CHAPTER 1

INTRODUCTION

1.1Chaos Theory

Chaos theory is the study of non linear dynamics. In mathematics chaos theory describes the behaviour of a certain dynamical systems i.e. system whose state evolves with time – that may exhibit dynamics that are highly sensitive to initial condition.

The two main components of the chaos theory are the following ideas-

- No matter how complex may be the system it will always depend on an underlying order.
- A very simple system can also cause very complex behaviour.

From the above 2 ideas, the second idea means the dependence on initial state when some sensitive situation or event occur. The second idea is the principle of chaos theory.

Edward Lorenz, a meteorologist who was performing some experiment to predict weather condition. While running some equations on a computer, he got a particular string of numbers. Again when he tried to execute the same process he rounded off the numbers to three decimal places (earlier the numbers were upto six decimal places), the result he got was extremely different. According to all scientific expectation of that time, the resulting sequence should not had been radically different because measurement to three decimal places was considered to be fairly precise based on the outcomes of this experiment Edward concluded that very minute deviation from initial condition(which is beyond human accountability) made predictions of past or future outcomes impossible. This is considered to be as the first experimental proof of chaos theory that was inadvertently given by Edward Lorenz in 1960.

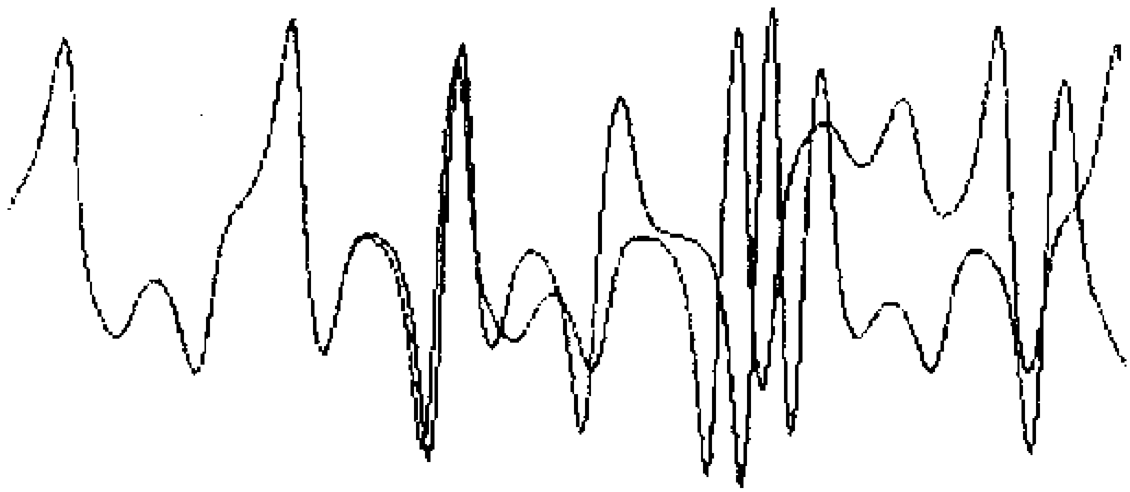


Fig1.1- Lorenz's experiment (the difference between the starting values of these curves is only .000127)

From this idea Lorenz stated that it is impossible to predict the weather accurately. However, this discovery led to Edward Lorenz to other aspects of what eventually called chaos theory.

Lorenz started to look for a simpler system that had sensitive dependence on initial condition. His first discovery had 12 equations and he wanted a much more simple version that still have this attribute. He took the equation of **convection** and stripped them down making them unrealistically simple. The system no longer had anything to do with the convection, but did have sensitive dependence on initial condition and there were only three equations this time. Later it was discovered that his equations precisely described a water wheel.

The equations for this system also seemed to give rise to entirely random behavior. However, when he graphed it, a surprising thing happened. The output always stayed on a curve, **a double spiral.** There were only two kinds of order previously known-

- A steady state, in which the variable never change
- A periodic behaviour, in which the system goes into a loop, repeating itself indefinitely.

Lorenz's equations were definitely ordered i.e. they always followed a spiral. They never settled down to a single point but since they never repeated the same thing, they weren't periodic either. He called the image he got when he graphed the equations, the Lorenz attractor.

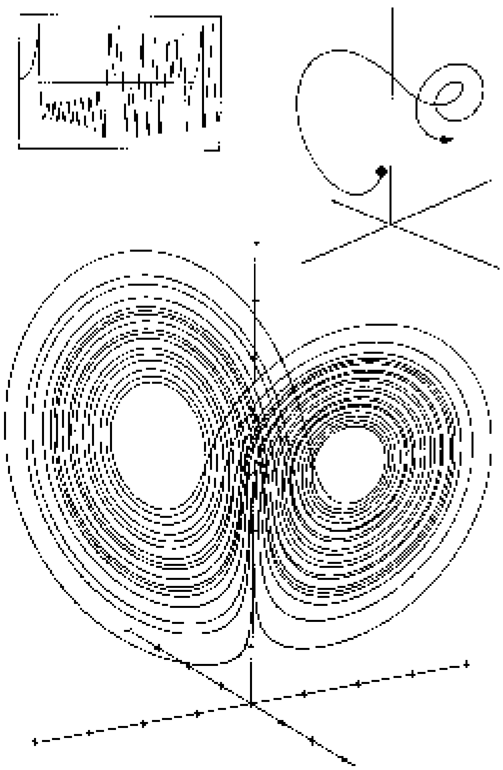


Fig1.2- the Lorenz's attractor: the pattern that Lorenz obtain after plotting the values he got from equations.

Years before this experimental proof of chaos theory, the mathematical evidence of chaos theory was given by H. Poincare in 1905. Poincare observed that in astronomical systems that consist of three or more astronomical bodies, very tiny errors in measurement would result in unpredictable result, far beyond what you expect mathematically. Poincare proved it mathematically that even if the initial set of measurement could be made a million times more precise.

1.2 Features Of Chaotic Systems

Chaotic systems are not random. Although, they may appear to be. They have some simple defining features-

- Chaotic systems are deterministic. This means that they have something determining their behaviour.

- Chaotic systems are very sensitive to initial condition. A very slight change in the starting point can lead to enormously different outcomes. This makes the system fairly unpredictable.

- Chaotic system appears to be disorderly, even random. But they are not. Beneath the random behaviour is a sense of order and pattern. Truly random systems are not chaotic. The orderly systems predicted by classical physics are the exceptions.

Chaos is characterized by a stretching and folding mechanism [1]. Nearby trajectories of a dynamical system are repeatedly pulled apart exponentially and folded back together. Since the discovery of Chua's circuit, it is always served as the main prototype circuit for studying chaos in electronic systems.

1.3 Chua's Circuit- An Overview

Chua's circuit is the simplest autonomous circuit that exhibits chaos. Chua's circuit was invented by Leon CHUA IN 1983. While on a visit to Japan, having witnessed a futile attempt at producing chaos in an electrical analog of Lorenz's equation [1]. Leon Chua was prompted to develop a chaotic electronic circuit. He realized that chaos could be produced in piecewise linear circuit if it possessed at least two unstable equilibrium points- one to provide stretching and the other to fold trajectories. With this insight, he systematically identified those third order piecewise linear circuits containing a single voltage controlled non linear resistor that could produce chaos. Specifying that the driving point characteristics of the voltage controlled non linear resistor N_R should be chosen to yield at least two unstable points, he invented Chua's circuit.

Soon after its conception, the rich dynamical behaviour of Chua's circuit was confirmed by computer simulation[2] and experiment[3]. Since then, there has been an intensive effort to understand every aspect of the dynamics of the circuit with a view to develop it as a paradigm for learning, understanding and teaching about non linear dynamics and chaos.

While differential equations and mechanical system provide convenient frameworks in which to examine bifurcation and chaos, electronic circuits are unique, in being easy to build, easy to measure and easy to model. Furthermore they operate in real time, and parameter values are easily adjusted. Just as the linear parallel RLC resonant circuits the simplest paradigm for understanding periodic steady state phenomena in linear circuits, so Chua's circuit presents an attractive paradigm for studying non periodic phenomena in nonlinear circuits. The importance of chua circuit and its relative is that they can exhibit every type of bifurcation and attractor that has been reported till date in third order continuous time dynamical systems. While exhibiting a rich variety of complex dynamical behavior, the circuit is simple enough to be constructed and modelled by using standard electronic parts and simulators.

Moreover, this remarkable circuit is the only physical system for which the presence of chaos has been proven mathematically.

1.4 Memristor Theory

The memristor's story starts nearly four decades ago with a flash of insight by IEEE fellow and non linear circuit pioneer Leon O Chua. Examining the relationship between charge and flux in resistors capacitors and inductors in a 1971 paper[34], Chua postulated the existence of a fourth element called the memory resistor. Such a device, he figured, would provide a similar relationship between magnetic flux and charge that a resistor gives between voltage and current. In practice that would mean it acted like a resistor whose value could vary according to the current passing through it and which would remember that value even after the current disappeared.

This hypothetical device was mostly written off as a mathematical dalliance. Thirty years later, HP senior fellow Stanley Williams and his group was working on a molecular electronics when they started to notice strange behavior in their devices. Then his HP collaborator Greg Snider rediscovered Chua's work from 1971. Williams spend several years reading and rereading Chua's papers. Then William realized their molecular device was really memristor. In April 2008, almost thirty seven years later from the memristor theory given by Chua, Hewlett Packard Scientists ,working at its laboratories in Palo Alto California reported the realization of a new nano-metre scale electric switch which remembers whether it is on or off after its power is turned off[35].Memristor created in HP labs based on a thin film of titanium di oxide part of which is doped to be missing some oxygen atoms. Researchers believe that memristor might become an important tool for constructing non volatile computer memory which is not lost even if the power goes off or for keeping the computer industry on pace to satisfy Moore's law. i.e. the exponential growth in processing power every 18 months.

The reason that the memristor is radically different from the other fundamental circuit element is that unlike others circuit elements ,it carries a memory of the past. When you turn off the voltage to the circuit, the memristor still remembers how much was applied before and for how long. That's an effect that cannot be duplicated by any other circuit combination.

1.5 Types of memristor

1. Molecular and Ionic Thin Film Memristive Systems
a. Titanium dioxide memristors
b. Polymeric (ionic) memristors
c. Manganite memristive systems
d. Resonant-tunneling diode memristors

2. Spin Based and Magnetic memristive systems
a. Spintronic Memristors
b. Spin Torque Transfer (STT) MRAM
3. 3-terminal memristors

1.6 Outline Of Thesis

In chapter 1 the introduction of chaos theory, chua's circuit and memristor theory has been discussed in detail.

In chapter 2, Literature review of project study about the research papers which are publish in different journal and conference.

In chapter 3, chua's circuit and its differential equations have been explained and v-I characteristics of chua's circuit is discussed.

In chapter 4, memdevices, memristance driven chua's circuit and canonical based chua's oscillator has been elaborated.

In chapter 5, simulation results and discussion on those results has been given .

Chapter 6 comprises of conclusion, future scope of project and references

CHAPTER-2

LITERATURE REVIEW

2.1 About Chua's Oscillator Circuit

Initially the theory of non linear systems was built on the foundations of the Poincare(1878-1900), Lyapnov(1893) and Birkhoff(1908-1944) results. But, the chaotic nature of chua's circuit was observed by Matsumoto in 1983 using computer simulation[4], following the instructions of chua who had invented this circuit and had explained its operating principles to Matsumoto moments before he was rushed to the hospital for a major surgery and who did not participated in the early phases of this research. In acknowledging his subsidiary role as a computer programmer, Matsumoto named this circuit Chua's circuit[4][5].

The first experimental chua's circuit which confirms the presence of chaos was due to Zhong and Ayrom in 1984[2][6]. A second experimental circuit was reported by Matsumoto shortly after[7] and was designed by Tokunaga who is also responsible for obtaining all of the experimental results presented in that paper. The global bifurcation landscape of chua's circuit[9] was obtained by Komuro and a team of students of Matsumoto. The colorful bifurcation landscape in this paper[9] was drawn by a professional artist.

The first rigorous proof of the chaotic nature of chua's circuit was given in[10], where the authors proved that there exists some parameters (α ,β) such that the chua's circuit satisfies Shil'nikov's theorem and therefore has infinitely many horseshoe maps[11]. Although the authors are [5] listed in alphabetical order as a compromise to Matsumoto's tradition of ordering his name first in earlier publications on chua's circuit, the rigorous proof of the main theorem is due to Komuro. However since the limiting cantor set from a horseshoe map is not an attractor, this result does not imply that the double scroll chua's attractor is directly related to the chaotic phenomenon associated with the Horseshoe map. This unsatisfactory situation has now been resolved by a recent proof that a 2-D geometrical model of chua's circuit give rise to **a double horse shoe map**,which generates strange attractors[12]. Another milestone was achieved in 1990 when a canonical circuit was discovered which is qualitative equivalent to a 21-parameter family C of continuous odd-symmetric piecewise linear vector fields[13].

Over the past two decades chaotic oscillators[14-16] have received increasing interest as a useful tool not only for investigation of non linear phenomenon, bifurcation and chaos but also for a variety of applications such as synchronizations, control[17] and chaos based communication systems[18]. Chua's circuit[16] is one of the best known chaotic oscillators using a piecewise-linear nonlinearity known as a 'chua diode'. Several techniques including the 'cubic' nonlinearity[19] and 'cubic like' non linearity[20,21]have alternately replaced the chua diode.

2.2 About Memristor

The concept of resistor having memory existed almost a decade before Leon Chua's publication on the memristor in 1971. In 1960, Prof. Bernard Widraw of Stanford University discovered a new circuit element named the 'memristor'. The resistance of this device was controlled by charge. Memristor was a three terminal device for which the conductance between two of the terminals was controlled by the time integral of the current into the third terminal.

In 1971, Chua mathematically introduced[22] a fourth element giving the relation between charge and flux. Again in 1976, Leon Chua along with Sung Mo Kang published a paper[23], generalizing the theory of memristors and memristive systems . Fifteen years later after this discovery of generalizing theory of memristor system was proposed, another paper[24] demonstrating a tungsten-oxide variable resistance device, which was reprogrammable. It was not clear whether the memristor device described has any relation with Chua's memristor[25]. In 1994, Buot and Rajgopal published an article entitled 'Binary information storage at zero bias in quantum-well diodes'[26]. In 2000 Beck of IBM's Zurich Research Laboratory, described reproducible resistance switching effects in thin oxide films[27]. One year later, researchers in the Space Vaccum Epitaxy center of the University of Houstan, presented results[28] during a non volatile memory conference held in San Diego, California achieve presented the importance of oxide bilayers to achieve high-to-low resistance ratio.

Apart from the devices mentioned above, some other devices were also developed between 1994 to 2008 that have same properties as memristor had, but only the HP scientists were successful in finding a link between their work and the one established by Chua[29]. Victor Erokhin and M.P. Fontana claimed to have developed a polymeric memristor[30] before the titanium-dioxide memristor developed by Stanley and group.
In 2008, J Joshua Yang, Matthew D. Pickett and R. Stanley Williams published an article[31] demonstrating the mechanism in nano devices and memristive switching behavior. In January 2009 Sung Hyun Jo, Kuk-Hwan Kim and Wei Lu of University of Michigan published an article[32] describing an amorphous silicon based memristive material capable of being integrated with CMOS devices. In june 2009, scientists at NIST[33] reported that they have fabricated non volatile memory using a flexible memristor that is both low power and inexpensive.

CHAPTER-3

CHUA'S CIRCUIT

3.1 General

Although Linear system theory provides inadequate characterization of sustained oscillation but it is still necessary to investigate these lower order systems with a view to classify behaviours and simplify models of higher order systems. It is mainly for this reason that the RLC resonant circuit is the established paradigm for understanding simple linear oscillatory behavior. Parallel RLC circuit is the system of lowest order that can model onset of oscillation in a dynamical system.

Consider the familiar parallel tuned RLC resonant circuit. It consists of two linear lossless passive energy storage elements that are positive (a linear inductor L and a linear capacitor C_2) and a linear resistor R with conductance G=1/R.

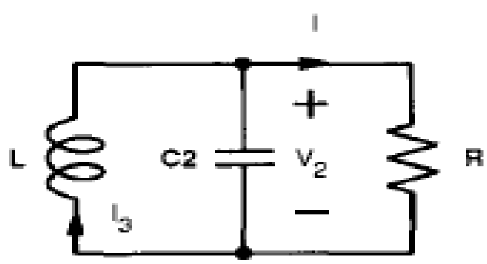


Fig3.1- linear parallel RLC resonant circuit

3.2 Qualitative Description Of RLC Model

Now, assuming that the current in the inductor at time t=0 is I_{30} and the capacitor is initially charged to a voltage V_{20}. Therefore, the total energy stored in the magnetic field of inductor and the electric field of the capacitor is given by-

$$\frac{1}{2}LI_{30}^2 + \frac{1}{2}C_2V_{20}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

When t tends to increase gradually from zero i.e. t>0, three cases arises-

- If G is positive, then resistor is said to be dissipative. The energy initially stored in the capacitor and inductor is dissipated as heat in the resistor as magnetic and electric fields collapse. Both the field approaches to zero in the form of exponentially decaying sinusoids.

- If G is negative, the resistor has negative dissipation i.e. it supplies energy to the rest of the circuit. Here the energy stored in the circuit increases with time. Both the fields have exponentially growing envelopes.

- If G is identically equal to zero, the circuit is said to be undamped. The energy that was initially stored in the capacitor and inductor cannot be dissipated but simply oscillates back and forth between these two elements. Both the fields are sinusoidal.

  The graphs of the above three cases are shown below-

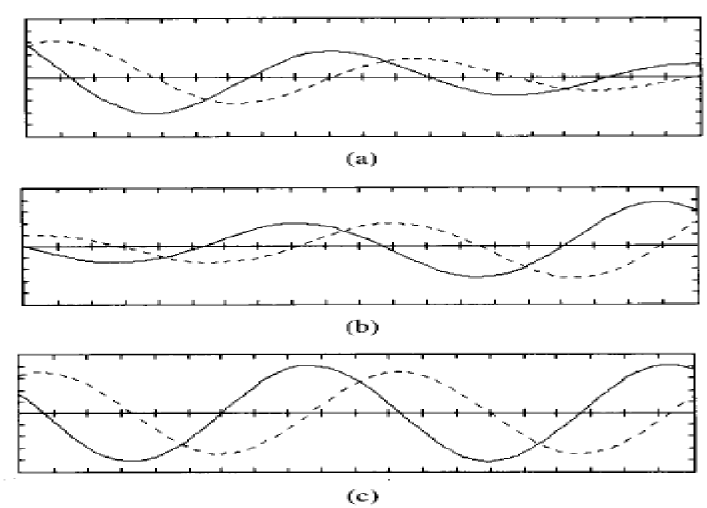


Fig3.2- the voltage and current waveforms for linear parallel RLC circuit with (a) positive damping (underdamped) (b) negative damping (underdamped) (c) zero damping(undamped).

The linear RLC circuit of fig-3.1 is characterized by a pair of ordinary differential equation and an initial state. Choosing V_2 and I_L as state variables.

$$\frac{dI_3}{dt} = -\frac{1}{L}V_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.2)$$

and

$$\frac{dV_2}{dt} = -\frac{1}{C_2}I_3 - \frac{1}{C_2}I$$

$$= -\frac{1}{C_2}I_3 - \frac{G}{C_2}V_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.3)$$

We know from experience that most real world oscillators are insensitive to small perturbations; they are structurally stable. Therefore, a linear RLC circuit provides a convenient model for analysis purpose, it represents a poor model of reality. A real oscillator must possess a non linearity to control the amplitude of oscillation.

3.3 Driving Point Characteristics of Non Linear Resistor N_R

In this section, we add a non linearity to the resistive part of the circuit in order to obtain sustained periodic oscillations. This we can done through piecewise linear modeling. Therefore we modify the parallel RLC circuit by placing in parallel with R a piecewise linear non linear resistor N_R as shown in fig below

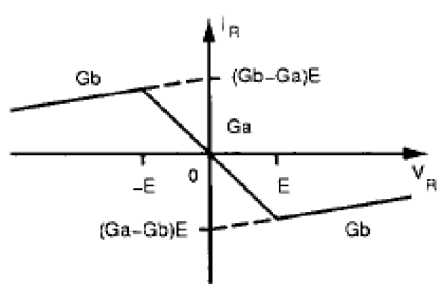


Fig3.3- driving point characteristic

The equations for the above circuit is given below. here for the following equations, we have values of $E>0$, $G_b>0$ and $G_a<0$

$I_R=f(V_R)=$

$$
\begin{cases}
G_bV_R+(G_b-G_a)E & \text{if } V_R< -E \\
G_aV_R & \text{if } -E<V_R<E \\
G_bV_R+(G_a-G_b)E & \text{if } V_R>E
\end{cases}
\quad \text{.....................................(3.4)}
$$

We can further simplify the above circuit by combining the two resistive elements R and N_R into a single non linear resistor N'_R that has the same driving point characteristics as the one shown in fig3.3-

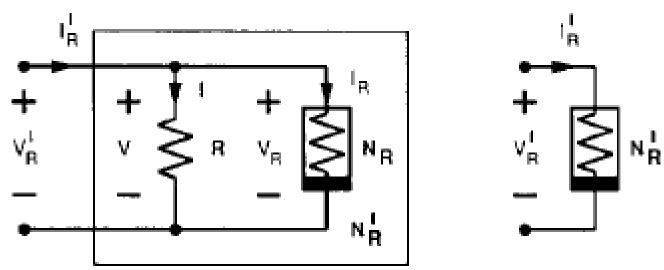


Fig-3.4 parallel combination of R and N_R to give the resultant as N'_R

Although we have the same driving point characteristics for N'_R, as we have for N_R, but still we can obtain them analytically by applying Kirchoff’s laws. Considering fig-3.4 , we get $V'_R=V=V_R$ and $I'_R=I+I_R$. Thus,

$$I'_R=GV'_R+f(V'_R)$$

$$=f'(V'_R)=$$

$$
\begin{cases}
G'_bV'_R+(G_b-G_a)E & \text{if } V'_R< -E \\
G'_aV'_R & \text{if } -E<V'_R<E \\
G'_bV'_R+(G_a-G_b)E & \text{if } V'_R>E
\end{cases}
\quad \text{..........................(3.5)}
$$

Where G'a=(G+Ga) and G'b=(G+Gb)

To determine the driving point characteristics of N'R graphically, we can add the characteristics of R and N'R vertically and for every value of V'R add the corresponding values of I and I'R. The entire procedure is explained graphically, with two special cases.

(i) When G'a<0

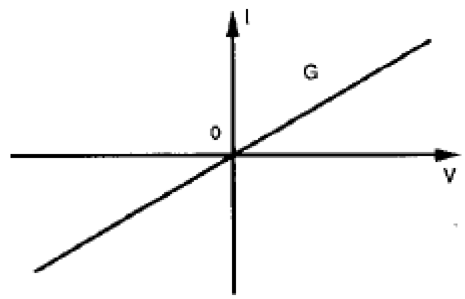


Fig3.5- characteristic of resistor R

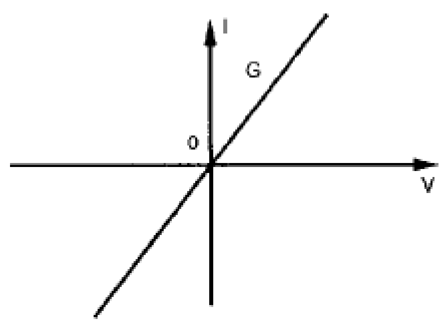


Fig3.6- characteristic of non linear resistor NR

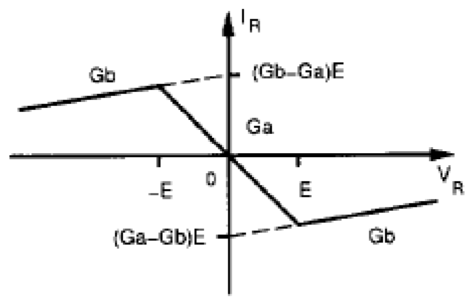


Fig-3.7 combined characteristic of R and N_R giving resultant characteristic of N'R. Non monotone V-I characteristic is obtained when G'a<0

(ii) When G'a>0

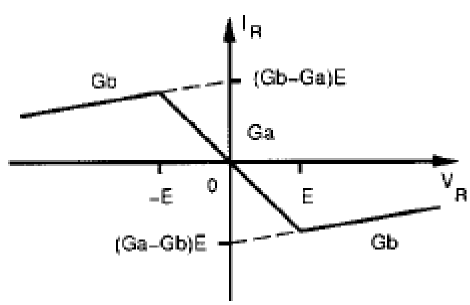


Fig-3.8 characteristic of resistor R

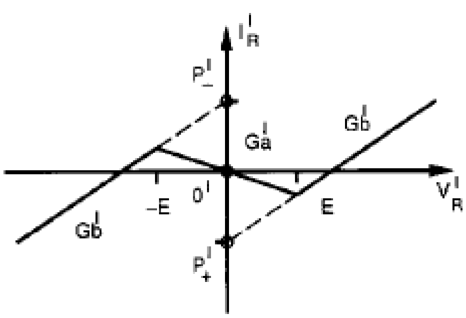


Fig-3.9 characteristic of non linear resistor NR

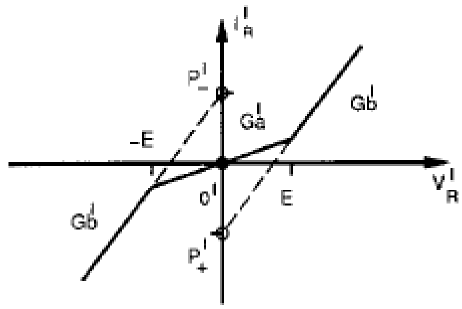


Fig-3.10 combined characteristic of R and N_R giving resultant characteristic of N'_R. Monotone v-i characteristic is obtained when Ga>0

3.4 Chua's Circuit Description

Chua's circuit is one of the most popular nonlinear dynamical system used to investigate the dynamic behavior in all possible states: Periodic, quasi-periodic and chaotic. It is very simple electronic circuit capable to exhibit a rich dynamical behavior. It was introduced in 1987 by Leon O. Chua ,since then, the ease of construction of the circuit has made it a natural real-world example of a chaotic system. It was the first physical implementation of chaos which has been rigorously proved . Any autonomous circuit made from standard components (resistors, capacitors, inductors) must satisfy three criteria in order to show chaotic behaviour :-

- at least one non linear element
- one or more locally active resistor
- three or more energy storage elements

Chua's circuit is the simplest electronic circuit meeting these criteria . This circuit has been studied extensively since its initial proposal, and serves as a test platform for many research areas that involve bifurcation processes including chaos. The robustness, ease of implementation, and its nonlinear behaviour contribute to the success between theoretical and experimental researches of chaos theory.

$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - f(v_1)$$

$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i_L \qquad \text{...(3.6)}$$

$$L \frac{di_L}{dt} = -v_2 - R_o i_L$$

The equations shown above are differential equations of chua's circuit

In the figure shown below it is shown how chua circuit looks like. Where C_1 and C_2 are the parameters of the conductors. R is the changeable resistance, R_0 is the internal resistance of the coil and L is the constant value of the coil.

N_R is the value of the negative resistance. The measured values are v_1 , v_2 and i_L. These values can be measured, which will give experimental results. But they can also theoretically be calculated by the differential equations above.

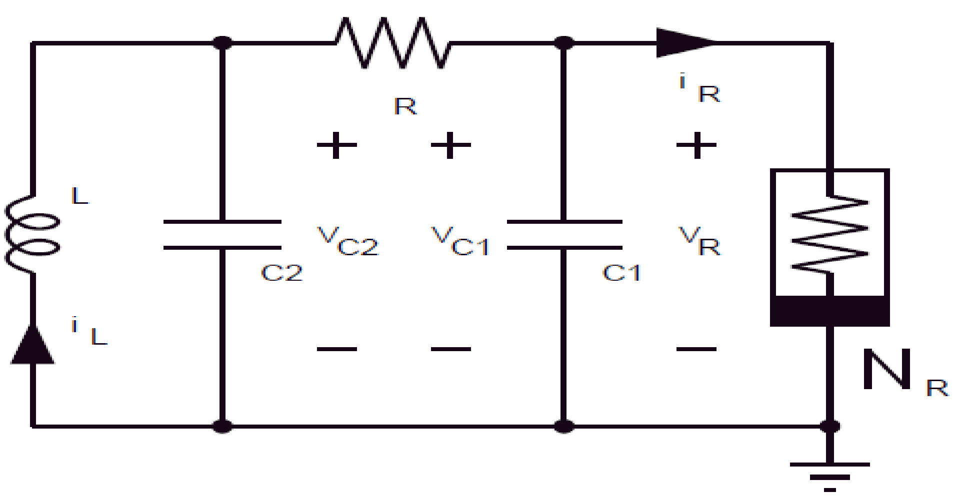
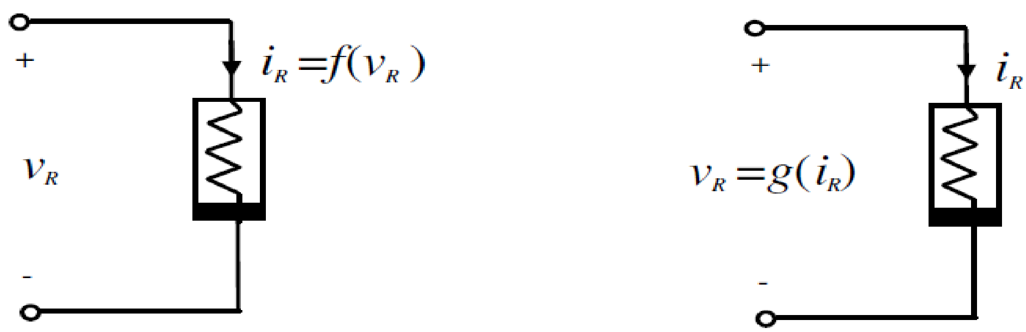


Fig- 3.11 circuit diagram of Chua's circuit

3.5 Chua Diode

The term chua's diode is a general description for a two terminal non linear resistor with a piecewise linear characteristics. In the literature chua's diode is defined in 2 forms. As shown below, the first type of chua diode is a voltage controlled non linear element characterized by $i_R=f(v_R)$ and the other type is a current controlled non linear element characterized by $v_R=g(i_R)$.



(a) VOLTAGE CONTROLLED

(b) CURRENT CONTROLLED

Fig-3.12 voltage and current controlled chua diode

The value of N_R depends on $f(v_1)$. Equation (3.7) is $f(v_1)$. The values of Ga, Gb and Bp are explained below

$$f(v_1)=Gbv_1+\frac{1}{2}(Ga-Gb)(|v_1+Bp|-|v_1-Bp|) \qquad \text{.....................................(3.7)}$$

now equation (3.7) can also be written according to the following equation, this is because Gb and Bp can have different values. The steepness of the graph and the breakpoints can have different values.

$f(v_1)=Gb(v_1)v_1+(Ga-Gb(v_1))f_1(v_1)$ ……….………(3.8)

for $f_1(v_1)$ is

$f_1(v_1)=\{ v_1 \quad \text{if} \mid v_1 \mid < Bp(v_1) \mid \text{sign}(v_1)Bp(v_1) \quad \text{otherwise} \}$ …..(3.9)

where

$Gb(v_1)=\frac{1}{2}(Gb+(1+\text{sign}(v_1))+Gb-(1-\text{sign}(v_1))$

$=\{Gb+ \quad if\ v_1 > 0 | Gb- \quad if\ v_1 < 0\}$..(3.10)

$Bp(v_1)=\frac{1}{2}(Bp+(1+\text{sign}(v_1))+Bp-(1-\text{sign}(v_1))$

$=\{Bp+ \quad if\ v_1 > 0 | Bp- \quad if\ v_1 < 0\}$ …………………………... (3.11)

These functions are implemented in MATLAB. The values for *Ga*,*G*b+, *Gb*-,*Bp+and Bp-* are taken from various implementations of chua's circuit and applied to get various V-I characteristics of chua's circuit. Results obtained after simulation of these functions and slopes has been incorporated in chapter 5

The figure is shown below, the value of *Ga*, *Gb*, and *Bp*, are also shown in the figure, as said above

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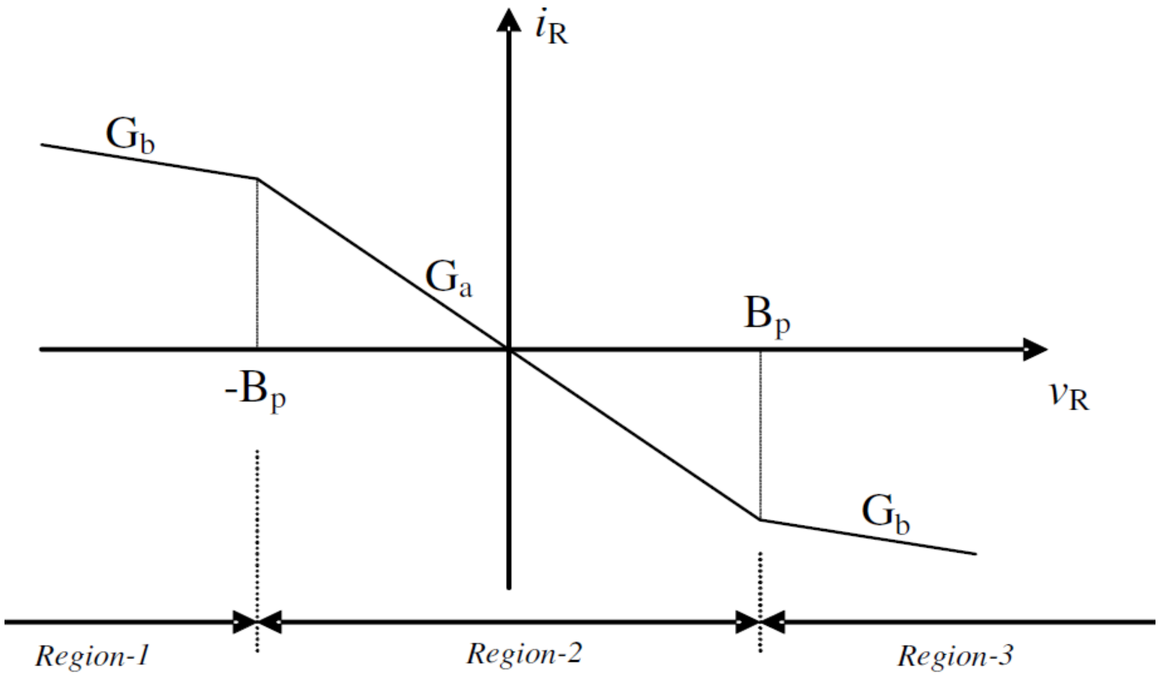


Fig-3.13 V-I characteristics of chua's diode

CHAPTER 4.

MEMRISTANCE DRIVEN CHUA'S CIRCUIT

4.1 General

Memristors are a class of passive two terminal circuit elements that maintain a functional relationship between the time integrals of current and voltage. This function is called **memristance**, is similar to variable resistance. Chua deduced the existence of memristors from the mathematical relationship between the circuit elements. The four circuit quantities i.e charge, current, voltage and magnetic flux, can be related to other in six ways. Two quantities are covered with basic physical laws and three are covered by known circuit elements. After analyzing the above theory of circuit elements, Chua proposed the memristor as a relationship element between charge and flux.

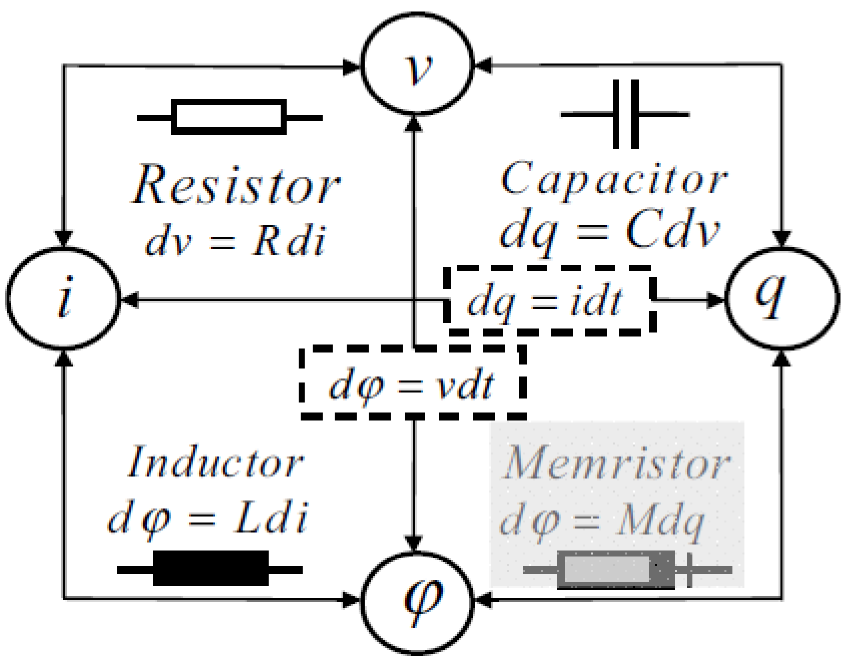


Fig-4.1 memristor as a relationship element between flux and charge

In memristor the resistance changes according to the voltage applied to the system via the migration of atomic defects which consists in the absence of some oxygen atoms in certain region of the film. Due to this effect when the power source is turned off the oxygen vacancies do not easily migrate back to the original position and the system maintains its new resistance state.

4.2 Memdevices

Apart from memristor, there are two more devices introduced after the discovery of memristor. Authors have generalized the concept of memory devices to capacitor and inductor[3]. So, two new memdevices were introduced in this way.

- Memcapacitor
- Meminductor

In these devices the capacitance depends on state of the system while the inductance rely on history of the system. Also these elements show pinched hysteric loops in the two constitutive variables-

Charge-voltage : for Memcapacitor

Current flux : for Meminductor

Current voltage : for Memristor

Symbols of all the three memdevices are shown below:-

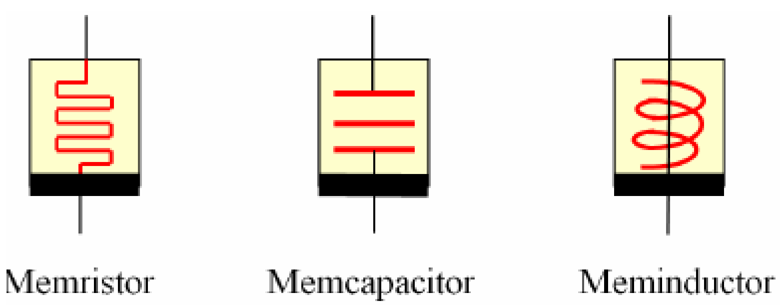


Fig-4.2 symbols of memdevices

4.3 Principle of Memristor

The memristor is composed of a thin (5 nm) titanium dioxide film between two electrodes . Initially there are two layers to the film, one of which has a slight depletion of oxygen atoms. The oxygen vacancies act as charge carriers, meaning that the depleted layer has a much lower resistance that the non-depleted layer. When an electric field is applied the oxygen vacancies drift changing the boundary between the high-resistance and low-resistance layers. Thus the resistance of the film as a whole is dependent on how much charge has been passed through it in a particular direction, which is reversible by changing the direction of current. Since the memristor displays fast ion conduction at nanoscale, it is considered a nanoionic device. Fig- 4.3 shows the final memristor component.

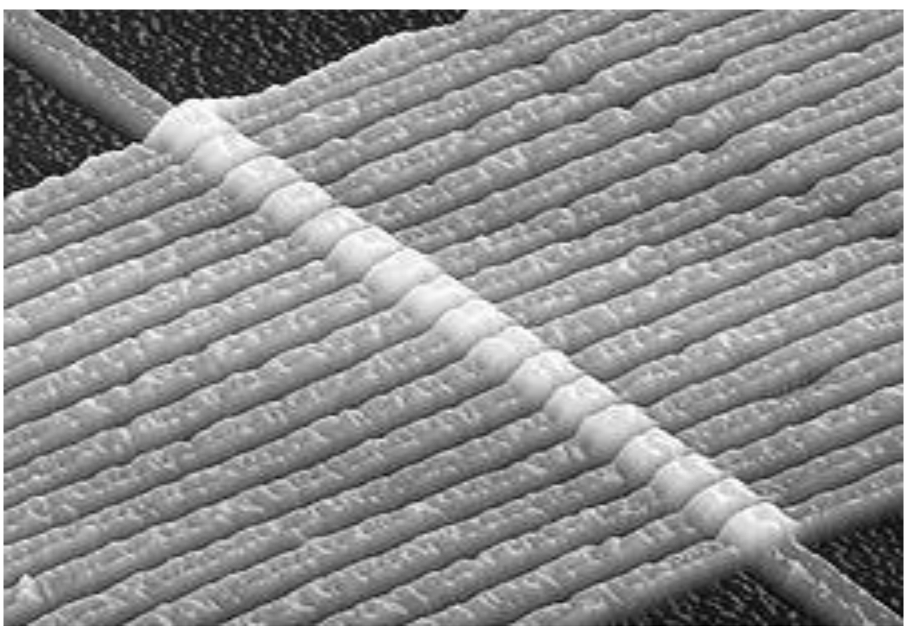


Fig-4.3 final memristor component

4.4 Analogy of Memristor

A common analogy for a resistor is a pipe that carries water. The water itself is analogous to electrical charge, the pressure at the input of the pipe is similar to voltage, and the rate of flow of the water through the pipe is like electrical current.

Just as with an electrical resistor, the flow of water through the pipe is faster if the pipe is shorter and/or it has a larger diameter. An analogy for a memristor is an interesting kind of pipe that expands or shrinks when water flows through it.

If water flows through the pipe in one direction, the diameter of the pipe increases, thus enabling the water to flow faster. If water flows through the pipe in the opposite direction, the diameter of the pipe decreases, thus slowing down the flow of water. If the water pressure is turned off, the pipe will retain it most recent diameter until the water is turned back on.

Thus, the pipe does not store water like a bucket (or a capacitor) – it remembers how much water flowed through it.

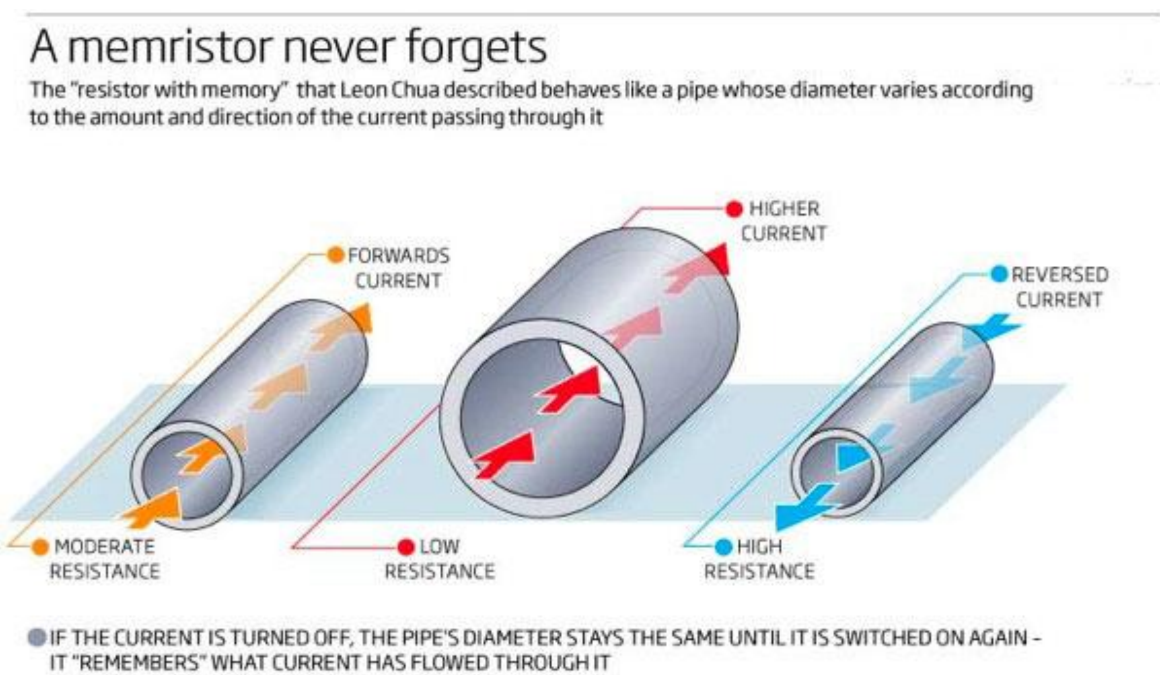


Fig-4.4 an analogy of memristor

4.5 Construction of Memristor

Titanium dioxide, also known as titanium(IV) oxide or titania, is the naturally occurring oxide of titanium, chemical formula TiO_2.

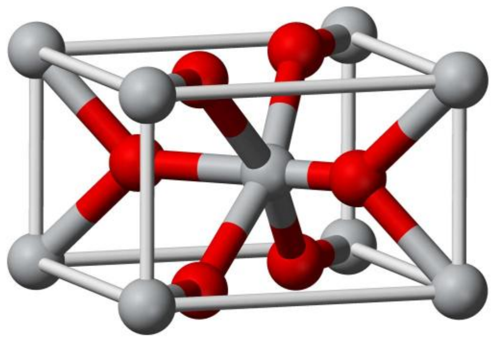


Fig-4.5 molecular structure of TiO_2

Titanium dioxide occurs in nature as well-known minerals rutile, anatase and brookite, and additionally as two high pressure forms, a monoclinic baddeleyite-like form and an orthorhombic α-PbO_2-like form, both found recently at the Ries crater in Bavaria. The most common form is rutile. which is also the most stable form. Anatase and brookite both convert to rutile upon heating. Rutile, anatase and brookite all contain six coordinated titanium.

Titanium dioxide is also used as a material in the memristor. It can be employed for solar energy conversion based on dye, polymer, or quantum dot sensitized nanocrystalline TiO_2 solar cells using conjugated polymers as solid electrolytes.

Semiconductors are doped to make them either p-type or n-type. For example, if silicon is doped with arsenic, it become n-type. However, when we apply an electric field to piece of n-type silicon, the ionized arsenics atoms sitting inside the silicon lattice will not move. We do not want them to move, in any case.

4.6 Working of memristor

Like silicon, titanium dioxide (TiO_2) is a semiconductor, and in its pure state it is highly resistive. However, it can be doped with other elements to make it very conductive. In TiO_2, the dopants don't stay stationary in a high electric field. They tend to drift in the direction of the current. Such mobility is poison to a transistor, but it turns out that's exactly what makes a memristor work. Putting a bias voltage across a thin film of TiO_2 semiconductor that has dopants only on one side causes them to move into the pure TiO_2 on the other side and thus lowers the resistance. Running current in the other direction will then push the dopants back into place, increasing the TiO_2's resistance.

Pure titanium dioxide (TiO_2), which is also a semiconductor, has high resistance, just as in the case of intrinsic silicon, and it can also be doped to make it conducting. If an oxygen atom, which is negatively charged, is removed from its substantial site in TiO_2, a positively charged oxygen vacancy is created(V_0^+) is created, which act as a donor of electrons. These positively charged oxygen vacancies (V_0^+) can be in the direction of current applying electric field. Taking advantage of this ionic transport, a sandwich of thin conducting and non-conducting layers of TiO_2 was used to release memristor.

Consider, we have two thin layers of TiO_2, one highly conducting layer with lots of oxygen vacancies(V_0^+) and the other layer undoped, which is highly resistive. Suppose that good ohmic contact are formed using platinum electrodes on either side of sandwich of TiO_2. The electronics barrier between the undoped TiO_2 and the metal looks broader. The situation remains the same, even when a negative potential is applied to electrode A, because the positively charged oxygen vacancies(V_0^+) are attracted towards electrode A and the length of undoped region increases. Under these conditions the electronics barrier at the undoped TiO_2 and the metal is still too wide and it will be difficult for the electrons to cross over the barrier.

However, when a positive potential is applied at electrode A the positively charged oxygen vacancies are repelled and moved into the undoped TiO_2. This ionic movement towards electrode B reduces the length of undoped region. When more positively charged oxygen vacancies(V_0^+) reach the TiO_2 metal interface, the potential barrier for the electrons become very narrow, as shown, making tunneling through the barrier a real possibility. This leads to a large current flow, making the device turn ON. In this case, the positively charged oxygen vacancies (V_0^+) are present across the length of device. When the polarity of the applied voltage is reversed, the oxygen vacancies can be pushed back into their original place on the doped side, restoring the broader electronic barrier at TiO_2 metal interface. This forces the device to turn OFF due to an increase in the resistance of the device and reduce possibility for carrier tunneling .

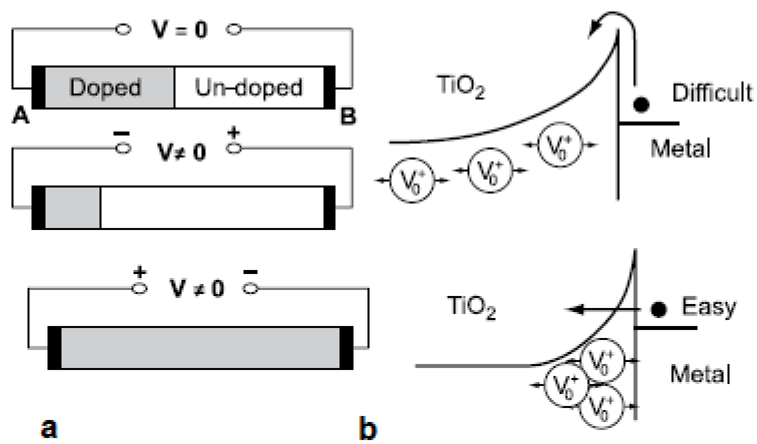


Fig-4.6 (a) broader electronic barrier when a negative potential is applied to electrode A (b) narrow electronic barrier when a positive potential is applied to electrode A

4.7 Basic Model of Memristor

The device is an electrically switchable semiconductor thin film sandwiched between two metal contacts. The semiconductor thin film has a certain length D, and consists of doped and undoped region. The internal state variable w, represents the length of the doped region. The doped region has a low resistance while the resistance of the undoped region is higher. When an external voltage bias is applied across the device, the length w will change due to charge dopant drifting. Hence the device's total resistivity changes.

- If the doped region extends to the full length D, that is w/D=1, the total resistivity of the device would be dominated by low resistivity region with the value measured to be Ron.
- Similarly, when the undoped region extends to full length D, that is w/D=0, the total resistance is denoted as Roff.

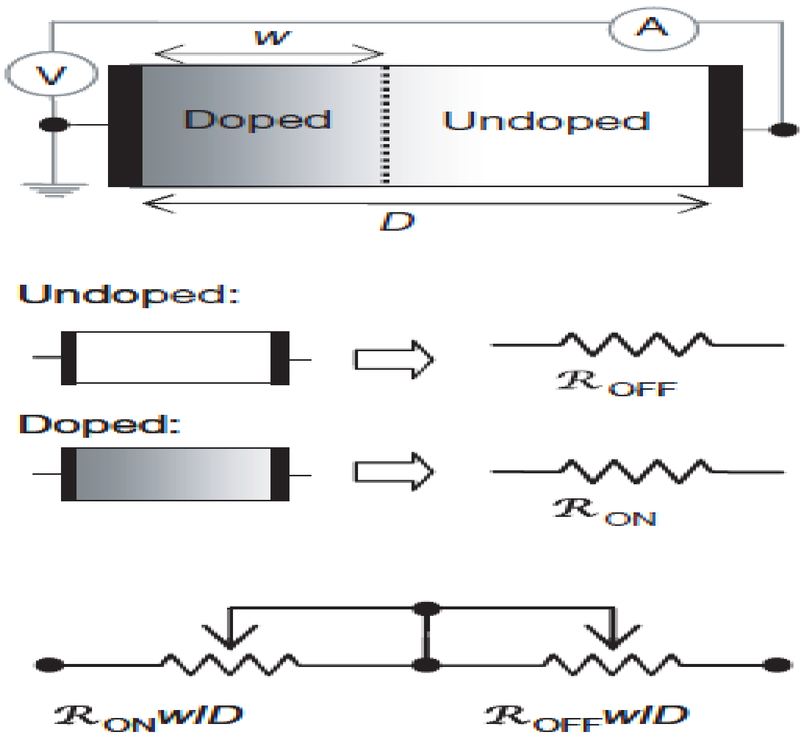


Fig-4.7 memristor equivalent circuit

4.8 memristor : an overview of piecewise linear characteristic

The HP memristor is a passive two terminal electronic device described by non linear constitutive relation between the device terminal voltage and terminal current.

$$v = M(q)i \quad \text{or} \quad i = W(\Phi)v \qquad \text{.........................(4.1)}$$

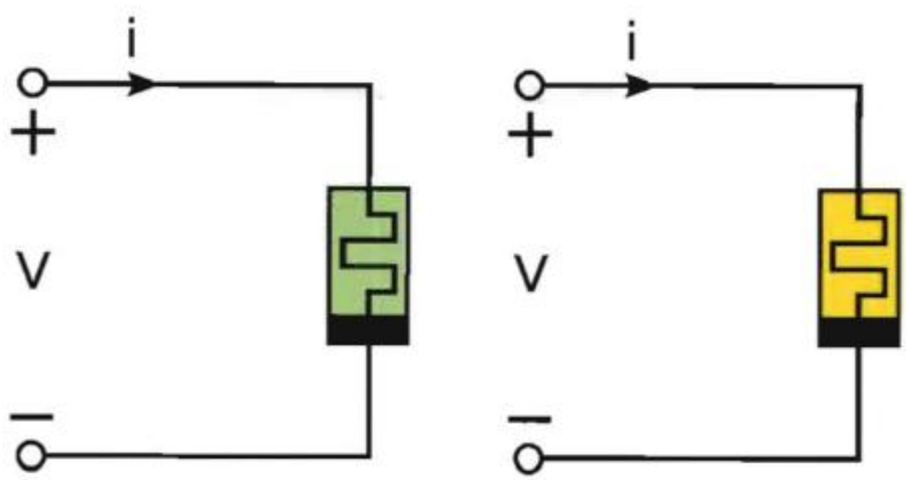
the two nonlinear functions M(q) and W(Φ), called the memristance and memductance are defined by-

$$M(q) \triangleq \frac{d\Phi}{dq}(q) \qquad \text{... ....(4.2)}$$

And

$$W(\Phi) \triangleq \frac{dq}{d\Phi}(\Phi) \qquad \text{....................(4.3)}$$

Representing the slope of a scalar function Φ=Φ(q) and q=q(Φ) respectively called the memristor constitutive relation.



(a) **V=M(q)i** **(b) i=W(Φ)V**

(a) Charge controlled memristor & (b) flux controlled memristor

Fig-4.8 symbols of charge controlled and flux controlled memristor

A memristor characterized by a differentiable q-Φ characteristic curve is passive if, and only if, its small signal memristance M(q) is non negative; i.e.

$$W(\Phi)=\frac{dq}{d\Phi}(\Phi) \geq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(4.4)}$$

Assuming that the memristor is characterized by 'monotoneincreasing' and piecewise linear characteristic as shown in fig-4.8and is given by-

$$q(\Phi)=b\Phi+0.5(a-b)(|\Phi+1|-|\Phi-1|) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(4.5)}$$

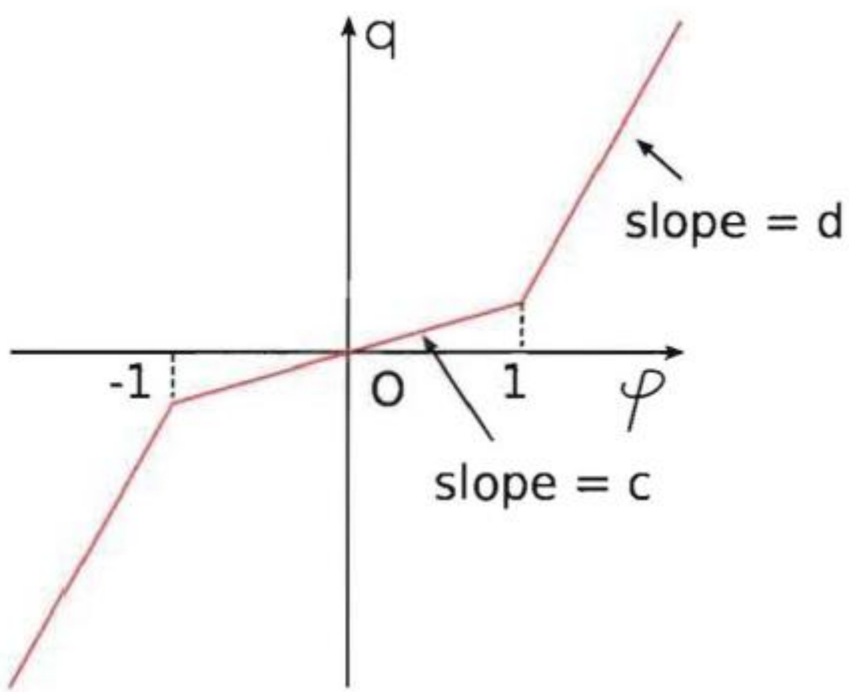


Fig-4.9 the constitutive relation of a monotonously increasing piecewise linear flux controlled memristor.

Where c,d>0 therefore ,the memductance W(Φ) is given by-

$$W(\Phi) = \frac{dq}{d\Phi}(\Phi) = c, |w| < 1 \quad \text{……………………….(4.6)}$$

$$W(\Phi) = \frac{dq}{d\Phi}(\Phi) = d, |w| > 1 \quad \text{……………………..(4.7)}$$

The instantaneous power dissipated by the above memductance element is given by-

$$p(t)=W(\Phi(t))v(t)^2 \geq 0 \quad \text{……………………………..(4.8)}$$

the energy flow into the memristor from time t_o to t satisfies

$$\int_0^t p(\tau)d\tau \geq 0 \quad \text{……………………………(4.9)}$$

Such that for all $t \geq t_o$. thus the memristor constitutive relation in fig-4.9 is passive. As we have seen above the nonlinearity in above section shown is passive non linearity. In the next section we will discuss the chua's circuit that contains active non linearity.

4.9 Canonical Memristor Oscillator- fourth order

Consider the canonical Chua's oscillator in fig shown below.

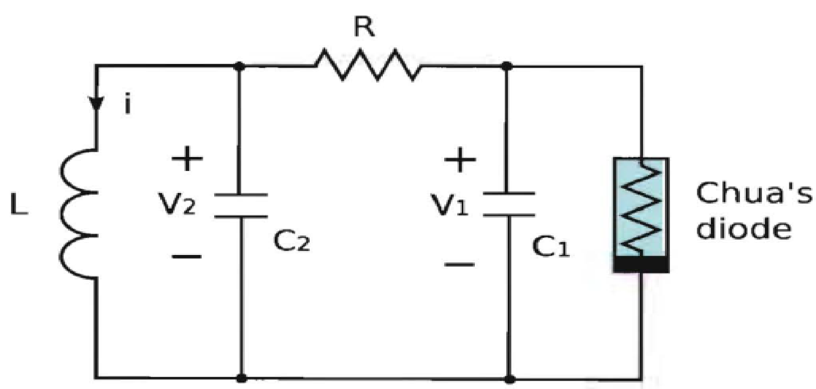


Fig-4.10 canonical chua's oscillator

On replacing the chua diode in the above fig. , with a flux controlled memristor, we can obtain the desired module of the chua's circuit.

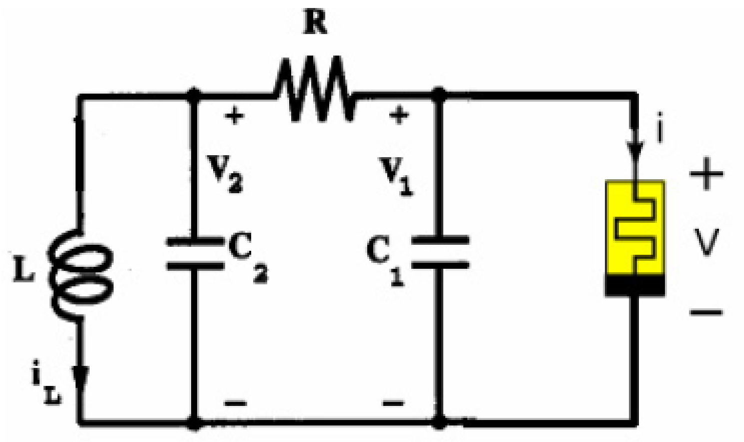


Fig- 4.11 chua's circuit with memristor(flux controlled)

Now on analyzing the above chua's circuit with memristance we obtain that active non linearity is still present(a mandatory condition for obtaining chaos) in the derived circuit. As on writing the differential equation of the above circuit we get the following four equations.

$$\frac{d\Phi}{dt}= v_1(t)$$

$$\frac{dv_1}{dt}=\frac{1}{C_1}[(v_2-v_1)–W(\Phi(t)).v_1(t)] \qquad \text{..........................(4.10)}$$

$$\frac{dv_2}{dt}=\frac{1}{C_2}[(v_1-v_2)/R – i_L(t)]$$

$$\frac{di_L}{dt}=\frac{v_2}{L}$$

Now on simulating the circuit with 2 set of values, following chaotic patterns are observed. The next chapter discusses the result obtained with different set of values i.e. two cases has been discussed. Original case with a particular set of value and then improved set of values.

CHAPTER-5

SIMULATION AND RESULTS

5.1 Simulation of V-I characteristics of chua's circuit using matlab simulink.

The plots shown below are the graphs of V-I characteristics of various implementations of chua's circuit that has been implemented so far by various authors. Chua's circuit can be implemented with various configurations of basic electronic components arranged to produce the desired chaotic pattern(proof of non linearity).

The piecewise linear characteristics have different values of slopes for different implementations. Using data from various implementations and simulating the values of slopes using matlab simulink software following results have been obtained.

Table A

Ga	-0.757mS
Gb+	-0.410mS
Gb-	-0.410mS
Bp+	1.70V
Bp-	1.70V

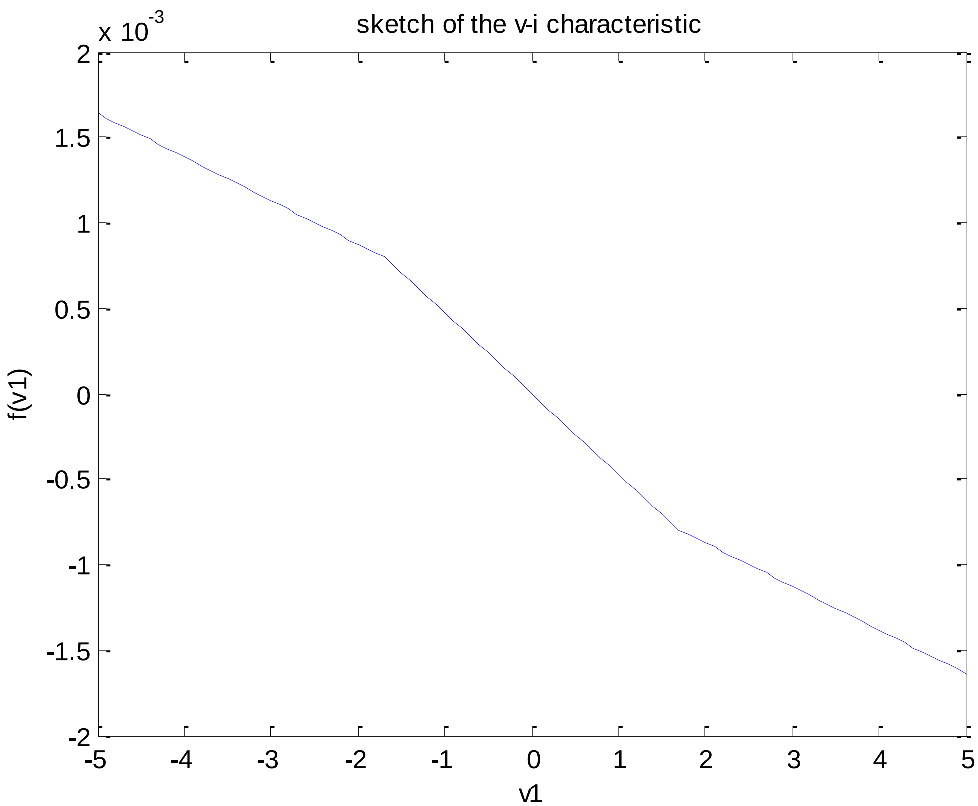


Fig-5.1 V-I characteristics of the chua's circuit model proposed by Santobani

Table B

Ga	-0.78
Gb+	-0.41
Gb-	-0.41
Bp+	0.70
Bp-	0.70

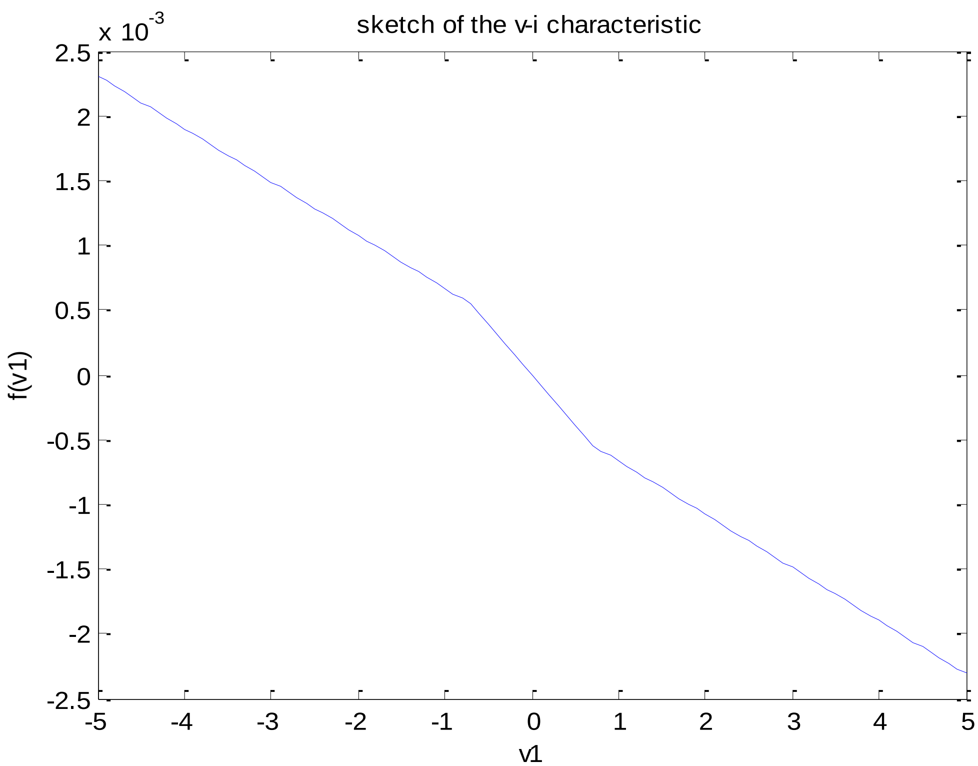


Fig-5.2 V-I characteristics of OTA by CRUZ and CHUA

Table C

Ga	-0.756
Gb+	-0.409
Gb-	-0.409
Bp+	1V
Bp-	1V

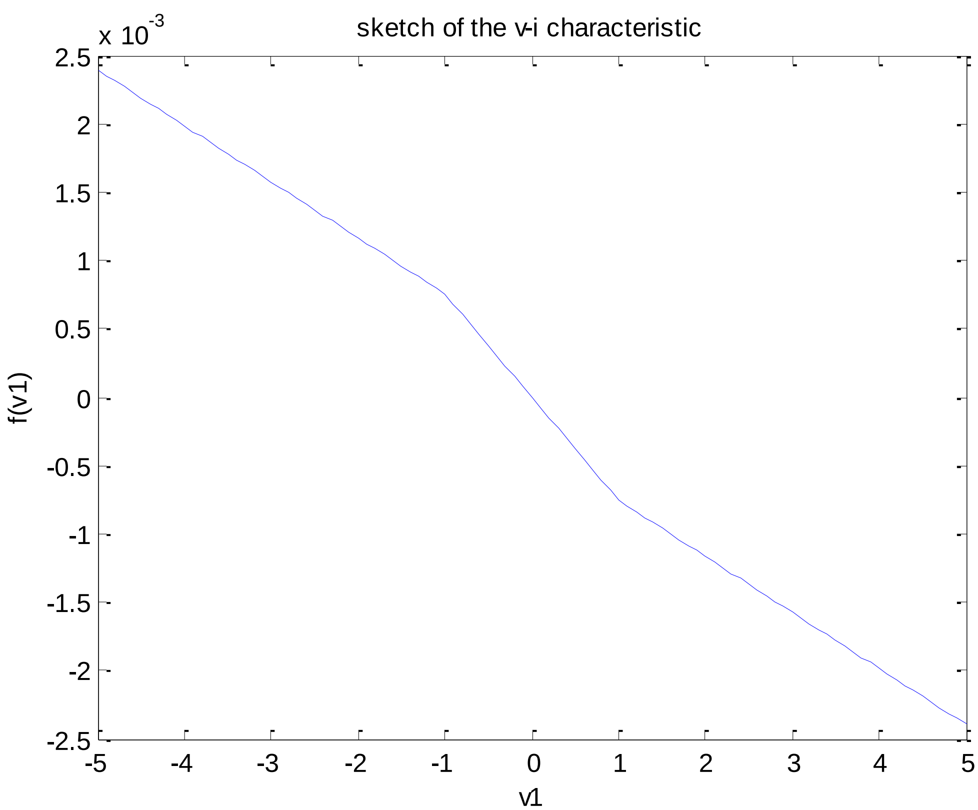


Fig-5.3 V-I Characteristics of chua's circuit by Kennedy

Table D

Ga	-0.8
Gb+	-0.5
Gb-	-0.5
Bp+	1V
Bp-	1V

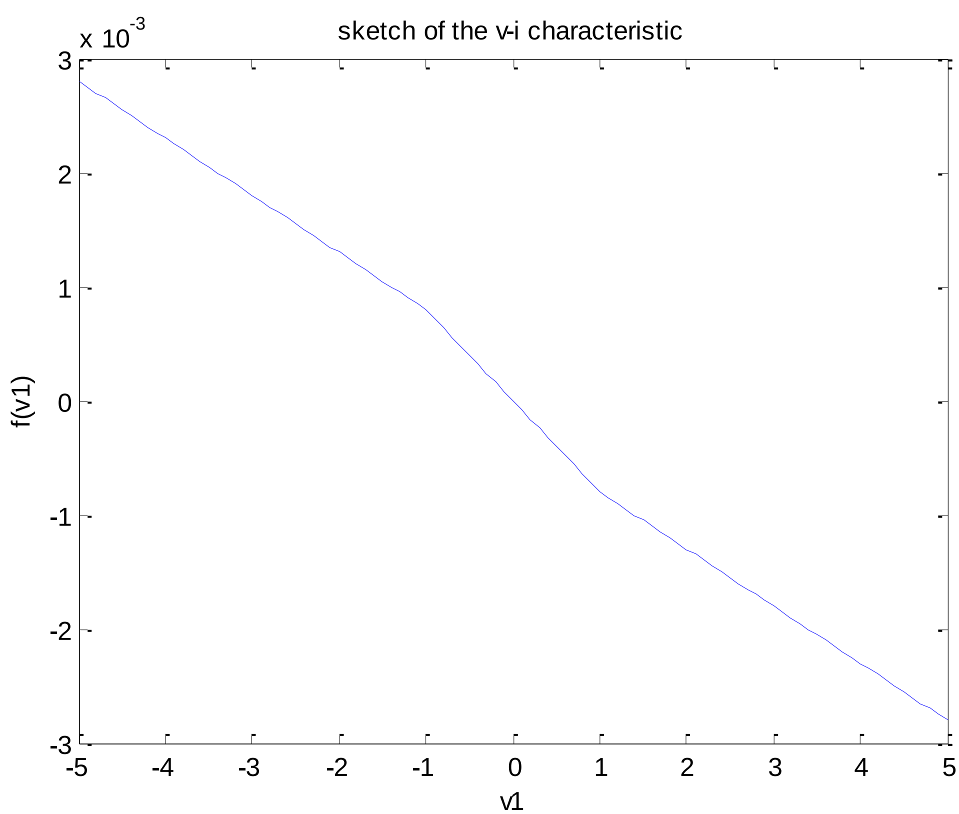


Fig-5.4 V-I characteristics of chua's circuit by Matsumoto

Table E

w	10 nano metre
D	100 nano $cm^2/s/V$
Ron	100 ohm
Roff	16Kohm
ω	3 rad/s

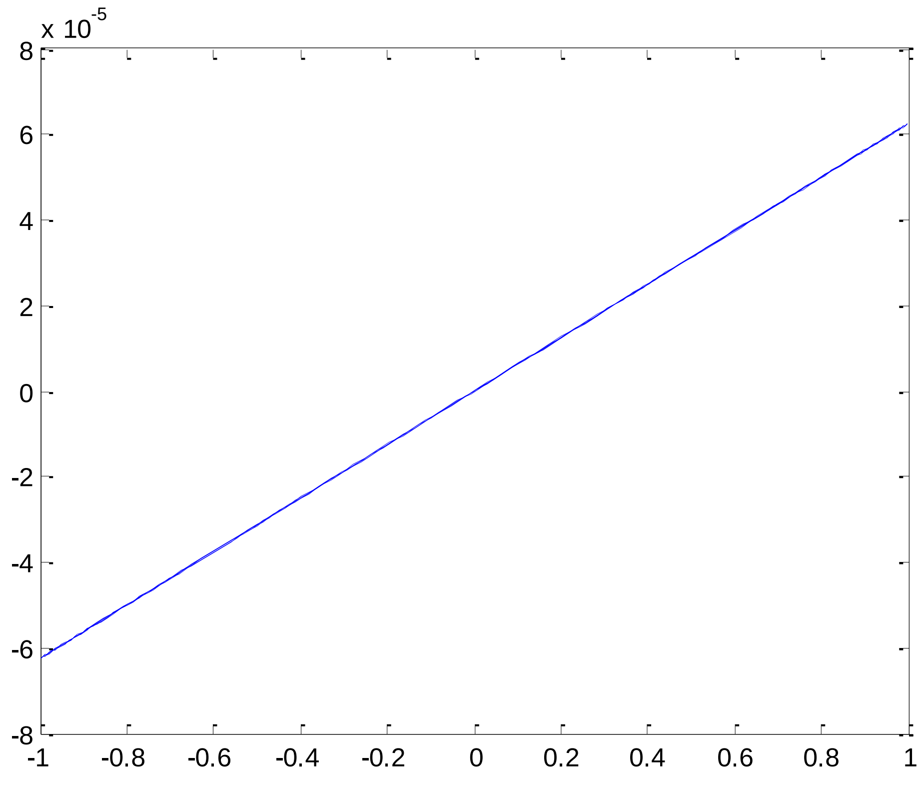


Fig-5.5 current vs. voltage plot of memristor

5.2 Simulation of canonical chua's circuit including a memristor with the following set of values.

1. **Case 1(original values)**

Table 1

C_1	100pico farad
C_2	1 nano farad
L	180 micro henry
R	1.6 kilo ohm
Φ(0)	0
$v_1=v_2$	0
i(0)	0.2

11. Simulation Results for case 1

On simulating the canonical chua's circuit having a memristor in place of chua diode, following plots of current, voltages and flux are obtained. These plots confirm the presence of chaos and a unique pattern is observed which made them highly sophisticated type and can be very useful in the areas of secure communications.

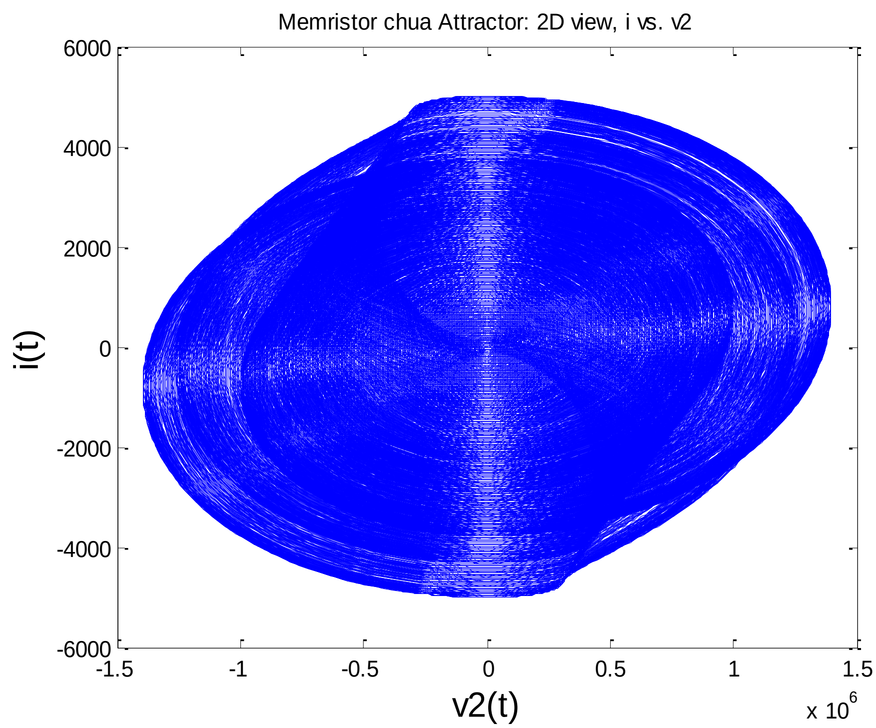


Fig-5.6 memristor plot of current vs. voltage2

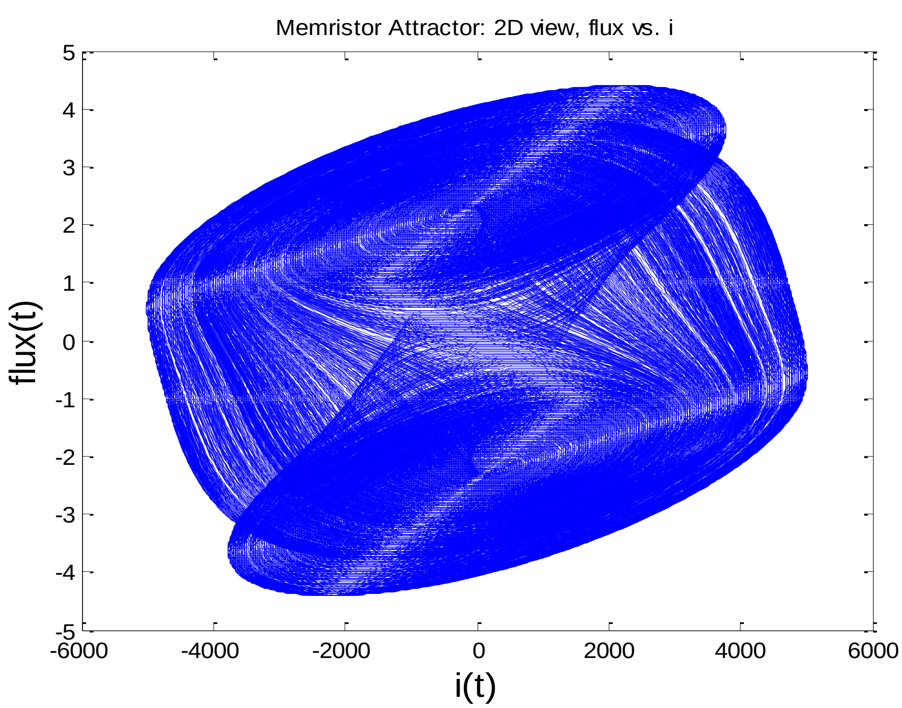


Fig- 5.7 memristor plot of flux vs. current

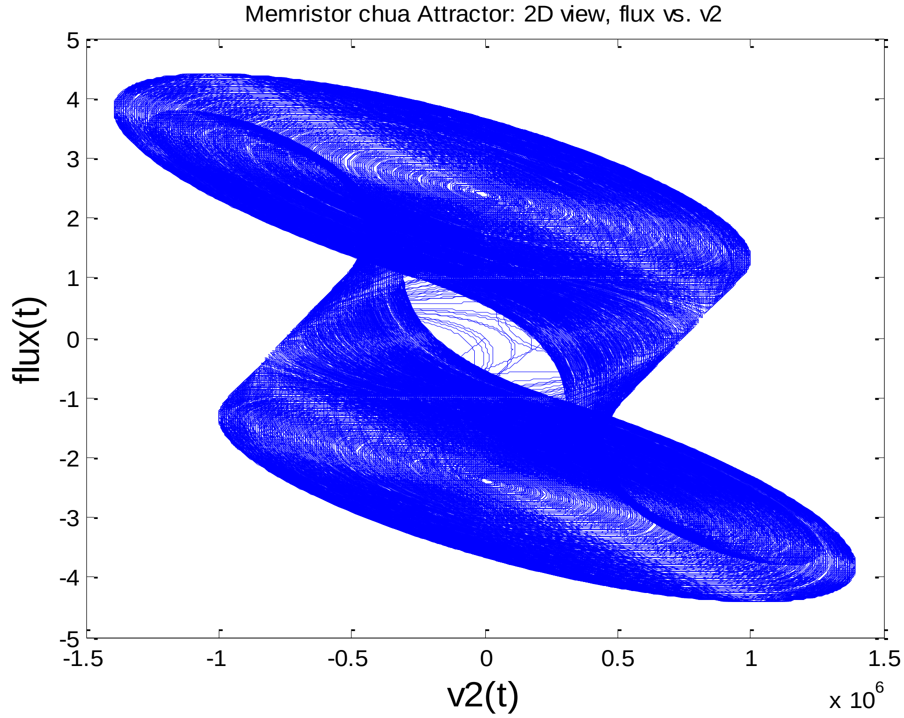


Fig-5.8 memristor plot of flux vs. voltage2

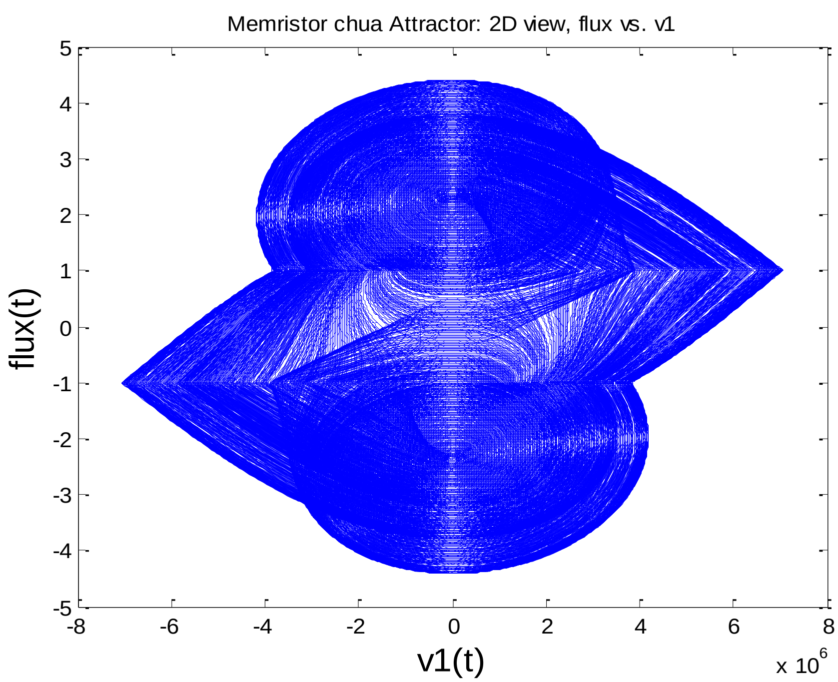


Fig-5.9 memristor plot of flux vs. voltage1

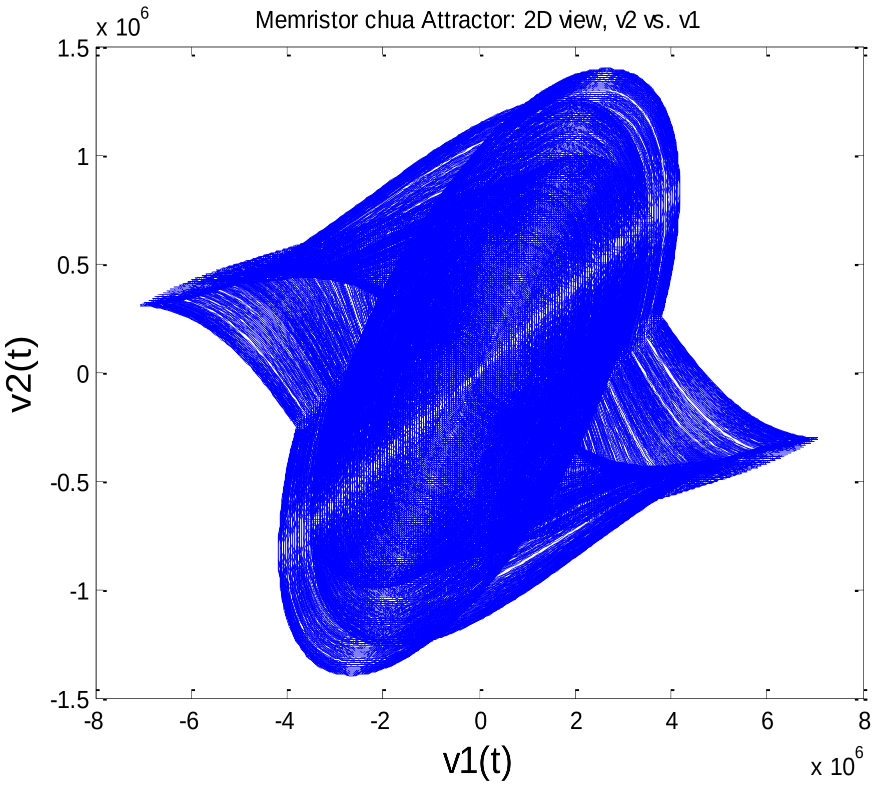


Fig- 5.10 memristor plot of voltage2 vs. voltage1

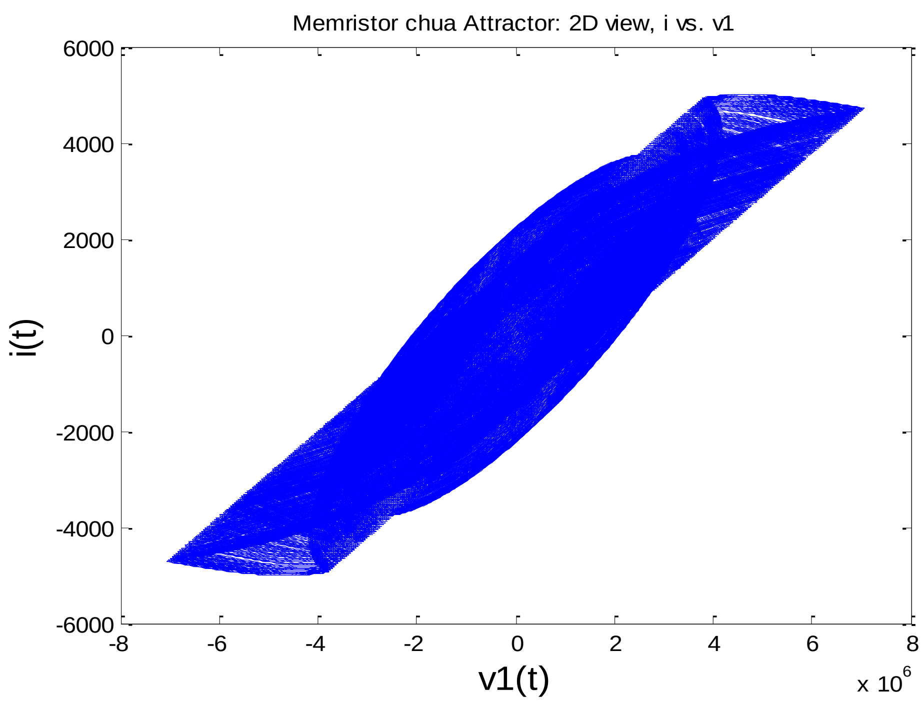


Fig-5.11 memristor plot of current vs. voltage

Case 11 (Improved values).

Table 2

C_1	5.5 nano farad
C_2	49.5 nano farad
L	7.07 milli henri
R	1428 ohms
$\Phi(0)$	0
$v_1=v_2$	0
i(0)	0.1

11 Simulation Results for case 11

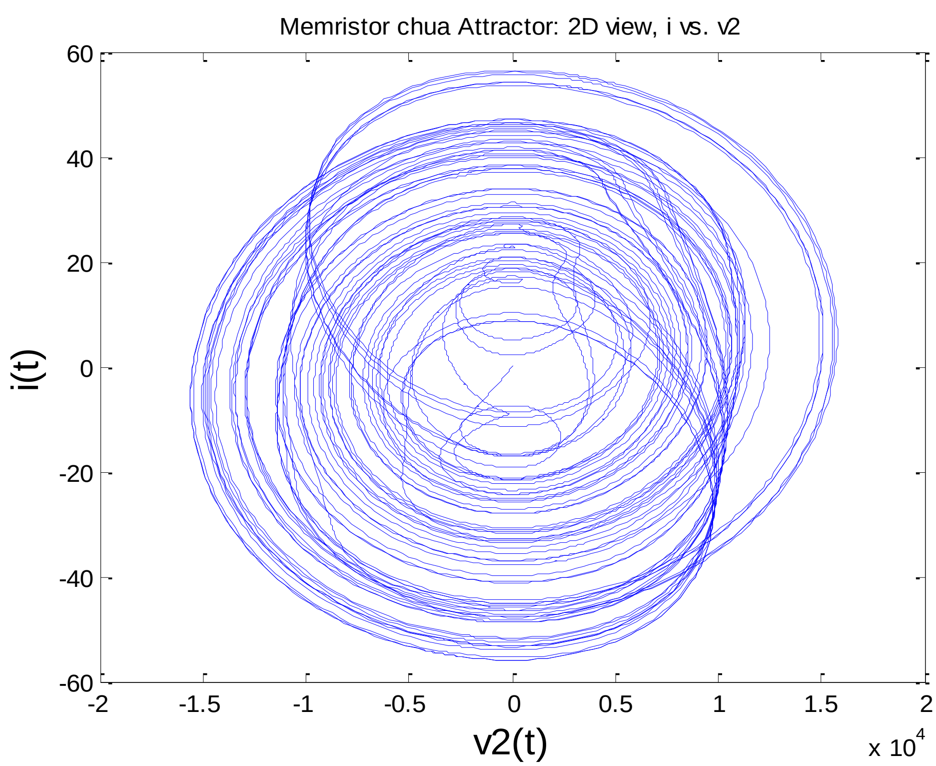


Fig-5.12 memristor plot of current vs. voltage

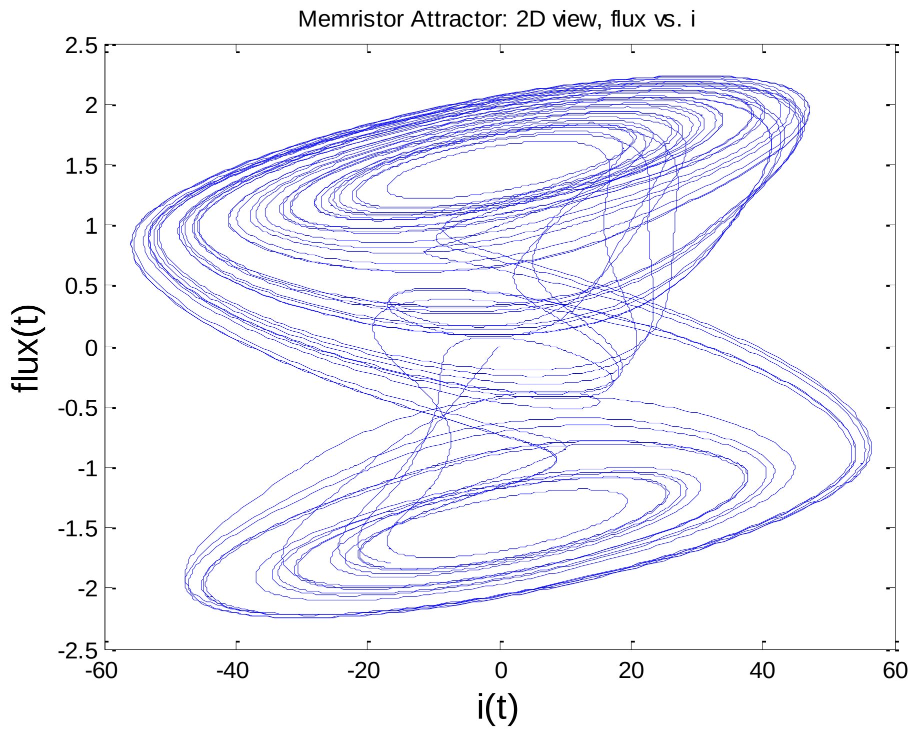


Fig-5.13 memristor plot of flux vs. current

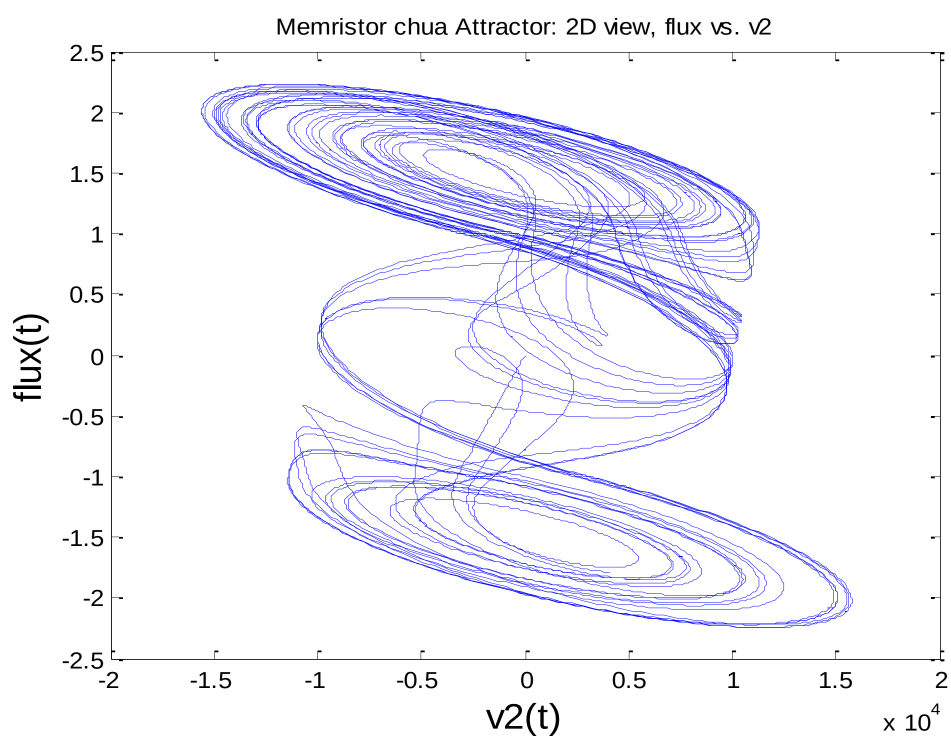


Fig-5.14 memristor plot of flux vs. voltage2

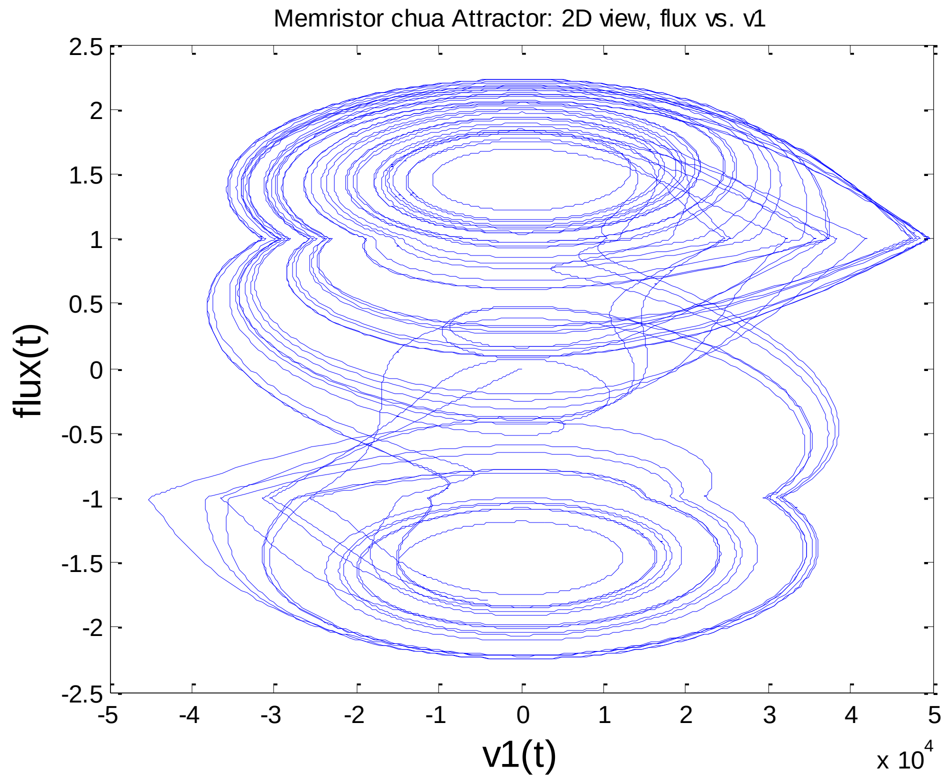


Fig-5.15 memristor plot of flux vs. voltage1

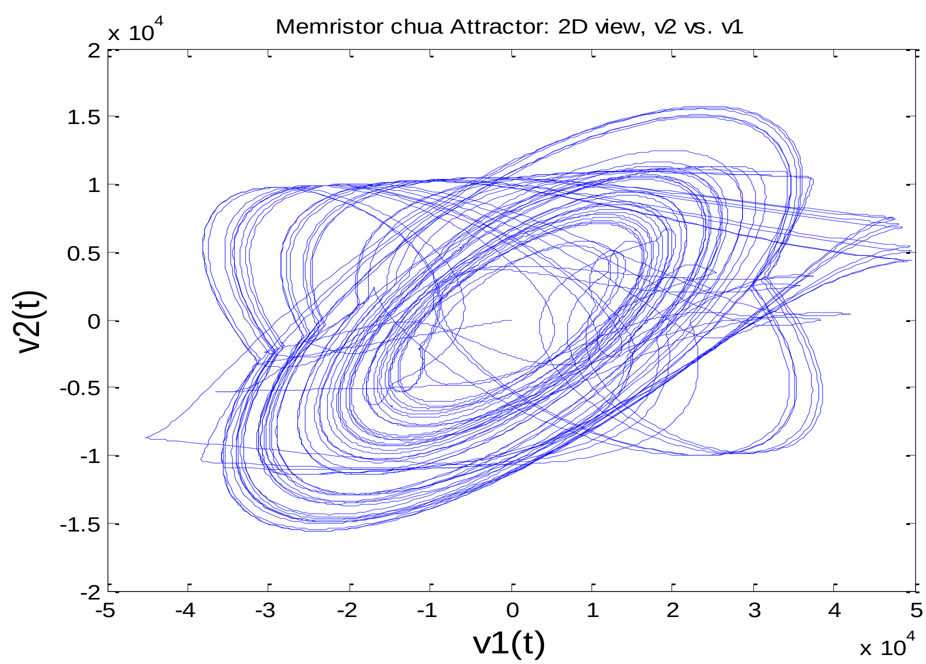


Fig- 5.16 memristor plot of voltage2 vs. voltage1

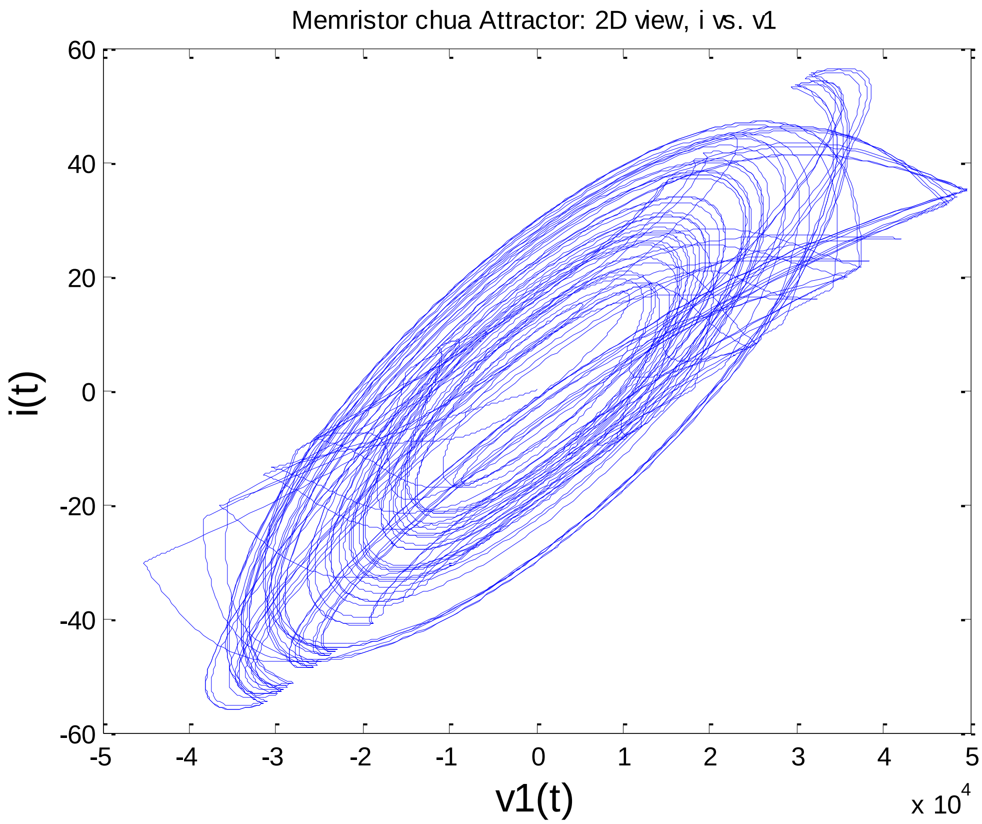


Fig-5.17 memristor plot of current and voltage

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

6.1 CONCLUSION

From the results obtained in chapter 5

We have simulated the chua diode N_R and derived the V-I characteristics of the non linear resistor by taking the data from various implementations of chua's circuit till date. Several other V-I characteristics can also be obtained by using different steepness of slopes and breakpoints values.

In the next section of my work, a brief case study is done of canonical based chua's circuit. Two cases has been discussed and different chaotic patterns has been observed on simulating the differential equations of the canonical based chua's circuit containing memristor.

Although, the patterns obtained in first case confirms the presence of chaos in the memristance based circuit but the second case provides us the proper chaotic patterns of memristance driven chaotic circuit.

6.2 FUTURE SCOPE

The attractors obtained from the above case studies have unique and sophisticated structure. So we can use these distinct pattern for secure communication applications. This field has enormous scope for future work in higher dimensional circuits.

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