further run of the algorithm
- e) [Test] If the end condition is satisfied, stop, and return the best Solution in current population
- **f)** [Loop] Go to step b

3.8 GENETIC OPERATORS[19]

Genetic operators used in genetic algorithms maintain genetic diversity. Genetic diversity or variation is a necessity for the process of evolution. Genetic operators are analogous to those which occur in the natural world:

- 3.8.1 Reproduction (or Selection)
- 3.8.2 Crossover (or Recombination)
- 3.8.3 Mutation.

3.8.1 Reproduction (or Selection)[19]

Reproductionisthe first operator applied on population. From the population, the chromosomes are selected which is calledparents and they crossover produce offspring. Themain problem is in selecting the chromosomes that is parents.

According to Darwin's evolution theory "survival of the fittest" – the bestOnes should survive and create new offspring.[19]

The Reproduction operators are also named as Selection operators. Selection means extraction of chromosomes from an existing population. Every gene has a meaning, soone can derive from the gene a kind of best measurement calledfitness function. According to this value, selection can be performed. Fitness function signifies the optimality of a solution (chromosome) so that a particular solution may be obtained against all the other solutions.[19]

The function performs the best result of a given 'solution'.. Theytake the current population in the strings of above average and inserttheir multiple copies in the mating pool in a probabilistic manner.

The most commonly used methods of selecting chromosomes for parentsto crossover are:[19]

- a) Rank selection
- b) Boltzmann selection, –
- c) Tournament selection

a) Rankselection [19]

Rankselection is also known as Roulettewheel Selection. It is a genetic operator, used for selecting potentially useful solutions.[19]

In rankselection, the chance of an individual's being selected is according to its fitness, greater or less than its competitors' fitness.[19]

b) Boltzmann Selection[19]

This method simulates the process of slow cooling of molten metal toachieve the minimum function value in a minimization problem. The coolingphenomenon is simulated by controlling a temperature likeparameter introduced with the concept of Boltzmann probability distribution. The system in thermal equilibrium at a temperature T has its energy distribution based on the probability defined by

$$\mathbf{P}(\mathbf{E}) = e^{\frac{-E}{kT}} [19]$$

Where,k is Boltzmann constant.

This equation means that a system at a higher temperature hasalmost uniform probability at any energy state, but atlowertemperature it has a small probability of being at a higher energy state. Thus, by controlling the temperature T and assuming that the searchprocess follows Boltzmann probability distribution, the convergence ofthe algorithm is controlled.[19]

3.8.2 Crossover[19]

Crossover operator is used to recombine two strings to get a better string. In crossover, recombination of two different individual strings takes place and makes a new string. In the crossover operator, new strings are created by exchanging information among strings of the mating pool. The two strings participating in crossover operation is called parent and resulting string is called as children. It is obvious from this construction that good sub-strings from parent strings can be combined to form a better child string, if an appropriate population is chosen. With a random site, the children strings produced may or may not have a combination of good sub-strings from parent strings, depending on whether the crossing has taken place that means it falls at anappropriately place or not. There will be more copies of them in the next mating pool generated by crossover.[19] It is clear from this discussion that the effect of cross over may be profitable. Thus, in order to preserve

some of the good strings that are already present in the mating pool, all strings in the mating pool are not used in crossover. When a crossover probability, defined here as Pc is used, only 100 Pc per cent strings in the population are used in the crossover operation and 100(1-Pc) percent of the population remains as they are in the current population. A crossover operator is mainly responsible for the search of new strings even though mutation operator is also used for this purpose sparingly. [19]

Crossover operators are of many types.

- a) One-string crossover.
- b) Two string, crossover.
- c) Uniform
- d) Arithmetic
- e) Heuristic crossovers

a) One-string Crossover

One-string crossover operator randomly selects one crossover point andthen copy everything before this point from the first parent and then copyeverything afterthe crossover point from the second parent. The Crossover process is as shown below.

String 1 011/01100 String 1011/ 11001

String 2 110/11001 String 2011/01100

Before crossover After crossover

Where, a vertical line, | is the chosen crossover point.

Figure 13 Crossover in one string

b) Two-strings Crossover

Two-Point crossover operator randomly selects two crossover points within chromosome then interchanges the two parent chromosomes betweenthese points to produce two new offspring.

String 1 011/011/00 String 1 011/110/00

String 2 110/110/01 String 2 011/011/01

Before crossover After[18] crossover

Figure- 14 Crossover in two strings

c) Uniform Crossover

Uniform crossover operator decides (with some probability known as the mixing ratio) in which parents will contribute the gene values in theoffspring chromosomes. The crossover operator allows the parent chromosomes to be mixed at the gene level rather than the segment level (as with one and two point crossover). [18]

d) Arithmetic

Arithmetic crossover operator linearly combines two parent chromosomevectors to produce two new offspring according to the equations:

```
Offspring1 = a \times Parent1 + (1-a) \times Parent2[19]
```

Offspring2 = $(1 - a) \times Parent1 + a \times Parent2[19]$

Where, a is a random weighting factor chosen before each crossoverOperation.

e) Heuristic crossovers

Heuristic crossover operator uses the fitness values of the two parent chromosomes to determine the direction of the search.

The offspring are created according to the equations:

Offspring1 = BestParent + $r \times (BestParent - WorstParent)[19]$

Offspring2 = BestParent

Where, r is a random number between 0 and 1.

One site crossover and two site crossover are the most common ones adopted. In most crossover operators, two strings are picked from the mating pool at random and some portion of the strings is exchanged between the strings. Crossover operation is done at string level by randomly selecting two strings for crossover operations. A one site crossover process is performed by randomly choosing a crossing areaacross the string and by exchanging all binary numbers on the right side of the crossing area.

3.8.3 Mutation[19]

Mutation is a process in which new information is added randomly to the genetic search process. It is an operator that displays the variation in the population whenever the population tends to become homogeneous due to repeated use of

reproduction and crossover operators. Mutation may cause the chromosomes of individuals to be different from those of their parent individuals.[19]

Mutation in a way is the process of randomly disturbing genetic information. They operate at the bit level; when the bits are being copied from the current string to the new string, there is probability that each bit may become mutated. This probability is usually a quite small value, called as mutation probabilityPm. A coin toss mechanism is employed; if random number between zero and one is less than the mutation probability, then the bit is inverted, so that zero becomes one and one becomes zero. This helps in introducing a bit of diversity to the population by scattering the occasional points. This random scattering would result in better optima, or even modify a part of genetic code that will be beneficial in later operations. On the other hand, it might produce a weak individual that will never be selected for further operations. [19]

The need for mutation is to create a point in the neighborhood of the current point, thereby achieving a local search around the current solution. The mutation is also used to maintain diversity in the population. For example, the following population having four eight bit strings may be considered: [19]

01101011

00111101

00010110

01111100

It can be noticed that all four strings have a 0 in the left most bit position. If the true optimum solution requires 1 in that position, then neither reproduction nor crossover operator described above will be able to create 1 in that position. The inclusion of mutation introduces probability Pm of turning 0 into 1.

These three operators are simple and straightforward. The reproduction operator selects good strings and the crossover operator recombines good sub-strings from good strings together, hopefully, to create a better sub-string. The mutation

operator alters a string locally expecting a better string. Even though none of these claims are guaranteed and/or tested while creating a string, it is expected that if bad strings are created they will be eliminated by the reproduction operator in the next generation and if good strings are created, they will be increasingly emphasized. Further insight into these operators, different ways of implementations and some mathematical foundations of genetic algorithms can be obtained from GA literature.[19]

Application of these operators on the current population creates a new population. This new population is used to generate subsequent populations and so on, yielding solutions that are closer to the optimum solution. The values of the objective function of the individuals of the new population are again determined by decoding the strings. These values express the fitness of the solutions of the new generations. This completes one cycle of genetic algorithm called a generation. In each generation if the solution is improved, it is stored as the best solution. This is repeated till convergence. The Mutation operators are of many types

- a) Flip Bit
- b) Boundary
- c) Non-Uniform,
- d) Uniform,
- e) Gaussian.

a) Flip Bit

The mutation operator simply inverts the value of the chosen genei.e. 0 goes to 1 and 1 goes to 0. This mutation operator can only be used for binary genes.

Consider the two original off-springs selected for mutation.[19]

Original offspring 1 1 1 0 1 1 1 1 0 0 0 0 1 1 1 1 1 0

Original offspring 2 1 1 0 1 1 0 0 1 0 0 1 1 0 1 1 0

Invert the value of the chosen gene as 0 to 1 and 1 to 0

The Mutated Off-spring produced are:

Mutated offspring 1

Mutated offspring 2

1 1 0 1 1 0 1 1 0 0 1 1 0 1 0 0 [19]

b) Boundary

The mutation operator replaces the value of the chosen gene with eitherthe upper or lower bound for that gene (chosen randomly).

This mutation operator can only be used for integer and float genes.

c) Non-Uniform

The mutation operator increases the probability such that the amount of the mutation will be close to 0 as the generation number increases. This mutation operator prevents the population from stagnating in the early stages of the evolution then allows the genetic algorithm to fine tune the solution in the later stages of evolution.[19] This mutation operator can only be used for integer and float genes.

d) Uniform

The mutation operator replaces the value of the chosen gene with auniform random value selected between the user-specified upper andlower bounds for that gene.[19]

This mutation operator can only be used for integer and float genes.

e) Gaussian

The mutation operator adds a unit Gaussian distributed random value to the chosen gene. The new gene value is clipped if it falls outside of theuser-specified lower or upper bounds for that gene. This mutation operator can only be used for integer and float genes.

3.9 PRACTICAL INSTRUCTIONS FOR SETTING THE GENETIC ALGORITHM PARAMETERS

Basic parameters of genetic algorithm are:

- a) Cross-over probability,
- b) Mutation probability,
- c) The number of cross-over points in the chromosomes,

- d) The maximum iteration of the genetic algorithm,
- e) The value of the criterion to end the optimization, and
- f) The number of subjects in the generation.
- a) The probability of cross-over p_c at standard genetic algorithms is between 0.6 and 1.0. The cross-over points are placed randomly within the chromosome length. A chromosome is a series of genes (ones or zeros) of the genetic material code and it describes a certain quality of the subject. The probability of cross-over in our example, computed in chapter 3.2.1, was the value 1.0, which means that the cross-over was performed every time. [19]
- b) The probability of mutation p_m is the probability of the individual gene value change from 1 to 0, or reverse. The mutation can be executed at all genes within all the chromosomes of the entire offspring generation. The probability of mutation is less than 0.1 at the standard genetic algorithms. In our simple example from the previous section, we have chosen the mutation probability value of 0.001.[19]
- c) The chromosomes can cross-over in several points. Within one chromosome, at least one cross-over has to be executed, but we can execute more of them, especially if the chromosome is long. If we have several chromosomes within one subject, we have to perform at least one cross-over per chromosome. We only selected one cross-over point for our example from the previous subchapter, because we only had one short chromosome.[19]
- d) The maximum iteration of genetic algorithm number is a parameter which means for how many generations the algorithm will be executed. It is theoretically possible that the genetic algorithm computes the optimum for an infinite number of iterations (generations), so it is pertinent that the operator foresees an automatic algorithm switch-off in such a case. [19]
- e) The criterion value to end the optimization is set by the operator, if he knows it. At multi-dimensional cases (with more chromosomes), the criterion value to end the optimization is often not known even approximately, so in those cases we only

rely on the maximum number of genetic algorithm iteration parameter values that would stop the algorithm execution.

optimums. The more optimums we foresee, the more subjects we need in a generation. However, it is true that the number of local optimums at multidimensional optimization cases is usually not known. A larger number of subjects, of course, demands greater computing power of the computer which is executing the algorithm, or a longer algorithm execution. A larger number of subjects also means that the positive change in the genetic material, caused by mutation or cross-over, is put into force with greater difficulty in the entire population of a generation (it takes a long time or many generations before the positive genes get transferred to all subjects of the population of a generation, the greater is the probability of the loss of the positive change in the genetic material of the entire generation population because of cross-over or mutation.[19]

3.10 GENETIC ALGORITHM OPTIMIZATION TOOL

To open the optimization tool, we write in command window optimtool('ga')

Then ENTER

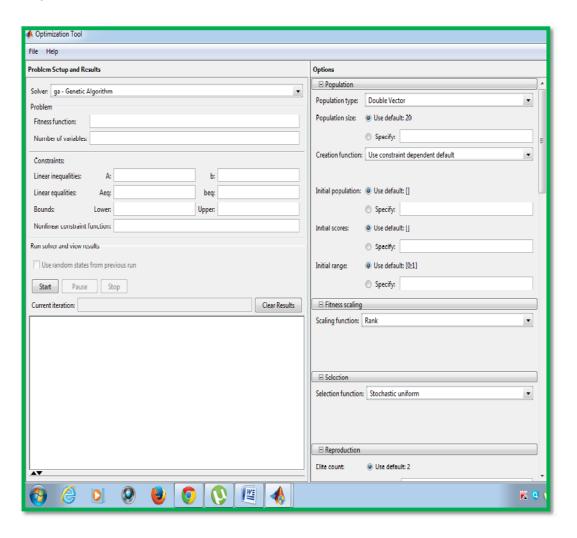


Figure 15Genetic algorithm Optimization tool

CHAPTER 4 NUMERICAL INVESTIGATION AND CALCULATION

4.1 FLOW CHART OF OPTIMIZATION TOOL

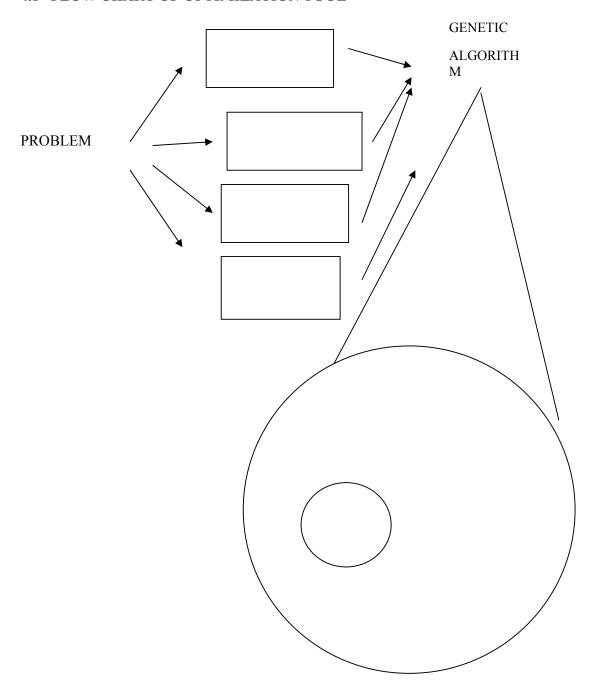


Figure 16Flow Chart of Optimization Tool

4.2 FLOW CHART OF GENETIC ALGORITHM

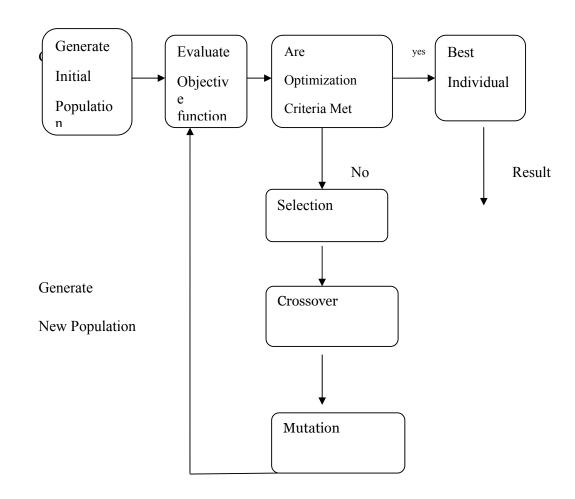


Figure 17 Flow Diagram of Genetic Algorithm

4.3RELATION OF SLOPE STABILITY PROBLEM WITH GENETIC ALGORITHM

In slope stability problem, population is initialized through soil's actual shear strength and increasing population shows that external forces are increasing. Then we calculate that how much external forces are greater than actual. This factor is called factor of safety for slopes. This factor of safety is related by best fitness value by using some fitness function in genetic algorithm.

Fitness function is related to slope stability formulaes. Actually first we see, which function will relate the slope stability formulae then we use the required fitness function and find the best fitness value that is factor of safety.

Optimization tools contains three parts

- a) Problem
- b) Constraints
- c) Run solver and view result

a) Problem

We mainly define fitness function in problem section.

Fitness function: In setting up problem, we have to decide to use which function is to be minimized and which function will be valid for problem.

Generally, in slope stability problem, rastrigins function is used to solve and view results.

Rastrigin's function

Rastrigins function has many local minima but only one global minimum. That global minimum value is called best fittness value.

The example of rastrigins fuction of two variable is given as

Ras (x) =
$$20 + x_1^2 + x_2^2 - 10$$
 ($\cos 2\pi x 1 + \cos 2\pi x 2$)

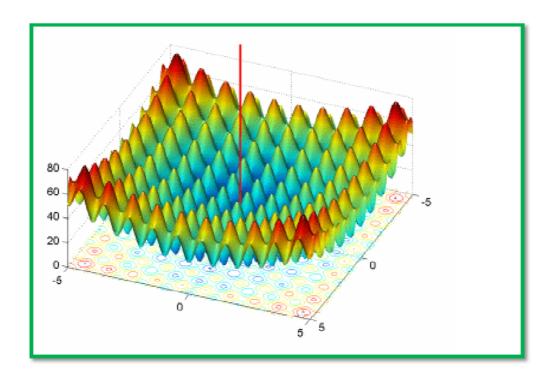


Figure 18Global Minimum Point inRastrigins Function[20]

b) Constraints

In constraints, the linear inequalities and linear equalities is considered. it is specified by matrix A and vector b as mentioned in optimization tool. in constraints, lower bounds and upper bounds are specified.

c) Run Solver and View Results

in this section, we run the solver and view the results which is also shown in graph of optimization tool. in this section, termination reasons also display and we know the reason of termination clearly.

4.4 MATLAB CODE FOR SOLVING SLOPE STABILITY PROBLEM IN SOILS BY USING MATLAB FUNCTION

```
functionfs =safety factor(c, h, theta, psoil,m, phi)
%fs = safety factor(c, h, theta, psoil, m, phi)
%this function calculates the factor of safety for slopes
%note that suggested units are in parenthesis below
%Variables:
        external
        c = cohesion (kg m^-1 s^-2)
        h = slide mass thickness (m)
         theta = slope angle in degrees
         psoil = soil density (kg m^-3)
%
        m = fraction of saturated thickness (such that <math>m = 0 if the water
                   table is just below the slide surface and m = 1 if the
응
                    water table is at the ground surface) (dimensionless)
응
         phi = angle of internal friction in degrees
응
         internal
         g = gravitational acceleration (m s^-2)
        pwater = water density (kg m^-3)
        driving = driving stresses
% resisting = resisting stresses
%set up a couple of constants
%others
          g = 9.81;
pwater = 1000;
%write the constants back to make sure that it looks right
f(c) = 1.3f, f(c)
phi = %.3f\n', c,h,theta,psoil,m,phi);
resisting=(c + h*g*(cos(rads(theta))^2)*(psoil-
(m*pwater))*tan(rads(phi)));
driving = (psoil*h*g*sin(rads(theta))*cos(rads(theta)));
fs=resisting/driving;
```

4.5 STATEMENT OF PROBLEM NO 1

A cohesive soil slope with its height 25meters has a slope ratio of 1 : 2. The soil unit weight γ is 20 KN/m3. The soil internal friction angle φ is 26.6 degrees, and cohesion is 10 KPA. Find the factor of safety of given slope.[21]

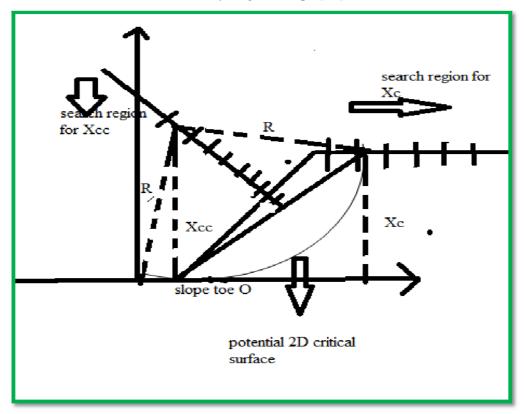


Figure 19 Failure surface of the given problem[21]

Where, Xcis the abscissa of a point on slope top surface.

Xccis the abscissa of the critical slip surface circle center.

4.6SOLUTION OF PROBLEM 1

4.6 .1 Solution by Random Search Using Optimization Tool

In GA, the parameters in the optimization problemare translated into chromosomes with a binary data string. If a solution is not obtained, a new population is created from the original(parent) chromosomes. This is obtained using "crossover"and "mutation"operations. Crossover involves gene exchange from two random (parent) solutions to form a new child (newsolution). Mutation involves the random switching

of a singlevariable in a chromosome and is used to maintain population diversity, as the process converges towards a solution.

In this problem, two variables are used in optimization tool. The fitness of each selected individual is determined using the objective function that the resulting safety factors should be lower enough, and the fitness of all solutions is compared, while the chromosomeof large safety factors shall be deleted and minimum value considered. first the random search process is tabulated and observed the best value of Xc and Xcc. then we find the value of best fitness value by optimization tool which are also shown below in form of steps.

a) Random Search Value Of Xc and Xcc

Table 4 Population

Xc	Xcc	Safety factor
50	25	2.8403621
51	30	3.6349872
51	0	1.3126389
60	10	1.4857964
60	25	1.8965329
100	10	2.3468952
100	25	2.2895643
100	70	4.3576249

In this table 4, population is initialized and value is taken from between 50 to 100. After random search, minimum safety factor obtained is considered for individual selection.

Table 5 Selected individual

Xc	Xcc	Factor of safety
51	0	1.3126389
60	10	1.4857964
60	25	1.8965329

In this table 5, factor of safety of selected individual is categorized .this selected individual is used for crossover operation.

Table 6 CrossoverResults

Xc	Xcc	Factor of safety
50	-10	1.379309
51	25	2.657834
51	0	1.468438
60	-10	1.398653
60	-20	1.459763
100	0	2.746984
100	-10	2.32795
100	25	2.36438

In this table 6 crossover results are shown and safety factor is obtained.

Table 7Mutation Results

Xc	Xcc	Safety factor
50	0	1.31246
50	-10	1.42658
51	21	1.76984
51	0	1.32658
55	-10	1.35938
55	20	1.65932

After crossover operation, mutation takes place and factor of safety is obtained in table 7.

Table 8 Selected Individuals

Xc	Xcc	Safety factor
51	0	1.310964
51	-10	1.369253
60	0	1.385743
50	0	1.343297
55	0	1.312796
55	-10	1.359874

Now after crossover and mutation result, again population is selected for crossover and mutation operation. This process continues until minimum fitness factor is obtained which is shown in table 8

Table 9 CrossoverResults

Xc	Xcc	Safety factor
51	0	1.310964
51	-10	1.369253
60	0	1.385743
50	0	1.343297
55	0	1.312796
55	-10	1.359874

Again crossover operation takes place and results are shown in this table 9.

Table 10 Mutation Results

Xc	Xcc	Safety factor
52	0	1.31009
52	-10	1.35743
53	0	1.31220
53	-10	1.35943
57	0	1.34964
57	-10	1.36984
52	11	1.35065
52	25	2.50439
52	100	4.89643
52	-15	1.46836

Aftercrossover, mutation operation is performed and fitness factor is obtained shown in table 10.

Table 11 Selected Individual

Xc	Xcc	Safety factor
51	0	1.32298
50	0	1.33476
55	0	1.32468
52	0	1.31568
53	0	1.31984
57	0	1.35732
52	11	1.34953

In this table again individual is selected and factor of safety is obtained.

Table 12 Crossover Results

Xc	Xcc	Safety factor
52	11	1.31568
53	11	1.31984
55	11	1.32298
51	11	1.32468
50	11	1.33476

After selecting individual, crossover results are performed and factor of safety is obtained.

Table 13Mutation Results

Xc	Xcc	Safety factor
53	23	1.68463
53	0	1.31875
53	15	1.35954
52	-10	1.37593
60	-10	1.45928

After crossover results, population are increased through mutation and factor of safety is obtained.

T

able 14 Selected Individuals

Xc	Xcc	Safety factor
50	0	1.34684
50	0	1.33520
53	0	1.31246
55	0	1.36830
52	0	1.31487
52	10	1.31672
54	0	1.32432

Finally individual is selected and minimum factor of safety obtained is being considered.

b) InRandom Search, Best Fitness Value that is Factor of Safety Obtained by Optimization Tool.

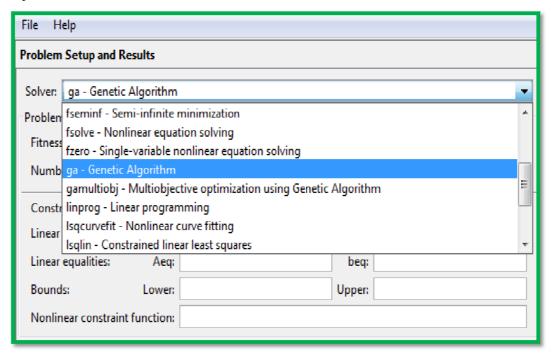


Figure 20 Problem setup in optimization tool

Step 1

In solver, different types of options are present

a) Semi Infinite Minimization

- b) Nonlinear Equation Solving
- c) Single Variable Non Linear Equation Solving

d) Genetic Algorithm

- e) Multiobjective Optimization Using Genetic Algorithm
- f) Linear Programming
- g) Nonlinear Curve Fitting
- h) Constrained Linear Least Squares
 Using Genetic Algorithm, we go to step 2.

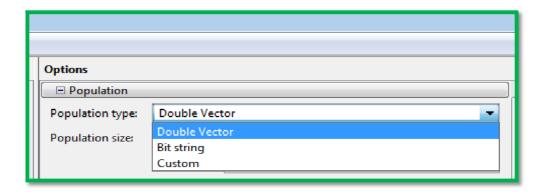


Figure 21 Population Type Selections.

Step 2

Three types of options are present for population type

a) Double vector

- b) Bit string
- c) Custom

Generally we use double vector for population type, then we go to creation function that is step3.

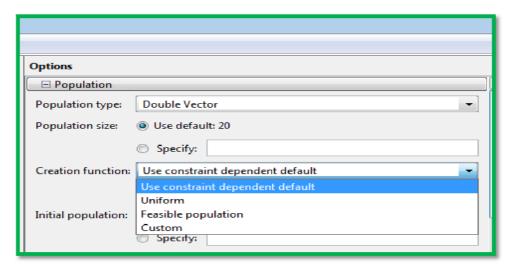


Figure 22 Creation Function

Step 3

In creation function, different options are available

a) Use constraint dependent default -

This option is used by default. Generally we create constraint dependent function in genetic algorithm.[19]

b) Uniform

Uniform creates a random initial population with a uniform distribution. If we assume uniform distribution for population then we will take uniform.[19]

c) Feasible population

Feasible population shows a random initial population that satisfies the limitation and linear dependencies.[19]

Custom

Custom provides own creation function, which must generate data of the type that is specified in Population type.[19]

We will use constraint dependent default for the above problem. Then go to next step

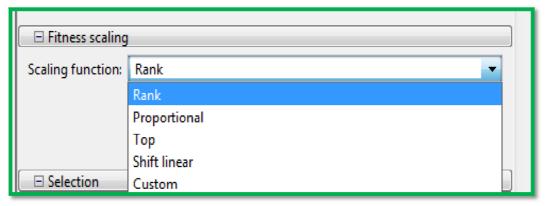


Figure 23Scaling function

Step 4

Scaling function specifies the function that performs the scaling. They are

a) Rank

It scales the raw scores based on the rank of each individual, rather than its score. The rank of an individual is its position in the sorted scores.[19] The rank of the fittest individual is 1, the next fittest is 2, and so on. Rank fitness scaling removes the effect of the spread of the raw scores.[19]

b) Proportional

It makes the expectation proportional to the raw fitness score. This strategy has weaknesses when raw scores are not in a "good" range.[19]

c) Top

It scales the individuals with the highest fitness values equally. If you select this option,

d) Shift linear

It scales the raw scores so that the expectation of the fittest individual is equal to a constant, which you can specify as Maximum survival rate, multiplied by the average.[19]

e) Custom

It enables to write own scaling function. Weuse rank for scaling function as it gives fittest individual. Then go to step 5.

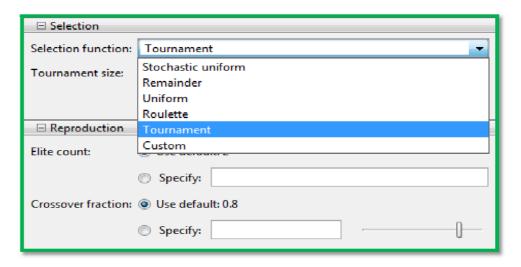


Figure 24Selection function

Step 5

The selection function chooses parents for the next generation based on their scaled values from the fitness scaling function.

- a) Stochastic Uniform— It exists in a line in which each parent corresponds to a section of the line of length proportional to its expectation.[19]. The algorithm moves along the line in steps of equal size, one step for each parent. At each step, the algorithm allocates a parent from the section it lands on. The first step is a uniform random number less than the step size.[19]
- **b) Remainder** It assigns parents deterministically from the integer part of each individual's scaled value.
- c) Uniform- It selects parents at random from a uniform distribution using the expectations and number of parents. This results in an undirected search.[19] Uniform selection is not a useful search strategy, but you can use it to test the genetic algorithm.
- **d) Shift linear-** It scales the raw scores so that the expectation of the fittest individual is equal to a constant, which you can specify as Maximum survival rate, multiplied by the average score.
- e) Roulette-It simulates a roulette wheel with the area of each segment proportional to its expectation. The algorithm then uses a random number to select one of the sections with a probability equal to its area.[19]

- f) Tournament— It selects each parent by choosing individuals at random, the number of which you can specify by Tournament size, and then choosing the best individual out of that set to be a parent.[19]
- **g)** Custom It enables to write your own selection function. Enter a function handle of the form @SelectFcn, where SelectFcn.m is a function file with syntax description.

Using tournament selection as we are searching solutions randomly. Now go to next step 6 for mutation function.

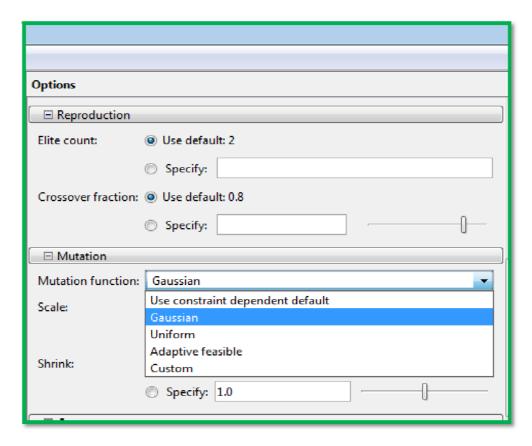


Figure 25Mutation Function

Step 6

Mutation functions make small random changes in the individuals in the population, which provide genetic diversity and enable the genetic algorithm to search a broader space.[20]

- a) Constraint dependent default chooses: It is a default selection for mutation function.
- b) Gaussian- It is used where there are no constraints.
- c) Adaptive feasible otherwise

using Gaussian as it adds a random number toeach vector entry of an individual. This random number is taken from a Gaussian distribution centered on zero. The standard deviation of this distribution can be controlled with two parameters. then go to step 7

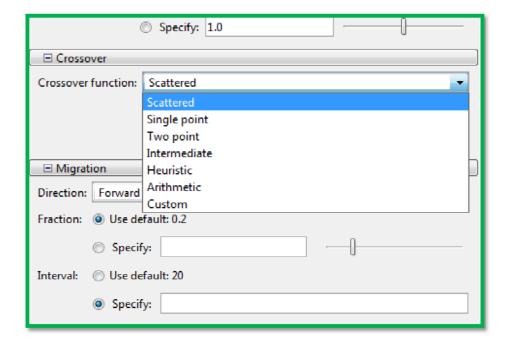


Figure 26Crossover Function

Step 7

Different type of options may be used for crossover function, they are

Scattered— It creates a random binary vector. It then selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child. For example:[20]

$$P_1 = [a b c d e f g h]$$

$$P_2 = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8]$$

Random crossover vector = $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Child =
$$[a b 3 4 e 6 7 8]$$

a) Single point - It chooses a random integer n between 1 and Number of variables, and selects the vector entries numbered less than or equal to n from the first parent, selects genes numbered greater than n from the second parent, and concentrates these entries to form the child. For example:

$$P_1 = [a b c d e f g h]$$

 $P_2 = [1 2 3 4 5 6 7 8]$

Random crossover point = 3

Child =
$$[a b c 4 5 6 7 8]$$

b) Two point– It selects two random integers m and n between 1 and Number of variables. The algorithm selects genes numbered less than or equal to m from the first parent, selects genes numbered from m+1 to n from the second parent, and selects genes numbered greater than n from the first parent. The algorithm then concatenates these genes to form a single gene. For example:[20]

$$P_1 = [a b c d e f g h]$$

 $P_2 = [1 2 3 4 5 6 7 8]$

Random crossover points = 3.6

Child =
$$[a b c 4 5 6 g h]$$

- c) Intermediate —It creates children by a random weighted average of the parents. Intermediate crossover is controlled by a single parameter Ratio.
- **d) Heuristic**—It creates children that randomly lie on the line containing the two parents, a small distance away from the parent with the better fitness value, in the direction away from the parent with the worse fitness value.
- **e) Arithmetic** It creates children that are a random arithmetic mean of two parents, uniformly on the line between the parents.
- f) Custom It enables you to write your own crossover function that satisfies any constraints specified.

using, scattered function for crossover because our problem id based on random search by using 2 variables, then 1 go to next step 8.

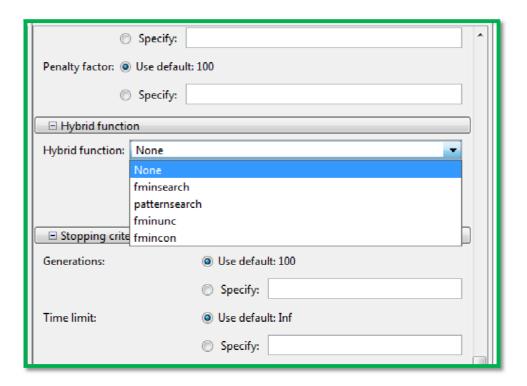


Figure 27Hybrid functions

Step 8

Hybrid Functions – It enables to specify another minimization function that runs after the genetic algorithm terminates. The choices available are:

- a) None
- b) fminsearch (unconstrained only)
- c) patternsearch (constrained or unconstrained)
- d) fminunc (unconstrained only)
- e) fmincon (constrained only)

Using none option because I have not used any function more except objective function, then go to step 9.

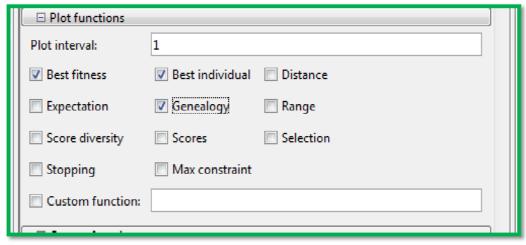


Figure 28 Plot Functions

Step 9 -

Then we select options for plotting graph. finally go to step 10

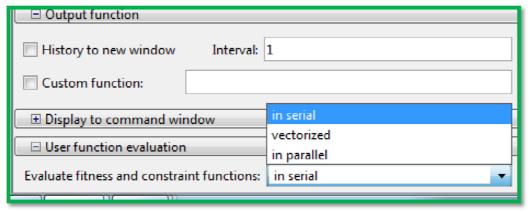


Figure 29 Fitness Function And Constraint Functions

Step 10

In last step, fitness function and constraints functions is kept in serial and then we run the optimization tool. finally we get the best fitness factor which is factor of safety.

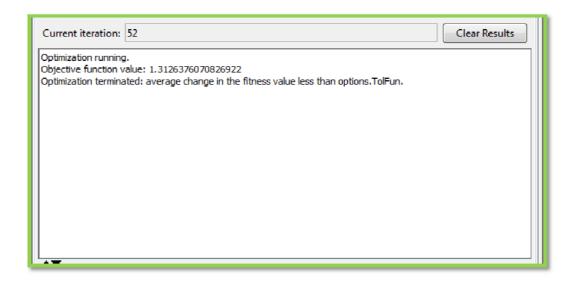


Figure-30 Result Obtained as Objective Function Value

4.6.2 Solution of Problem 1 by Conventional Method

a) By ordinary method of slices, (fellenius method)

Given that,

Height = 25 m,

Slope ratio = 1:2

 $\gamma = 20 \text{ KN/m}^3$

$$\Phi = 26.6^{\ 0}$$

Applying formula,

Factor of safety =
$$\frac{\sum [c'\Delta l + (W\cos\alpha - u\Delta l\cos\alpha * \cos\alpha)\tan\theta]}{\sum W\sin\alpha}$$

Where, $\alpha = \text{ angle of slope} = 26.56^{\circ}$

Safety Factor = 1.34

b) By Janbu's method

Factor of safety =
$$\frac{\sum (c'l + (P-ul)tan\Phi')sec\alpha}{\sum Wtan\alpha}$$

Where,

 $sec\alpha = 1.24$

 $tan\alpha = 0.4998$

Safety Factor = 1.33

4.7 STATEMENT OF PROBLEM 2

A soil slope is located in national highway NO. 107 K8+615-641. The height ofslope is 9m, the length is 26m, the angle is 760, the soil layer is alluvial sedimentsclay layer.[22]The soil parameters of this slope are obtained by laboratory test. The cohesion force c is 20kPa, and its standard deviation is 7.7kPa. The internal friction angle is 19⁰, and its standard deviation is 3.950.[22] The soil bulk density is 19kN/m3. Find out factor of safety.

4.8 SOLUTION OF PROBLEM 2

4.8.1 By Optimization Tool

This problem is solved by taking both 2 and 3 variables and observes the difference in value obtained as factor of safety. In this problem, the only difference is in taking the Selection function.

Selection function is Stochastic Uniformwhich lays out a line in which each parent corresponds to a section of the line of length proportional to its expectation. The algorithm moves along the line in steps of equal size, one step for each parent. At each step, the algorithm allocates a parent from the section it lands on. The first step is a uniform random number less than the step size.

This problem is solved by 2 variables and 3 variables keeping all parameters same.

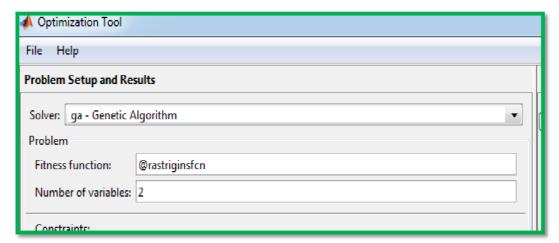


Figure 31 Problem setup in 2 variables

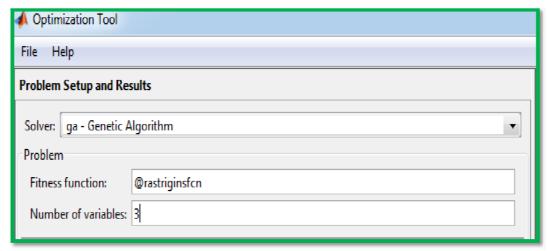


Figure 32Problem setup in 3 variables

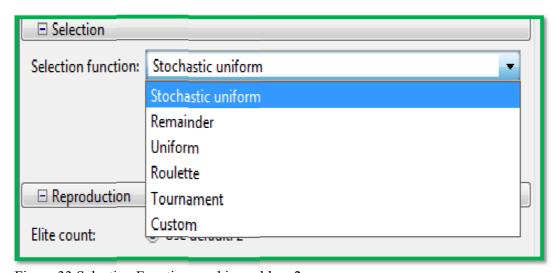


Figure 33 Selection Function used in problem 2

4.8.2 Solution of Problem 2 by Conventional Method

a) By Ordinary Method of slices, (Fellenius method)

Given that,

Ht of slope = 9m

Length =26 m,

c = 20 KPa

 $\Phi = 19^{0}$

 $\gamma = 19KN/m^3$,

Safety Factor =
$$\frac{\sum [c'\Delta l + (W\cos\alpha - u\Delta l\cos\alpha * \cos\alpha) tan\theta]}{\sum W\sin\alpha}$$

Safety Factor calculated = 1.1328

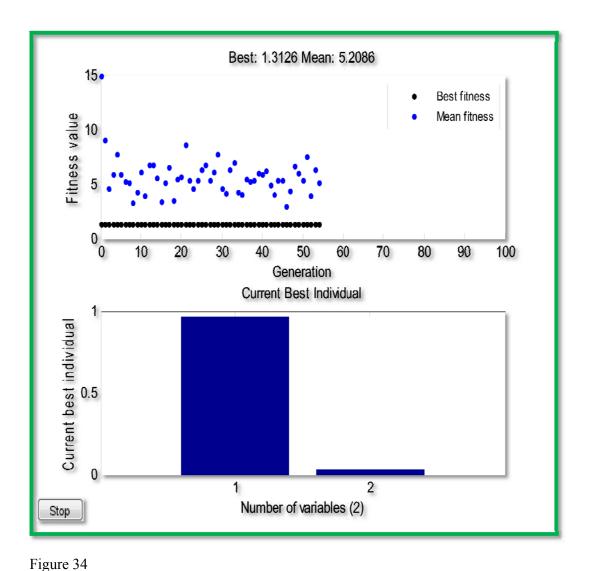
b) By Janbu's Method,

Safety Factor=
$$\frac{\sum (c'l + (P-ul)tan\Phi')sec\alpha}{\sum Wtan\alpha}$$

Safety Factor Calculated= 1.12628

CHAPTER 5 RESULTS AND ANALYSIS

5.1 RESULTS OF PROBLEM 1 SHOWN BY GRAPH



In this figure, best fitness factor for problem no 1 is shown 1.3126. In this figure two graphs are obtained by running the optimization tool. First one is the graph between fitness value and generation. Another one is the graph between no of variables and

current best individual.

5.2 RESULT OF PROBLEM 2 USING TWO VARIABLES

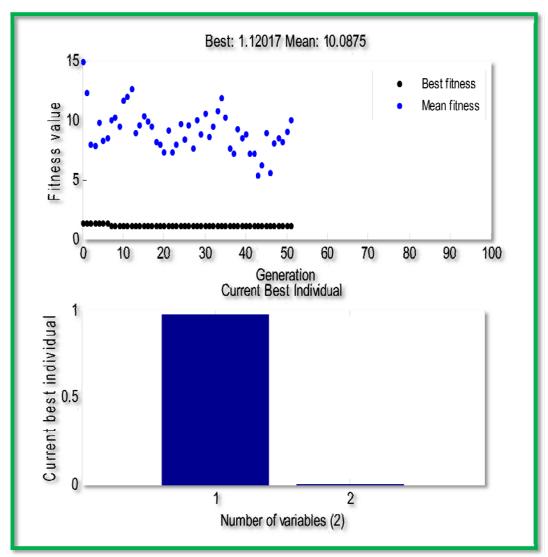


Figure 35

In this figure, safety factor is obtained for problem 2 by using two variables. Two graphs are shown in this figure. First one is the graph between fitness value and generation. Another one is the graph between no of variables and current best individual.

5.3 RESULT OF PROBLEM 2 USING THREE VARIABLES

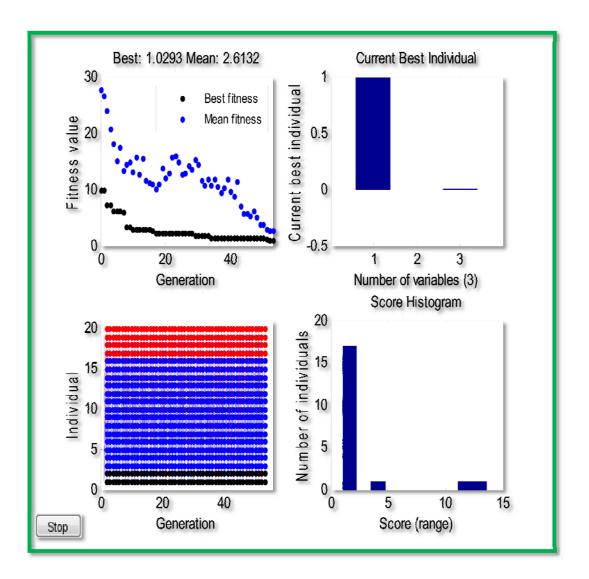


Figure 36
In this figure, safety factor is obtained for problem 2 by using three variables.
Fourgraphs are shown in this figure. First one is the graph between fitness value and generation. Second one is the graph between no of variables and current best individual. Third one is the graph between generation and individual. Last one is the graph between score and score histogram.

5.4 RESULT ANALYSIS OF PROBLEM 1

- a) In problem 1, first the problem 1 is solved by genetic algorithm optimization tool in which selection is taken as tournament that means the load or external forces are increasing tournamentally. Tournament selection means that the selection of each parent individually. in problem 1, failure surface is randomly searched through the population size and load is increased by crossover and mutation operation. In crossover function, scattered function is used because of random searching. In mutation function, Gaussian function is used because in given problem, no constraints have been assumed. Since problem 1 is two dimensional problems. That'swhy; I have used two independent variablesin optimization tool. Finally we obtain the results as safety factor for given slope which is approximately equal to the safety factor obtained.
- b) I have solved the problem 1 also by conventional method. In conventional method, I have solved the problems by two methods. One is method of slices and another one is Janbu's method. Janbu's method is more accurate than method of slices because in method of slices, many external forces has been ignored. That'swhy;Janbu's method gives more accurate result than fellenius method that is method of slices. Many other type of conventional methods are also available in which different type of failure surfaces has been assumed.
- c) Finally the result obtained by Optimization tool is slightly different from Conventional method.
 - From conventional method, factor of safety is obtained 1.34 and 1.33 by Ordinary method of Slices and Janbu's method respectively. That means there is the variation of 0.01 in factor of safety.
 - Whereas by Genetic Algorithm of optimization tool of random search, keeping two independent variables is found to be 1.3126.
 - as a whole , the variation in the value of factor of safety is apporixmately observed that 0.0274 and 0.0174.

5.5 RESULT ANALYSIS OF PROBLEM 2

- a) Like previous problem, I have solved problem 2 by using optimization tool. For solving problem in optimization tool, I have used selection function stochastic uniform. Other parameters are kept same like mutation and crossover function. I have used two and three variables for getting factor of safety in which their graphs is showing numerical variation of 0.1 obtaining graphs shows first best fitness factor and mean fitness factor. Also graph shows the relation between generation and individual.
- b) I have also solved the problem by conventional methods in which Janbu's method is giving more accurate result than Fellenius method.
- c) From conventional method, Factor of safety isobtained 1.13 and 1.12628 by Ordinary method of slices and Janbu's method respectively.
- d) From optimization tool of genetic algorithm, using two variables the factor of safety is equal to 1.12 but using 3 variables the safety factor is 1.0293. Hencethere is variation in factor of safety value is 0.0031 in two conventional methods that is Fellenius method and Janbu's method.
- e) There is variation in factor of safety by using two and three variables by optimization tool is 0.903. That means in two dimensional and three dimensional, factor of safety differs due to dimension.

CHAPTER 6 CONCLUSION AND FUTURE SCOPE

6.1 CONCLUSION

The present work is an effective method for analyzing slope stability problem and finding factor of safety. The conclusions are made after obtaining the results which are given below

- a) It is observed that Genetic Algorithm tool box is very effective method for finding Factor of safety. It works fast and gives immediate results in comparison to conventional method.
- b) It is observed that in problem 1, the variation in factor of safety using conventional method that is method of slices and Janbu's method is observed, the variation is approximately 0.01 that means Janbu's method is more accurate than method of slices. This variation may be observed due to ignoring some forces in method of slices.
- c) It is observed that there is variation in the value of factor of safety is approximately 0.0274 and 0.0174 between conventional methods and artificial intelligence method in problem 1.
- d) It is also observed that there is variation about 0.01 using two variables and about 0.11 using three variables by using conventional method and artificial intelligence method in problem 2.
- e) It is observed that there is variation in 0.903 in results by using two variables and three variables. This change cannot be observed by conventional method.
- f) In conventional method, only those factor of safety is observed of which soil's strength is given. But by genetic algorithm optimization tool, we can obtain Factor of Safety at each point with given range.

6.2 Future Scope

a) Due to absence of actual field data, I have not checked accuracy of safety factor obtained by artificial intelligence method. In future, we can check the accuracy of the safety factor by comparing the field data and obtaining data from optimization tool.

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