CHAPTER-4

TCSC Powerflow model:

Load flow studies are one of the most important aspects of power system planning and operation. The load flow gives us the sinusoidal steady state of the entire system – voltages, real and reactive power generated and absorbed and line losses. Since the load is a static quantity and it is the power that flows through transmission lines, the purists prefer to call this ***Power Flow studies*** rather than load flow studies. We shall however stick to the original nomenclature of load flow.

Through the load flow studies we can obtain the voltage magnitudes and angles at each bus in the steady state. This is rather important as the magnitudes of the bus voltages are required to be held within a specified limit. Once the bus voltage magnitudes and their angles are computed using the load flow, the real and reactive power flow through each line can be computed. Also based on the difference between power flow in the sending and receiving ends, the losses in a particular line can also be computed. Furthermore, from the line flow we can also determine the over and under load conditions.

The steady state power and reactive powers supplied by a bus in a power network are expressed in terms of nonlinear algebraic equations. We therefore would require iterative methods for solving these equations. In this chapter we shall discuss two of the load flow methods. We shall also delineate how to interpret the load flow results.

4.1 REAL AND REACTIVE POWER INJECTED IN A BUS

For the formulation of the real and reactive power entering a bus, we need to define the following quantities. Let the voltage at the i^{th} bus be denoted by

$$V_i = |V_i|\angle\delta_i = |V_i|(\cos\delta_i + j\sin\delta_i) \qquad (4.1)$$

Also let us define the self admittance at bus-*i* as

$$Y_{ii} = |Y_{ii}|\angle\theta_{ii} = |Y_{ii}|(\cos\theta_{ii} + j\sin\theta_{ii}) = G_{ii} + jB_{ii} \tag{4.2}$$

Similarly the mutual admittance between the buses *i* and *j* can be written as

$$Y_{ij} = |Y_{ij}|\angle\theta_{ij} = |Y_{ij}|(\cos\theta_{ij} + j\sin\theta_{ij}) = G_{ij} + jB_{ij} \tag{4.3}$$

Let the power system contains a total number of *n* buses. The current injected at bus-*i* is given as

$$\begin{aligned} I_i &= Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n \\ &= \sum_{k=1}^{n} Y_{ik}V_k \end{aligned} \tag{4.4}$$

It is to be noted we shall assume the current entering a bus to be positive and that leaving the bus to be negative. As a consequence the active power and reactive power entering a bus will also be assumed to be positive. The complex power at bus-*i* is then given by

$$\begin{aligned} P_i - jQ_i &= V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik}V_k \\ &= |V_i|(\cos\delta_i - j\sin\delta_i)\sum_{k=1}^{n}|Y_{ik}V_k|(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k) \\ &= \sum_{k=1}^{n}|Y_{ik}V_iV_k|(\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k) \end{aligned} \tag{4.5}$$

Note that

$$\begin{aligned} &(\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k) \\ &\quad = (\cos\delta_i - j\sin\delta_i)[\cos(\theta_{ik} + \delta_k) + j\sin(\theta_{ik} + \delta_k)] \\ &\quad = \cos(\theta_{ik} + \delta_k - \delta_i) + j\sin(\theta_{ik} + \delta_k - \delta_i) \end{aligned}$$

Therefore substituting in (4.5) we get the real and reactive power as

$$P_i = \sum_{k=1}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{4.6}$$

$$Q_i = -\sum_{k=1}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{4.7}$$

4.2 CLASSIFICATION OF BUSES

For load flow studies it is assumed that the loads are constant and they are defined by their real and reactive power consumption. It is further assumed that the generator terminal voltages are tightly regulated and therefore are constant. The main objective of the load flow is to find the voltage magnitude of each bus and its angle when the powers generated and loads are pre-specified. To facilitate this we classify the different buses of the power system as listed below.

1. ***Load Buses***: In these buses no generators are connected and hence the generated real power P_{Gi} and reactive power Q_{Gi} are taken as zero. The load drawn by these buses are defined by real power $-P_{Li}$ and reactive power $-Q_{Li}$ in which the negative sign accommodates for the power flowing out of the bus. This is why these buses are sometimes referred to as P-Q bus. The objective of the load flow is to find the bus voltage magnitude $|V_i|$ and its angle δ_i.

2. ***Voltage Controlled Buses***: These are the buses where generators are connected. Therefore the power generation in such buses is controlled through a prime mover while the terminal voltage is controlled through the generator excitation. Keeping the input power constant through turbine-governor control and keeping the bus voltage constant using automatic voltage regulator, we can specify constant P_{Gi} and $|V_i|$ for these buses. This is why such buses are also referred to as P-V buses. It is to be noted that the reactive power supplied by the generator Q_{Gi} depends on the system configuration and cannot be specified in advance. Furthermore we have to find the unknown angle δ_i of the bus voltage.

3. ***Slack or Swing Bus***: Usually this bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses. Since it is the angle difference between

two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important. However it sets the reference against which angles of all the other bus voltages are measured. For this reason the angle of this bus is usually chosen as 0°. Furthermore it is assumed that the magnitude of the voltage of this bus is known.

Now consider a typical load flow problem in which all the load demands are known. Even if the generation matches the sum total of these demands exactly, the mismatch between generation and load will persist because of the line I^2R losses. Since the I^2R loss of a line depends on the line current which, in turn, depends on the magnitudes and angles of voltages of the two buses connected to the line, it is rather difficult to estimate the loss without calculating the voltages and angles. For this reason a generator bus is usually chosen as the slack bus without specifying its real power. It is assumed that the generator connected to this bus will supply the balance of the real power required and the line losses.

There three methods for load flow studies mainly

#Gauss siedel method

Newton-Raphson method

Fast decoupled method.

In this thesis we are implementing and using the Newton raphson method to solve the network equations before and after incorporating TCSC.

4.3 NEWTON-RAPHSON METHOD

This method is used to solve a set of nonlinear equations through an iterative process. Let us consider that we have a set of n nonlinear equations of a total number of n variables x_1, x_2, …, x_n. Let these equations be given by

$$
\begin{aligned}
f_1(x_1,\cdots,x_n) &= \eta_1 \\
f_2(x_1,\cdots,x_n) &= \eta_2 \\
&\vdots \\
f_n(x_1,\cdots,x_n) &= \eta_n
\end{aligned}
\tag{4.8}
$$

where f_1, …, f_n are functions of the variables x_1, x_2, …, x_n. We can then define another set of functions g_1, …, g_n as given below

$$
\begin{aligned}
g_1(x_1,\cdots,x_n) &= f_1(x_1,\cdots,x_n) - \eta_1 = 0 \\
g_2(x_1,\cdots,x_n) &= f_2(x_1,\cdots,x_n) - \eta_2 = 0 \\
&\vdots \\
g_n(x_1,\cdots,x_n) &= f_n(x_1,\cdots,x_n) - \eta_n = 0
\end{aligned}
\tag{4.9}
$$

Let us assume that the initial estimates of the n variables are $x_1^{(0)}$, $x_2^{(0)}$, …, $x_n^{(0)}$. Let us add corrections $\Delta x_1^{(0)}$, $\Delta x_2^{(0)}$, …, $\Delta x_n^{(0)}$ to these variables such that we get the correct solution of these variables defined by

$$
\begin{aligned}
x_1^* &= x_1^{(0)} + \Delta x_1^{(0)} \\
x_2^* &= x_2^{(0)} + \Delta x_2^{(0)} \\
&\vdots \\
x_n^* &= x_n^{(0)} + \Delta x_n^{(0)}
\end{aligned}
\tag{4.10}
$$

The functions in (4.9) then can be written in terms of the variables given in (4.10) as

$$
g_k(x_1^*,\cdots,x_n^*) = g_k\left(x_1^{(0)} + \Delta x_1^{(0)},\cdots,x_n^{(0)} + \Delta x_n^{(0)}\right), \quad k = 1,\ldots,n \tag{4.11}
$$

We can then expand the above equation in Taylor's series around the nominal values of $x_1^{(0)}$, $x_2^{(0)}$, …, $x_n^{(0)}$. Neglecting the second and higher order terms of the series, the expansion of g_k, $k = 1$, …, n is given as

$$g_k\left(x_1^*,\cdots,x_n^*\right)=g_k\left(x_1^{(0)},\cdots,x_n^{(0)}\right)+\Delta x_1^{(0)}\left.\frac{\partial g_k}{\partial x_1}\right|^{(0)}+\Delta x_2^{(0)}\left.\frac{\partial g_k}{\partial x_2}\right|^{(0)}+\cdots+\Delta x_n^{(0)}\left.\frac{\partial g_k}{\partial x_n}\right|^{(0)} \quad (4.12)$$

where $\partial g_k/\partial x_i|^{(0)}$ is the partial derivative of g_k evaluated at $x_2^{(0)}, \ldots, x_n^{(0)}$.

Equation (4.12) can be written in vector-matrix form as

$$\begin{bmatrix} \partial g_1/\partial x_1 & \partial g_1/\partial x_2 & \cdots & \partial g_1/\partial x_n \\ \partial g_2/\partial x_1 & \partial g_2/\partial x_2 & \cdots & \partial g_2/\partial x_n \\ \vdots & \vdots & \ddots & \vdots \\ \partial g_n/\partial x_1 & \partial g_n/\partial x_2 & \cdots & \partial g_n/\partial x_n \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix} = \begin{bmatrix} 0-g_1\left(x_1^{(0)},\cdots,x_n^{(0)}\right) \\ 0-g_2\left(x_1^{(0)},\cdots,x_n^{(0)}\right) \\ \vdots \\ 0-g_n\left(x_1^{(0)},\cdots,x_n^{(0)}\right) \end{bmatrix} \quad (4.13)$$

The square matrix of partial derivatives is called the Jacobian matrix J with $J^{(0)}$ indicating that the matrix is evaluated for the initial values of $x_2^{(0)}$, …, $x_n^{(0)}$. We can then write the solution of (4.13) as

$$\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix} = \left[J^{(0)}\right]^{-1} \begin{bmatrix} \Delta g_1^{(0)} \\ \Delta g_2^{(0)} \\ \vdots \\ \Delta g_n^{(0)} \end{bmatrix} \quad (4.14)$$

Since the Taylor's series is truncated by neglecting the 2nd and higher order terms, we cannot expect to find the correct solution at the end of first iteration. We shall then have

$$\begin{aligned} x_1^{(1)} &= x_1^{(0)} + \Delta x_1^{(0)} \\ x_2^{(1)} &= x_2^{(0)} + \Delta x_2^{(0)} \\ &\vdots \\ x_n^{(1)} &= x_n^{(0)} + \Delta x_n^{(0)} \end{aligned} \quad (4.15)$$

These are then used to find $J^{(1)}$ and $\Delta g_k^{(1)}$, k = 1, …, n. We can then find $\Delta x_2^{(1)}$, …, $\Delta x_n^{(1)}$ from an equation like (4.14) and subsequently calculate $x_2^{(1)}$, …, $x_n^{(1)}$. The process continues till Δg_k, k = 1, …, n becomes less than a small quantity.

4.4 LOAD FLOW BY NEWTON-RAPHSON METHOD

Let us assume that an n-bus power system contains a total number of n_p P-Q buses while the number of P-V (generator) buses be n_g such that $n = n_p + n_g + 1$. Bus-1 is assumed to be the slack bus. The approach to Newton-Raphson load flow is similar to that of solving a system of nonlinear equations using the Newton-Raphson method: at each iteration we have to form a Jacobian matrix and solve for the corrections from an equation of the type given in (4.13). For the load flow problem, this equation is of the form

$$J\begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \\ \dfrac{\Delta|V_2|}{|V_2|} \\ \vdots \\ \dfrac{\Delta|V_{1+n_p}|}{|V_{1+n_p}|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \vdots \\ \Delta Q_{1+n_p} \end{bmatrix} \tag{4.16}$$

where the Jacobian matrix is divided into submatrices as

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \tag{4.17}$$

It can be seen that the size of the Jacobian matrix is $(n + n_p - 1) \times (n + n_p - 1)$. The dimensions of the submatrices are as follows:

J_{11}: $(n - 1) \times (n - 1)$, J_{12}: $(n - 1) \times n_p$, J_{21}: $n_p \times (n - 1)$ and J_{22}: $n_p \times n_p$

The submatrices are

$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} \tag{4.18}$$

$$J_{12} = \begin{bmatrix} |V_2| \frac{\partial P_2}{\partial |V_2|} & \cdots & |V_{1+n_p}| \frac{\partial P_2}{\partial |V_{1+n_p}|} \\ \vdots & \ddots & \vdots \\ |V_2| \frac{\partial P_n}{\partial |V_2|} & \cdots & |V_{1+n_p}| \frac{\partial P_n}{\partial |V_{1+n_p}|} \end{bmatrix} \tag{4.19}$$

$$J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_{1+n_p}}{\partial \delta_2} & \cdots & \frac{\partial Q_{1+n_p}}{\partial \delta_n} \end{bmatrix} \tag{4.20}$$

$$J_{22} = \begin{bmatrix} |V_2| \frac{\partial Q_2}{\partial |V_2|} & \cdots & |V_{1+n_p}| \frac{\partial Q_2}{\partial |V_{1+n_p}|} \\ \vdots & \ddots & \vdots \\ |V_2| \frac{\partial Q_{1+n_p}}{\partial |V_2|} & \cdots & |V_{1+n_p}| \frac{\partial Q_{1+n_p}}{\partial |V_{1+n_p}|} \end{bmatrix} \tag{4.21}$$

4.4.1 Load Flow Algorithm

The Newton-Raphson procedure is as follows:

Step-1: Choose the initial values of the voltage magnitudes $|V|^{(0)}$ of all n_p load buses and $n - 1$ angles $\delta^{(0)}$ of the voltages of all the buses except the slack bus.

Step-2: Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total $n - 1$ number of injected real power $P_{calc}^{(0)}$ and equal number of real power mismatch $\Delta P^{(0)}$.

Step-3: Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total n_p number of injected reactive power $Q_{calc}^{(0)}$ and equal number of reactive power mismatch $\Delta Q^{(0)}$.

Step-3: Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to formulate the Jacobian matrix $J^{(0)}$.

Step-4: Solve (4.16) for $\Delta\delta^{(0)}$ and $\Delta|V|^{(0)} \div |V|^{(0)}$.

Step-5: Obtain the updates from

$$\delta^{(1)} = \delta^{(0)} + \Delta\delta^{(0)} \tag{4.22}$$

$$|V|^{(1)} = |V|^{(0)}\left[1 + \frac{\Delta|V|^{(0)}}{|V|^{(0)}}\right] \tag{4.23}$$

Step-6: Check if all the mismatches are below a small number. Terminate the process if yes. Otherwise go back to step-1 to start the next iteration with the updates given by (4.22) and (4.23).

4.4.2 Formation of the Jacobian Matrix

We shall now discuss the formation of the submatrices of the Jacobian matrix. To do that we shall use the real and reactive power equations of (4.6) and (4.7). Let us rewrite them with the help of (4.2) as

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{4.24}$$

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{4.25}$$

A. Formation of J_{11}

Let us define J_{11} as

$$J_{11} = \begin{bmatrix} L_{22} & \cdots & L_{2n} \\ \vdots & \ddots & \vdots \\ L_{n2} & \cdots & L_{nn} \end{bmatrix} \tag{4.26}$$

It can be seen from (4.32) that M_{ik}'s are the partial derivatives of P_i with respect to δ_k. The derivative P_i (4.38) with respect to k for $i \neq k$ is given by

$$L_{ik} = \frac{\partial P_i}{\partial \delta_k} = -\left|Y_{ik} V_i V_k\right| \sin\left(\theta_{ik} + \delta_k - \delta_i\right), \quad i \neq k \tag{4.27}$$

Similarly the derivative P_i with respect to k for $i = k$ is given by

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} \left|Y_{ik} V_i V_k\right| \sin\left(\theta_{ik} + \delta_k - \delta_i\right)$$

Comparing the above equation with (4.25) we can write

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - \left|V_i\right|^2 B_{ii} \tag{4.28}$$

B. Formation of J_{21}

Let us define J_{21} as

$$J_{21} = \begin{bmatrix} M_{22} & \cdots & M_{2n} \\ \vdots & \ddots & \vdots \\ M_{n_p 2} & \cdots & M_{n_p n} \end{bmatrix} \tag{4.29}$$

From (4.20) it is evident that the elements of J_{21} are the partial derivative of Q with respect to δ. From (4.25) we can write

$$M_{ik} = \frac{\partial Q_i}{\partial \delta_k} = -|Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k \tag{4.30}$$

Similarly for $i = k$ we have

$$M_{ii} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = P_i - |V_i|^2 G_{ii} \tag{4.31}$$

The last equality of (4.31) is evident from (4.24).

C. Formation of J_{12}

Let us define J_{12} as

$$J_{12} = \begin{bmatrix} N_{22} & \cdots & N_{2n_p} \\ \vdots & \ddots & \vdots \\ N_{n2} & \cdots & N_{nn_p} \end{bmatrix} \tag{4.32}$$

As evident from (4.19), the elements of J_{21} involve the derivatives of real power P with respect to magnitude of bus voltage $|V|$. For $i \neq k$, we can write from (4.24)

$$N_{ik} = |V_k| \frac{\partial P_i}{\partial |V_k|} = |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = -M_{ik} \quad i \neq k \tag{4.33}$$

For $i = k$ we have

$$N_{ii} = |V_i| \frac{\partial P_i}{\partial |V_i|} = |V_i| \left[2|V_i| G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \right]$$

$$= 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = 2|V_i|^2 G_{ii} + M_{ii} \qquad (4.34)$$

D. Formation of J_{22}

For the formation of J_{22} let us define

$$J_{22} = \begin{bmatrix} O_{22} & \cdots & O_{2n_p} \\ \vdots & \ddots & \vdots \\ O_{n_p 2} & \cdots & O_{n_p n_p} \end{bmatrix} \qquad (4.35)$$

For $i \neq k$ we can write from (4.25)

$$O_{ik} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = -|V_i| |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = L_{ik}, \quad i \neq k \qquad (4.36)$$

Finally for $i = k$ we have

$$O_{ii} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = |V_i| \left[-2|V_i| B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \right]$$

$$= -2|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = -2|V_i|^2 B_{ii} - L_{ii} \qquad (4.37)$$

We therefore see that once the submatrices J_{11} and J_{21} are computed, the formation of the submatrices J_{12} and J_{22} is fairly straightforward. For large system this will result in considerable saving in the computation time.

4.5 CHARACTERISTICS OF NEWTON RAPHSON METHOD

With sparse programming techniques and optimally ordered factorization, the Newton method for solving load flow has become faster than other methods for large systems. The number iterations are virtually independent of system size (for a flat voltage start and with no automatic adjustments) due to the quadratic characteristic of convergence. Most systems are solved in 5-10 iterations without no acceleration factors being necessary. The accuracy of the power flow solution is limited only by the round-off error of the direct solution of the system of simultaneous equations.

With good programming, the time per iteration rises nearly linearly with the number of system buses N. One Iteration is equal to about seven Gauss-Seidel iterations. For a 500-bus system, the conventional Gauss-Siedel takes about 500 iterations and the speed advantage of the Newton method is then 15:1.Storage requirements of the Newton method are greater, however, but increase linearly with system size. It is, therefore, attractive for large systems.

The Newton method is very reliable in system solving, given good starting approximations. Heavily loaded systems with phase shifts upto 90' can be solved. The methods not troubled by ill-conditioned systems and the location of slack bus are not critical. Due to the quadratic convergence of bus voltages, high accuracy (near exact solution) is obtained in only a few iterations .This is important for the use of load flow in short circuit and stability studies. The method is really extended to include tap-changing transformers, variable constraints on bus voltages, and reactive, optimal power scheduling. Network modifications are easily made.

4.6 POWER FLOW ANALYSIS WITH TCSC

TCSC vary the electrical length of the compensated transmission line which enables it to be used to provide fast active power flow regulation. It also increases the stability margin of the system and has proved very effective in damping Sub Synchronous Resonance (SSR) and power oscillations. The simpler TCSC model exploits the concept of a variable series reactance. The series reactance is adjusted automatically, within limits, to satisfy a specified amount of active power flow through it. The more advanced model uses directly the TCSC reactance–firing angle characteristic, given in the form of a nonlinear relation as in equation [25, 28]. The TCSC firing angle is chosen to be the state variable in the Newton–Raphson power flow solution.

4.6.1 FIRING ANGLE MODEL OF TCSC

The equivalent circuit of TCSC for firing angle model is shown in Fig. 2, which consists of an anti-parallel connection of thyristors and the combination of inductor and capacitor. The thyristor combination is fired at various angles to obtain the required power flow in the line. The capacitor is used to supply the reactive power during heavy loaded conditions and the inductor will take care of the power during light loaded conditions. This is a series connected device which can have various roles in power systems such as sheduling power flow , reducing unsymmetrical components, reducing net loss, providing voltage support, reducing short circuit currents, mitigating SSR, enhancing transient stability. The fundamental frequency equivalent reactance X_{TCSC} of the TCSC module shown in Fig. 4.1 is given by

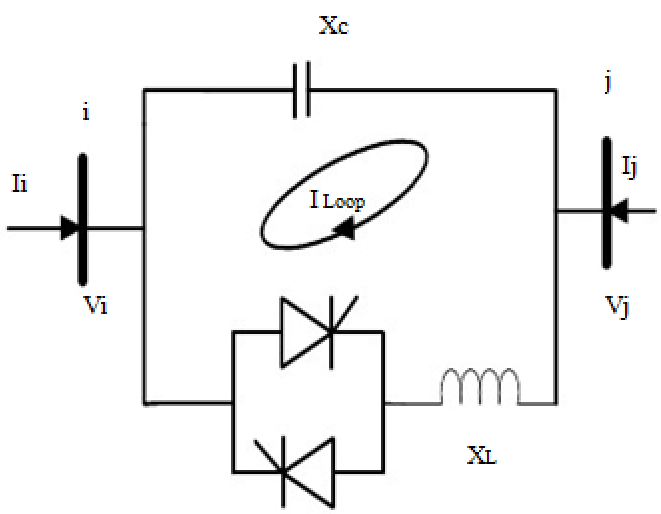

Fig. 4.1.Equivalent circuit for firing angle model

$$X_{TCSC} = -X_C + C1\{2(\pi-\alpha)+\sin[(\pi-\alpha)]\} - C2\cos^2(\pi-\alpha)\{K\tan[K(\pi-\alpha)]-\tan(\pi-\alpha)\} \quad (4.38)$$

Where the constants are

$$C_1 = \frac{X_C + X_{LC}}{\pi}$$

$$C_2 = \frac{4X_{LC}^2}{\pi X_L}$$

$$X_{LC} = \frac{X_C * X_L}{X_C - X_L}$$

$$K = \sqrt{\frac{X_C}{X_L}}$$

The above mentioned equation can also be written as shown below by eliminating the term X_L and Substituting the other values as usual.

$$X_{TCSC} = -X_C + \frac{K^2 X_C}{\pi(K^2-1)}[2(\pi-\alpha) + Sin2(\pi-\alpha)] + \frac{4K^2 X_C Cos^2(\pi-\alpha)}{\pi(K^2-1)^2}[K \tan\{K(\pi-\alpha)\} - Tan(\pi-\alpha)] \quad (4.39)$$

The Selection of K value is of critical importance and is governed by many factors, detailed analysis is given in [29], K value is chosen between 2.4-2.75 to avoid multiple resonance points and also operating region distribution in my project I have chosen 2.75,[13] gives a study of practical installations of the TCSC projects and also emphasizes in the selection of K value and how the performance varies by selection of K.

TCSC operating range of firing angle is in between 90^o – 180^o .The capacitive and inductive region of operation depends on the firing angle. It will operate in inductive region when the firing angle is from 90^o – 140^o, and in capacitive region for firing angle ranging from 140^o to 180^o. The maximum and minimum value of firing angles should be selected in such a way as to avoid the TCSC operating in high impedance region (at resonance) which results in high voltage drop across the TCSC.

The active and reactive power injections at bus "i" are shown in Fig. 2, where the TCSC is connected between bus "i" and bus "j" are given by

$$P_i = V_i V_j B_{ij} Sin(\theta_i - \theta_j) \quad (4.40)$$

$$Q_i = V_i^2 B_{ii} - V_i V_j B_{ij} Cos(\theta_i - \theta_j) \quad (4.41)$$

$$P_j = V_i V_j B_{ij} Sin(\theta_j - \theta_i) \quad (4.42)$$

$$Q_j = V_j^2 B_{jj} - V_i V_j B_{ij} Cos(\theta_j - \theta_i) \quad (4.43)$$

Where

V_i &θ_i are voltage and angle at i^{th} bus and

V_j &θ_j are voltage and angle at j^{th} bus

When TCSC controls the active power from bus “i” to bus “j”, at a specified value, the set of linearised power flow equations are given below

$$\begin{bmatrix} \Delta P_i \\ \Delta P_j \\ \Delta Q_i \\ \Delta Q_j \\ \Delta P_{ij} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial V_j} & \frac{\partial P_i}{\partial \alpha} \\ \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial V_j} & \frac{\partial P_j}{\partial \alpha} \\ \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial V_j} & \frac{\partial Q_i}{\partial \alpha} \\ \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial V_j} & \frac{\partial Q_j}{\partial \alpha} \\ \frac{\partial P_{ij}^{\alpha}}{\partial \theta_i} & \frac{\partial P_{ij}^{\alpha}}{\partial \theta_j} & \frac{\partial P_{ij}^{\alpha}}{\partial V_i} & \frac{\partial P_{ij}^{\alpha}}{\partial V_j} & \frac{\partial P_{ij}^{\alpha}}{\partial \alpha} \end{bmatrix} \begin{bmatrix} \Delta \theta_i \\ \Delta \theta_j \\ \frac{\Delta V_i}{V_i} \\ \frac{\Delta V_j}{V_j} \\ \Delta \alpha \end{bmatrix} \tag{4.44}$$

The Jacobian elements for the series reactance as a function of the firing angle α_{TCSC} is given by

$$\frac{\partial P_i}{\partial \alpha} = P_i B_{TCSC} \frac{\partial X_{TCSC}}{\partial \alpha} \tag{4.45}$$

$$\frac{\partial Q_i}{\partial \alpha} = Q_i B_{TCSC} \frac{\partial X_{TCSC}}{\partial \alpha} \tag{4.46}$$

$$\frac{\partial B_{TCSC}}{\partial \alpha} = B_{TCSC}^2 \frac{\partial X_{TCSC}}{\partial \alpha} \tag{4.47}$$

$$\frac{\partial X_{TCSC}}{\partial \alpha} = -2C_1[1 + \mathrm{Cos}(2\alpha)] + C_2\mathrm{Sin}(2\alpha)\{K\,\mathrm{Tan}[K(\pi - \alpha)]\} + C_2\{K^2 \frac{\mathrm{Cos}^2(\pi - \alpha)}{\mathrm{Cos}^2[K(\pi - \alpha)]} - 1\}$$

$$\Delta \alpha_{TCSC} = \alpha_{TCSC}^{(i-1)} - \alpha_{TCSC}^{(i)} \tag{4.48}$$

During each iteration firing angle values are modified using above equation, the Jacobian matrix is changed, and the corresponding power mismatches are calculated until it reaches the desired tolerance or the maximum number of iterations. From [25] it is clear that the reactance of the TCSC is a function of the firing angle, and the jacobian matrix is also made firing angle dependent.

4.6.2 Susceptance model of TCSC

In case of the variable impedance model or the Susceptance model the Susceptance is first found out after carrying out the load flows and then a separate Newton raphson loop is formulated and from that the firing angle is found out, here Newton Raphson method usage is justified as the relation between both the parameters listed above is non-linear and method with good convergence is Newton raphson method.

For inductive mode of operation

$$B_{ii} = B_{jj} = \frac{1}{X_{TCSC}}$$

$$B_{ij} = B_{ji} = -\frac{1}{X_{TCSC}}$$

For capacitive mode of operation

$$B_{ii} = B_{jj} = -\frac{1}{X_{TCSC}}$$

$$B_{ij} = B_{ji} = \frac{1}{X_{TCSC}}$$

Where ‘i’ and ‘j’ are bus numbers.